

## 13.5 Curl and Divergence

**A** [Click here for answers.](#)

**1–15** Find (a) the curl and (b) the divergence of the vector field.

1.  $\mathbf{F}(x, y, z) = xy \mathbf{i} + yz \mathbf{j} + zx \mathbf{k}$
2.  $\mathbf{F}(x, y, z) = (x - 2z) \mathbf{i} + (x + y + z) \mathbf{j} + (x - 2y) \mathbf{k}$
3.  $\mathbf{F}(x, y, z) = xe^y \mathbf{j} + ye^z \mathbf{k}$
4.  $\mathbf{F}(x, y, z) = \frac{x}{z} \mathbf{i} + \frac{y}{z} \mathbf{j} - \frac{1}{z} \mathbf{k}$
5.  $\mathbf{F}(x, y, z) = xe^{yz} \mathbf{i} + ye^{xz} \mathbf{j} + ze^{xy} \mathbf{k}$
6.  $\mathbf{F}(x, y, z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$
7.  $\mathbf{F}(x, y, z) = x^2y \mathbf{i} + yz^2 \mathbf{j} + zx^2 \mathbf{k}$
8.  $\mathbf{F}(x, y, z) = yz \mathbf{i} + xz \mathbf{j} + xy \mathbf{k}$
9.  $\mathbf{F}(x, y, z) = y^2z \mathbf{i} - x^2yz \mathbf{k}$
10.  $\mathbf{F}(x, y, z) = xy \mathbf{j} + xyz \mathbf{k}$
11.  $\mathbf{F}(x, y, z) = \sin x \mathbf{i} + \cos x \mathbf{j} + z^2 \mathbf{k}$
12.  $\mathbf{F}(x, y, z) = e^{xz} \mathbf{i} - 2e^{yz} \mathbf{j} + 3xe^{yz} \mathbf{k}$
13.  $\mathbf{F}(x, y, z) = (x + 3y - 5z) \mathbf{i} + (z - 3y) \mathbf{j} + (5x + 6y - z) \mathbf{k}$
14.  $\mathbf{F}(x, y, z) = xe^y \mathbf{i} - ze^{-y} \mathbf{j} + y \ln z \mathbf{k}$
15.  $\mathbf{F}(x, y, z) = e^{xyz} \mathbf{i} + \sin(x - y) \mathbf{j} - \frac{xy}{z} \mathbf{k}$

**S** [Click here for solutions.](#)

**16–27** Determine whether or not the given vector field is conservative. If it is conservative, find a function  $f$  such that  $\mathbf{F} = \nabla f$ .

16.  $\mathbf{F}(x, y, z) = y \mathbf{i} + x \mathbf{j} + \mathbf{k}$
17.  $\mathbf{F}(x, y, z) = x \mathbf{i} + y \mathbf{j} + x \mathbf{k}$
18.  $\mathbf{F}(x, y, z) = yz \mathbf{i} - z^2 \mathbf{j} + x^2 \mathbf{k}$
19.  $\mathbf{F}(x, y, z) = z \mathbf{i} + 2yz \mathbf{j} + (x + y^2) \mathbf{k}$
20.  $\mathbf{F}(x, y, z) = \cos y \mathbf{i} + \sin x \mathbf{j} + \tan z \mathbf{k}$
21.  $\mathbf{F}(x, y, z) = x \mathbf{i} + e^y \sin z \mathbf{j} + e^y \cos z \mathbf{k}$
22.  $\mathbf{F}(x, y, z) = yz \mathbf{i} + (y^2 + xz) \mathbf{j} + xy \mathbf{k}$
23.  $\mathbf{F}(x, y, z) = zx \mathbf{i} + xy \mathbf{j} + yz \mathbf{k}$
24.  $\mathbf{F}(x, y, z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$
25.  $\mathbf{F}(x, y, z) = xy^2z^3 \mathbf{i} + 2x^2yz^3 \mathbf{j} + 3x^2y^2z^2 \mathbf{k}$
26.  $\mathbf{F}(x, y, z) = e^x \mathbf{i} + e^z \mathbf{j} + e^y \mathbf{k}$
27.  $\mathbf{F}(x, y, z) = yze^{xz} \mathbf{i} + e^{xz} \mathbf{j} + xye^{xz} \mathbf{k}$

