

 Answers

[E Click here for exercises.](#)

1. $\frac{1}{54} (10^{3/2} - 1)$

2. $\frac{1}{1215} [19(13)^{3/2} + 64]$

3. $\frac{3\sqrt{13}}{2}$

4. 48

5. 0

6. $\frac{32\sqrt{3}+66}{5}$

7. $\frac{9\sqrt{13}}{4}\pi$

8. $\frac{\sqrt{13}}{6}$

9. $3\sqrt{35}$

10. $\frac{864}{35}$

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11. $\frac{16}{11}$

12. $\frac{23}{28}$

13. $\frac{77}{6}$

14. 0

15. $-\frac{19}{143}$

16. $-\frac{59}{30}$

17. $\frac{\pi^6}{192}$

18. 0.052

19. 0.396

20. $\frac{23}{88}$

Solutions

E Click here for exercises.

$$1. \int_C x \, ds = \int_0^1 (t^3) \sqrt{9t^4 + 1} \, dt = \frac{1}{54} (9t^4 + 1)^{3/2} \Big|_0^1 \\ = \frac{1}{54} (10^{3/2} - 1)$$

$$2. \int_C y \, ds = \int_0^1 (t^2) \sqrt{9t^4 + 4t^2} \, dt = \int_0^1 t^3 \sqrt{9t^2 + 4} \, dt \\ = \int_0^1 3t^3 \sqrt{t^2 + (\frac{2}{3})^2} \, dt \\ = 3 \left(\frac{1}{5} t^2 - \frac{8}{135} \right) (t^2 + \frac{4}{9})^{3/2} \Big|_0^1 \\ = \frac{19}{45} \left(\frac{13}{9} \right)^{3/2} + \frac{8}{45} \left(\frac{8}{27} \right) \\ = \frac{1}{1215} [19(13)^{3/2} + 64]$$

3. The line is $3y - 2x = 5$, so $x = x$, $y = \frac{1}{3}(2x + 5)$, $-1 \leq x \leq 2$. Then

$$\int_C xy \, ds = \int_{-1}^2 \frac{1}{3} x (2x + 5) \cdot \frac{\sqrt{13}}{3} \, dx \\ = \frac{\sqrt{13}}{9} \int_{-1}^2 (2x^2 + 5x) \, dx = \frac{\sqrt{13}}{9} \left(\frac{46}{3} - \frac{11}{6} \right) = \frac{3\sqrt{13}}{2}$$

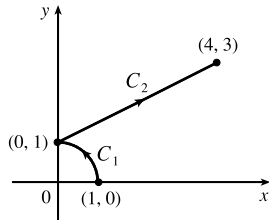
4. $x = x$, $y = x^2$, $-2 \leq x \leq 1$. Then

$$\int_C (x - 2y^2) \, dy = \int_{-2}^1 (x - 2x^4) 2x \, dx \\ = \int_{-2}^1 (2x^2 - 4x^5) \, dx = \frac{2}{3} [x^3 - x^6]_{-2}^1 = 48$$

5. Choosing y as the parameter, we have

$$x = y^4, \, y = y, \, -1 \leq y \leq 1. \text{ Then} \\ \int_C \sin x \, dx = \int_{-1}^1 (\sin y^4) (4y^3) \, dy = -\cos y^4 \Big|_{-1}^1 = 0.$$

6.



$$\text{On } C_1: x = \cos t \Rightarrow dx = -\sin t \, dt, \, y = \sin t \Rightarrow \\ y = \cos t \, dt, \, 0 \leq t \leq \frac{\pi}{2}.$$

$$\text{On } C_2: x = 4t \Rightarrow dx = 4 \, dt, \, y = 2t + 1 \Rightarrow \\ dy = 2 \, dt, \, 0 \leq t \leq 1. \text{ Then}$$