## Answers

**E** Click here for exercises.

- (a)  $-y\mathbf{i} - z\mathbf{j} - x\mathbf{k}$  (b)  $x + y + z$
- (a)  $-3\mathbf{i} - 3\mathbf{j} + \mathbf{k}$  (b) 2
- (a)  $e^z\mathbf{i} + e^y\mathbf{k}$  (b)  $xe^y + ye^z$
- (a)  $\frac{y}{z^2}\mathbf{i} - \frac{x}{z^2}\mathbf{j}$  (b)  $\frac{2z+1}{z^2}$
- (a)  $x(ze^{xy} - ye^{xz})\mathbf{i} + y(xe^{yz} - ze^{xy})\mathbf{j}$   
 $+ z(ye^{xz} - xe^{yz})\mathbf{k}$   
 (b)  $e^{yz} + e^{xz} + e^{xy}$
- (a)  $\mathbf{0}$  (b) 3
- (a)  $-(2yz\mathbf{i} + 2xz\mathbf{j} + x^2\mathbf{k})$   
 (b)  $2xy + z^2 + x^2$
- (a)  $\mathbf{0}$  (b) 0
- (a)  $-x^2z\mathbf{i} + (y^2 + 2xyz)\mathbf{j} - 2yz\mathbf{k}$   
 (b)  $-x^2y$
- (a)  $xz\mathbf{i} - yz\mathbf{j} + y\mathbf{k}$  (b)  $x(1+y)$
- (a)  $-\sin x\mathbf{k}$   
 (b)  $\cos x + 2z$
- (a)  $(3xe^y + 2ye^{yz})\mathbf{i} + (xe^{xz} - 3e^y)\mathbf{j}$   
 (b)  $ze^{xz} - 2ze^{yz}$

**S** Click here for solutions.

- (a)  $5\mathbf{i} - 0\mathbf{j} - 3\mathbf{k}$  (b)  $-3$
- (a)  $(e^{-y} + \ln z)\mathbf{i} - xe^y\mathbf{k}$   
 (b)  $e^y + ze^{-y} + \frac{y}{z}$
- (a)  $-\frac{x}{z}\mathbf{i} + \left(xye^{xyz} + \frac{y}{z}\right)\mathbf{j} + [\cos(x-y) - xze^{xyz}]\mathbf{k}$   
 (b)  $yz e^{xyz} - \cos(x-y) + \frac{xy}{z^2}$
- $f(x, y, z) = xy + z + K$
- Not conservative
- Not conservative
- $f(x, y, z) = xz + y^2z + K$
- Not conservative
- $f(x, y, z) = \frac{1}{2}x^2 + e^y \sin z + K$
- $f(x, y, z) = xyz + \frac{1}{3}y^3 + K$
- Not conservative
- $f(x, y, z) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2 + K$
- Not conservative
- Not conservative
- $f(x, y, z) = ye^{xz} + K$

## Solutions

[Click here for exercises.](#)

$$\begin{aligned} 1. \text{ (a) } \operatorname{curl} \mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & yz & zx \end{vmatrix} \\ &= \left[ \frac{\partial}{\partial y}(zx) - \frac{\partial}{\partial z}(yz) \right] \mathbf{i} - \left[ \frac{\partial}{\partial x}(zx) - \frac{\partial}{\partial z}(xy) \right] \mathbf{j} \\ &\quad + \left[ \frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial y}(xy) \right] \mathbf{k} \\ &= (0 - y) \mathbf{i} - (z - 0) \mathbf{j} + (0 - x) \mathbf{k} \\ &= -y \mathbf{i} - z \mathbf{j} - x \mathbf{k} \end{aligned}$$

$$\begin{aligned} \text{(b) } \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(yz) + \frac{\partial}{\partial z}(zx) \\ &= y + z + x = x + y + z \end{aligned}$$

$$\begin{aligned} 2. \text{ (a) } \operatorname{curl} \mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x - 2z & x + y + z & x - 2y \end{vmatrix} \\ &= (-2 - 1) \mathbf{i} - (1 + 2) \mathbf{j} + (1 - 0) \mathbf{k} \\ &= -3 \mathbf{i} - 3 \mathbf{j} + \mathbf{k} \end{aligned}$$

$$\begin{aligned} \text{(b) } \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} \\ &= \frac{\partial}{\partial x}(x - 2z) + \frac{\partial}{\partial y}(x + y + z) + \frac{\partial}{\partial z}(x - 2y) \\ &= 1 + 1 + 0 = 2 \end{aligned}$$

$$\begin{aligned} 3. \text{ (a) } \operatorname{curl} \mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & xe^y & ye^z \end{vmatrix} \\ &= (e^z - 0) \mathbf{i} - (0 - 0) \mathbf{j} + (e^y - 0) \mathbf{k} \\ &= e^z \mathbf{i} + e^y \mathbf{k} \end{aligned}$$

$$\begin{aligned} \text{(b) } \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(0) + \frac{\partial}{\partial y}(xe^y) + \frac{\partial}{\partial z}(ye^z) \\ &= xe^y + ye^z \end{aligned}$$

$$\begin{aligned} 4. \text{ (a) } \operatorname{curl} \mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x/z & y/z & -1/z \end{vmatrix} \\ &= \left(0 + \frac{y}{z^2}\right) \mathbf{i} - \left(0 + \frac{x}{z^2}\right) \mathbf{j} + (0 - 0) \mathbf{k} \\ &= \frac{y}{z^2} \mathbf{i} - \frac{x}{z^2} \mathbf{j} \end{aligned}$$

$$\begin{aligned} \text{(b) } \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} \\ &= \frac{\partial}{\partial x} \left(\frac{x}{z}\right) + \frac{\partial}{\partial y} \left(\frac{y}{z}\right) + \frac{\partial}{\partial z} \left(-\frac{1}{z}\right) \\ &= \frac{1}{z} + \frac{1}{z} + \frac{1}{z^2} = \frac{2z + 1}{z^2} \end{aligned}$$

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$$\begin{aligned} 5. \text{ (a) } \operatorname{curl} \mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xe^{yz} & ye^{xz} & ze^{xy} \end{vmatrix} \\ &= (xze^{xy} - xye^{xz}) \mathbf{i} - (yze^{xy} - xye^{yz}) \mathbf{j} \\ &\quad + (yze^{xz} - xze^{yz}) \mathbf{k} \\ &= x(ze^{xy} - ye^{xz}) \mathbf{i} + y(xe^{yz} - ze^{xy}) \mathbf{j} \\ &\quad + z(ye^{xz} - xe^{yz}) \mathbf{k} \end{aligned}$$

$$\begin{aligned} \text{(b) } \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} \\ &= \frac{\partial}{\partial x}(xe^{yz}) + \frac{\partial}{\partial y}(ye^{xz}) + \frac{\partial}{\partial z}(ze^{xy}) \\ &= e^{yz} + e^{xz} + e^{xy} \end{aligned}$$

$$\begin{aligned} 6. \text{ (a) } \operatorname{curl} \mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} \\ &= (0 - 0) \mathbf{i} + (0 - 0) \mathbf{j} + (0 - 0) \mathbf{k} = \mathbf{0} \end{aligned}$$

$$\begin{aligned} \text{(b) } \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z) \\ &= 1 + 1 + 1 = 3 \end{aligned}$$

$$\begin{aligned} 7. \text{ (a) } \operatorname{curl} \mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & yz^2 & zx^2 \end{vmatrix} \\ &= (0 - 2yz) \mathbf{i} + (0 - 2xz) \mathbf{j} + (0 - x^2) \mathbf{k} \\ &= -(2yz \mathbf{i} + 2xz \mathbf{j} + x^2 \mathbf{k}) \end{aligned}$$

$$\begin{aligned} \text{(b) } \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(yz^2) + \frac{\partial}{\partial z}(zx^2) \\ &= 2xy + z^2 + x^2 \end{aligned}$$

$$\begin{aligned} 8. \text{ (a) } \operatorname{curl} \mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & xz & xy \end{vmatrix} \\ &= (x - x) \mathbf{i} + (y - y) \mathbf{j} + (z - z) \mathbf{k} = \mathbf{0} \end{aligned}$$

$$\begin{aligned} \text{(b) } \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(yz) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(xy) \\ &= 0 + 0 + 0 = 0 \end{aligned}$$