$$\int_C x\sqrt{y} \, dx + 2y\sqrt{x} \, dy \\ = \int_{C_1} x\sqrt{y} \, dx + 2y\sqrt{x} \, dy + \int_{C_2} x\sqrt{y} \, dx + 2y\sqrt{x} \, dy \\ = \int_0^{\pi/2} \left[-\cos t (\sin t)^{3/2} + 2 \sin t (\cos t)^{3/2} \right] dt \\ + \int_0^1 [16t\sqrt{2t+1} + 8(2t+1)\sqrt{t}] \, dt \\ = \left[-\frac{2}{5} (\sin t)^{5/2} - \frac{4}{5} (\cos t)^{5/2} \right]_0^{\pi/2} \\ + \left[\frac{16}{3} t(2t+1)^{3/2} - \frac{16}{15} (2t+1)^{5/2} \right. \\ \left. + 8 \left(\frac{4}{5} t^{5/2} + \frac{2}{3} t^{3/2} \right) \right]_0^1 \\ = \frac{2}{5} + \frac{16}{3} \cdot 3\sqrt{3} - \frac{16}{15} \cdot 3^2 \cdot \sqrt{3} + \frac{16}{15} + 8 \left(\frac{4}{5} + \frac{2}{3} \right) \\ = \frac{32\sqrt{3} + 66}{5}$$

A Click here for answers.

$$7. \int_C xyz \, ds = \int_0^{\pi/2} (18t \sin t \cos t) \sqrt{4+9} \, dt \\ = 18\sqrt{13} \int_0^{\pi/2} (t \sin t \cos t) \, dt \\ = 18\sqrt{13} \int_0^{\pi/2} \frac{1}{2} t \sin 2t \, dt \\ = 9\sqrt{13} \left[-\frac{1}{2} t \cos 2t + \frac{1}{4} \sin 2t \right]_0^{\pi/2} = \frac{9\sqrt{13}}{4} \pi$$

$$8. \int_C x^2 z \, ds = \int_0^{\pi/4} (\sin^2 2t \cos 2t) \sqrt{4+9} \, dt \\ = \sqrt{13} \left[\frac{1}{6} \sin^3 2t \right]_0^{\pi/4} = \frac{\sqrt{13}}{6}$$

9. $x = -t + 1$, $y = 3t$, $z = 5t + 1$, $0 \leq t \leq 1$.

$$\int_C xyz \, ds = \int_0^1 (1-t)(9t^2)(5t+1) \sqrt{1+9+25} \, dt \\ = 9\sqrt{35} \int_0^1 (t^2 + 4t^3 - 5t^4) \, dt = 3\sqrt{35}$$

$$10. \sqrt{(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2} \\ = \sqrt{36 + 36(2)t^2 + 36t^4} = 6\sqrt{(t^2+1)^2} = 6(t^2+1)$$

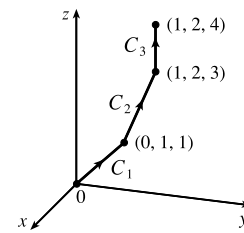
Then

$$\int_C xz \, ds = \int_0^1 72t^4 (t^2+1) \, dt = 72 \left[\frac{1}{7} t^7 + \frac{1}{5} t^5 \right]_0^1 \\ = 72 \left(\frac{12}{35} \right) = \frac{864}{35}$$

$$11. \int_C x^3 y^2 z \, dz = \int_0^1 (2t)^3 (t^2)^2 (t^2) (2t) \, dt \\ = \int_0^1 (8t^3) (t^4) (t^2) (2t) \, dt = \int_0^1 16t^{10} \, dt \\ = \frac{16}{11} t^{11} \Big|_0^1 = \frac{16}{11}$$

$$12. \int_C yz \, dy + xy \, dz = \int_0^1 (t) (t^2) \, dt + \int_0^1 \sqrt{t} (t) 2t \, dt \\ = \int_0^1 (t^3 + 2t^{5/2}) \, dt = \left[\frac{1}{4} t^4 + \frac{4}{7} t^{7/2} \right]_0^1 = \frac{23}{28}$$

13.