$$\begin{aligned} 9. \text{ (a) } \operatorname{curl} \mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2z & 0 & -x^2yz \end{vmatrix} \\ &= (-x^2z - 0) \mathbf{i} + (y^2 + 2xyz) \mathbf{j} + (0 - 2yz) \mathbf{k} \\ &= -x^2z \mathbf{i} + (y^2 + 2xyz) \mathbf{j} - 2yz \mathbf{k} \end{aligned}$$

$$\begin{aligned} \text{(b) } \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(y^2z) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(-x^2yz) \\ &= 0 + 0 - x^2y = -x^2y \end{aligned}$$

$$10. (a) \operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & xy & xyz \end{vmatrix}$$

$$= xz \mathbf{i} - yz \mathbf{j} + y \mathbf{k}$$

$$(b) \operatorname{div} \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(0) + \frac{\partial}{\partial y}(xy) + \frac{\partial}{\partial z}(xyz)$$

$$= 0 + x + xy = x(1 + y)$$

$$11. (a) \operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \sin x & \cos x & z^2 \end{vmatrix}$$

$$= (-\sin x + 0) \mathbf{k} = -\sin x \mathbf{k}$$

$$(b) \operatorname{div} \mathbf{F} = \nabla \cdot \mathbf{F}$$

$$= \frac{\partial}{\partial x}(\sin x) + \frac{\partial}{\partial y}(\cos x) + \frac{\partial}{\partial z}(z^2)$$

$$= \cos x + 2z$$

$$12. (a) \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{xz} & -2e^{yz} & 3xe^y \end{vmatrix}$$

$$= (3xe^y + 2ye^{yz}) \mathbf{i} + (xe^{xz} - 3e^y) \mathbf{j}$$

$$(b) \nabla \times \mathbf{F} = \frac{\partial}{\partial x}(e^{xz}) + \frac{\partial}{\partial y}(-2e^{yz}) + \frac{\partial}{\partial z}(3xe^y)$$

$$= ze^{xz} - 2ze^{yz}$$

$$13. (a) \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x + 3y - 5z & z - 3y & 5x + 6y - z \end{vmatrix}$$

$$= (6 - 1) \mathbf{i} + (-5 - 5) \mathbf{j} + (0 - 3) \mathbf{k}$$

$$= 5 \mathbf{i} - 0 \mathbf{j} - 3 \mathbf{k}$$

$$(b) \nabla \times \mathbf{F} = \frac{\partial}{\partial x}(x + 3y - 5z) + \frac{\partial}{\partial y}(z - 3y)$$

$$+ \frac{\partial}{\partial z}(5x + 6y - z)$$

$$= 1 - 3 - 1 = -3$$

$$14. (a) \operatorname{curl} \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xe^y & -ze^{-y} & y \ln z \end{vmatrix}$$

$$= (e^{-y} + \ln z) \mathbf{i} - xe^y \mathbf{k}$$

$$(b) \operatorname{div} \mathbf{F} = \frac{\partial}{\partial x}(xe^y) + \frac{\partial}{\partial y}(-ze^{-y}) + \frac{\partial}{\partial z}(y \ln z)$$

$$= e^y + ze^{-y} + \frac{y}{z}$$

$$15. (a) \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{xyz} & \sin(x-y) & -xy/z \end{vmatrix}$$

$$= -\frac{x}{z} \mathbf{i} + \left(xy e^{xyz} + \frac{y}{z}\right) \mathbf{j}$$

$$+ (\cos(x-y) - xze^{xyz}) \mathbf{k}$$

$$(b) \nabla \times \mathbf{F} = \frac{\partial}{\partial x}(e^{xyz}) + \frac{\partial}{\partial y} \sin(x-y) + \frac{\partial}{\partial z} \left(-\frac{xy}{z}\right)$$

$$= yze^{xyz} - \cos(x-y) + \frac{xy}{z^2}$$

$$16. \operatorname{curl} \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & x & 1 \end{vmatrix} = \mathbf{0} \text{ and } \mathbf{F} \text{ is defined on all}$$

of  $\mathbb{R}^3$  with component functions which have continuous partial derivatives, so by (4),  $\mathbf{F}$  is conservative. Thus there exists  $f$  such that  $\mathbf{F} = \nabla f$ . Then  $f_x(x, y, z) = y$  implies  $f(x, y, z) = xy + g(y, z)$  and  $f_y(x, y, z) = x + g_y(y, z)$ . But  $f_y(x, y, z) = x$ , so  $g(y, z) = h(z)$  and  $f(x, y, z) = xy + h(z)$ . Thus  $f_z(x, y, z) = h'(z)$  but  $f_z(x, y, z) = 1$  so  $h(z) = z + K$ . Hence a potential for  $\mathbf{F}$  is  $f(x, y, z) = xy + z + K$ .

$$17. \operatorname{curl} \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & x \end{vmatrix} = -\mathbf{j} \neq \mathbf{0}$$

Hence  $\mathbf{F}$  isn't conservative.

$$18. \operatorname{curl} \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -z^2 & x^2 \end{vmatrix}$$

$$= 2z \mathbf{i} + (y - 2x) \mathbf{j} - z \mathbf{k} \neq \mathbf{0}$$

Hence  $\mathbf{F}$  isn't conservative.

19. Since  $\operatorname{curl} \mathbf{F} = (2y - 2y) \mathbf{i} + (1 - 1) \mathbf{j} + (0 - 0) \mathbf{k} = \mathbf{0}$ ,  $\mathbf{F}$  is defined on all of  $\mathbb{R}^3$ , and the partial derivatives of the components of  $\mathbf{F}$  are continuous,  $\mathbf{F}$  is conservative.

Thus there exists  $f$  such that  $\nabla f = \mathbf{F}$ . Then  $f_x(x, y, z) = z$  implies  $f(x, y, z) = xz + g(y, z)$  and  $f_y(x, y, z) = g_y(y, z)$ . But  $f_y(x, y, z) = 2yz$ , so  $g(y, z) = y^2z + h(z)$  and  $f(x, y, z) = xz + y^2z + h(z)$ . Then  $f_z(x, y, z) = x + y^2 + h'(z)$ , but  $f_z(x, y, z) = x + y^2$  so  $h(z) = K$  and  $f(x, y, z) = xz + y^2z + K$ .

$$20. \operatorname{curl} \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \cos y & \sin x & \tan z \end{vmatrix} = (\cos x - \sin y) \mathbf{k} \neq \mathbf{0}$$

Hence  $\mathbf{F}$  isn't conservative.

21. Since

$$\begin{aligned} \operatorname{curl} \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & e^y \sin z & e^y \cos z \end{vmatrix} \\ &= (e^y \cos z - e^y \cos z) \mathbf{i} + (0 - 0) \mathbf{j} + (0 - 0) \mathbf{k} = \mathbf{0} \end{aligned}$$

$\mathbf{F}$  is defined on  $\mathbb{R}^3$ , and since the partial derivatives of the components of  $\mathbf{F}$  are continuous,  $\mathbf{F}$  is conservative.

Thus there exists  $f$  such that  $\nabla f = \mathbf{F}$ . Then

$$\begin{aligned} f_x(x, y, z) = x &\text{ implies } f(x, y, z) = \frac{1}{2}x^2 + g(y, z) \text{ and} \\ f_y(x, y, z) = g_y(y, z). &\text{ But } f_y(x, y, z) = e^y \sin z, \\ \text{so } g(y, z) = e^y \sin z + h(z) &\text{ and} \end{aligned}$$

$$f(x, y, z) = \frac{1}{2}x^2 + e^y \sin z + h(z). \text{ Thus}$$

$$\begin{aligned} f_z(x, y, z) = e^y \cos z + h'(z). &\text{ But } f_z(x, y, z) = e^y \cos z \\ \text{implies } h(z) = K \text{ and } f(x, y, z) = &\frac{1}{2}x^2 + e^y \sin z + K \text{ is} \\ \text{a potential for } \mathbf{F}. & \end{aligned}$$

22. Since

$$\begin{aligned} \operatorname{curl} \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & y^2 + xz & xy \end{vmatrix} \\ &= (x - x) \mathbf{i} + (y - y) \mathbf{j} + (z - z) \mathbf{k} = \mathbf{0} \end{aligned}$$

$\mathbf{F}$  is defined on  $\mathbb{R}^3$ , and since the partial derivatives of the components of  $\mathbf{F}$  are continuous,  $\mathbf{F}$  is conservative.