$$\text{On } C_1: x = 0 \Rightarrow dx = 0 \, dt, \, y = t \Rightarrow dy = dt, \\ z = t \Rightarrow dz = dt, \, 0 \leq t \leq 1.$$

$$\text{On } C_2: x = t \Rightarrow dx = dt, \, y = t + 1 \Rightarrow dy = dt, \\ z = 2t + 1 \Rightarrow dz = 2 \, dt, \, 0 \leq t \leq 1.$$

$$\text{On } C_3: x = 1 \Rightarrow dx = 0 \, dt, \, y = 2 \Rightarrow dy = 0 \, dt, \\ z = t + 3 \Rightarrow dz = dt, \, 0 \leq t \leq 1.$$

Then

$$\int_C z^2 \, dx - z \, dy + 2y \, dz \\ = \int_0^1 (0 - t + 2t) \, dt \\ + \int_0^1 [(2t+1)^2 - (2t+1) + 2(t+1)(2)] \, dt \\ + \int_0^1 (0 + 0 + 4) \, dt \\ = \frac{1}{2} + \left[\frac{4}{3} t^3 + 3t^2 + 4t \right]_0^1 + 4 = \frac{77}{6}$$

14. C_1 : $(0, 0, 0)$ to $(2, 0, 0)$: $x = 2t, y = z = 0, 0 \leq t \leq 1$.

C_2 : $(2, 0, 0)$ to $(1, 3, -1)$: $x = -t + 2, y = 3t, z = -t, 0 \leq t \leq 1$.

C_3 : $(1, 3, -1)$ to $(1, 3, 0)$: $x = 1, y = 3, z = t - 1, 0 \leq t \leq 1$.

Then

$$\begin{aligned} & \int_C yz \, dx + xz \, dy + xy \, dz \\ &= 0 + \int_0^1 [(3t^2) + 3(t^2 - 2t) - 3(2t - t^2)] \, dt + \int_0^1 3 \, dt \\ &= [3t^3 - 6t^2]_0^1 + 3 = 0 \end{aligned}$$

15. $\mathbf{F}(\mathbf{r}(t)) = t^{10}\mathbf{i} - t^7\mathbf{j}$, $\mathbf{r}'(t) = 3t^2\mathbf{i} + 4t^3\mathbf{j}$.

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 (3t^{12} - 4t^{10}) \, dt = \frac{3}{13} - \frac{4}{11} = -\frac{19}{143}$$

16. $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 \langle t^2 + t^4, -t^2, -4t^4 \rangle \cdot \langle 1, 2t, 4t^3 \rangle \, dt$

$$= \int_0^1 (t^2 + t^4 - 2t^3 - 16t^7) \, dt$$

$$= \frac{1}{3} + \frac{1}{5} - \frac{1}{2} - 2 = -\frac{59}{30}$$

17. $\int_C \mathbf{F} \cdot d\mathbf{r}$

$$= \int_0^{\pi/2} \langle \sin^2 t, \sin t \cos t, t^4 \rangle \cdot \langle \cos t, -\sin t, 2t \rangle \, dt$$

$$= \int_0^{\pi/2} (\sin^2 t \cos t - \sin^2 t \cos t + 2t^5) \, dt$$

$$= \left[\frac{1}{3} t^6 \right]_0^{\pi/2} = \frac{\pi^6}{192}$$

18. A calculator or CAS gives

$$\begin{aligned} \int_C x \sin y \, ds &= \int_1^2 \ln t \sin(e^{-t}) \sqrt{(1/t)^2 + (-e^{-t})^2} \, dt \\ &\approx 0.052 \end{aligned}$$

19. A calculator or CAS gives

$$\begin{aligned} \int_C z^2 \ln(1 + x^2 + y^2) \, ds \\ &= \int_0^1 t^6 \ln(1 + t^2 + t^4) \sqrt{1 + (2t)^2 + (3t^2)^2} \, dt \approx 0.396 \end{aligned}$$

20. $W = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 \langle t^6, -t^5, -t^7 \rangle \cdot \langle 2t, -3t^2, 4t^3 \rangle \, dt$

$$= \int_0^1 (5t^7 - 4t^{10}) \, dt = \frac{5}{8} - \frac{4}{11} = \frac{23}{88}$$