Thus there exists  $f$  such that  $\nabla f = \mathbf{F}$ . Then

$$\begin{aligned} f_x(x, y, z) = yz &\text{ implies } f(x, y, z) = xyz + g(y, z) \text{ and} \\ f_y(x, y, z) = xz + g_y(y, z). &\text{ But } f_y(x, y, z) = xz + y^2 \text{ so} \\ g(y, z) = \frac{1}{3}y^3 + h(z) &\text{ and } f(x, y, z) = xyz + \frac{1}{3}y^3 + h(z). \end{aligned}$$

Then  $f_z(x, y, z) = xy + h'(z)$ . But  $f_z(x, y, z) = xy$  so  $h(z) = K$ . Hence  $f(x, y, z) = xyz + \frac{1}{3}y^3 + K$  is a potential for  $\mathbf{F}$ .

$$23. \operatorname{curl} \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ zx & xy & yz \end{vmatrix} = z \mathbf{i} + x \mathbf{j} + y \mathbf{k} \neq \mathbf{0}, \text{ so}$$

$\mathbf{F}$  isn't conservative.

24.  $\operatorname{curl} \mathbf{F} = \mathbf{0}$  by Problem 6(a),  $\mathbf{F}$  is defined on all of  $\mathbb{R}^3$ , and the partial derivatives of the component functions are continuous, so  $\mathbf{F}$  is conservative. Thus there exists a function  $f$  such that  $\nabla f = \mathbf{F}$ . Then  $f_x(x, y, z) = x$  implies

$$f(x, y, z) = \frac{1}{2}x^2 + g(y, z) \text{ and } f_y(x, y, z) = g_y(y, z).$$

$$\text{But } f_y(x, y, z) = y, \text{ so } g(y, z) = \frac{1}{2}y^2 + h(z) \text{ and}$$

$$f(x, y, z) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + h(z). \text{ Thus } f_z(x, y, z) = h'(z)$$

$$\text{but } f_z(x, y, z) = z \text{ so } h(z) = \frac{1}{2}z^2 + K \text{ and}$$

$$f(x, y, z) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2 + K.$$

$$\begin{aligned} 25. \operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy^2z^3 & 2x^2yz^3 & 3x^2y^2z^2 \end{vmatrix} \\ &= (6x^2yz^2 - 6x^2yz^2) \mathbf{i} - (6xy^2z^2 - 3xy^2z^2) \mathbf{j} \\ &\quad + (4xyz^3 - 2xyz^3) \mathbf{k} \neq \mathbf{0} \end{aligned}$$

so  $\mathbf{F}$  isn't conservative.

$$\begin{aligned} 26. \operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^x & e^z & e^y \end{vmatrix} \\ &= (e^y - e^z) \mathbf{i} - (0 - 0) \mathbf{j} + (0 - 0) \mathbf{k} \neq \mathbf{0} \end{aligned}$$

so  $\mathbf{F}$  isn't conservative.

$$\begin{aligned} 27. \operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yze^{xz} & e^{xz} & xye^{xz} \end{vmatrix} \\ &= (xe^{xz} - xe^{xz}) \mathbf{i} \\ &\quad - [(xyze^{xz} + ye^{xz}) - (xyze^{xz} + ye^{xz})] \mathbf{j} \\ &\quad + (ze^{xz} - ze^{xz}) \mathbf{k} \\ &= \mathbf{0} \end{aligned}$$

$\mathbf{F}$  is defined on all of  $\mathbb{R}^3$ , and the partial derivatives of the component functions are continuous, so  $\mathbf{F}$  is conservative.

Thus there exists a function  $f$  such that  $\nabla f = \mathbf{F}$ . Then

$$f_x(x, y, z) = yze^{xz} \text{ implies } f(x, y, z) = ye^{xz} + g(y, z)$$

$$\text{and } f_y(x, y, z) = e^{xz} + g_y(y, z). \text{ But } f_y(x, y, z) = e^{xz}, \text{ so}$$

$$g(y, z) = h(z) \text{ and } f(x, y, z) = ye^{xz} + h(z). \text{ Thus}$$

$$f_z(x, y, z) = xye^{xz} + h'(z) \text{ but } f_z(x, y, z) = xye^{xz} \text{ so}$$

$$h(z) = K \text{ and } f(x, y, z) = ye^{xz} + K.$$