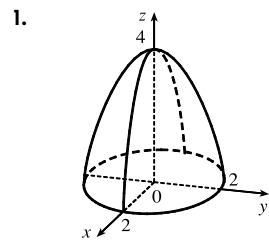




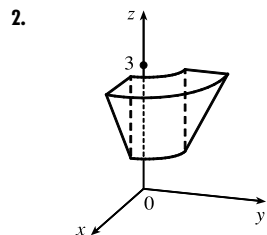
# Answers

**E** [Click here for exercises.](#)

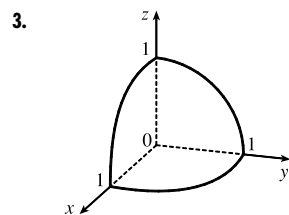
**S** [Click here for solutions.](#)



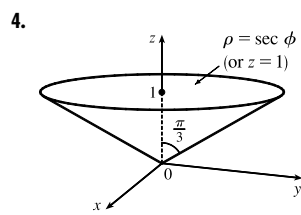
$8\pi$



$\frac{5\pi}{3}$



$\frac{\pi}{6}$



$\pi$

5.  $24\pi$

6.  $\frac{324\pi}{5}$

7. 0

8. 0

9.  $\frac{1}{16}\pi (e^{16} - e)$

10.  $4\pi (2 - \sqrt{3})$

11.  $121\pi \left(\frac{8-5\sqrt{2}}{30}\right)$

12. (0, 0, 2.1)

## Solutions

[Click here for exercises.](#)

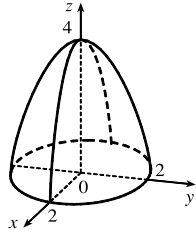
1. The region of integration is given in

cylindrical coordinates by

$$E = \{(r, \theta, z) \mid 0 \leq \theta \leq 2\pi, 0 \leq r \leq 2, 0 \leq z \leq 4 - r^2\}.$$

This represents the solid region bounded above by

$z = 4 - r^2 = 4 - x^2 - y^2$ , a paraboloid, and below by the  $xy$ -plane.



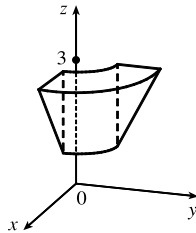
$$\begin{aligned} \int_0^{2\pi} \int_0^2 \int_0^{4-r^2} r \, dz \, dr \, d\theta &= \int_0^{2\pi} \int_0^2 (4r - r^3) \, dr \, d\theta \\ &= \int_0^{2\pi} [2r^2 - \frac{1}{4}r^4]_{r=0}^{r=2} \, d\theta = \int_0^{2\pi} (8 - 4) \, d\theta = 4\theta \Big|_0^{2\pi} = 8\pi \end{aligned}$$

2. The region of integration is given in cylindrical coordinates

by  $E = \{(r, \theta, z) \mid 0 \leq \theta \leq \frac{\pi}{2}, 1 \leq r \leq 3, r \leq z \leq 3\}$ .

This represents the solid in the first octant between the cylinders  $r = 1$  and  $r = 3$  and bounded below by

$z = r = \sqrt{x^2 + y^2}$ , a cone, and above by the plane  $z = 3$ .

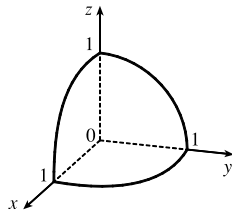


$$\begin{aligned} \int_1^3 \int_0^{\pi/2} \int_r^3 r \, dz \, d\theta \, dr &= \int_1^3 \int_0^{\pi/2} (3r - r^2) \, d\theta \, dr \\ &= \int_1^3 \frac{\pi}{2} (3r - r^2) \, dr = \frac{\pi}{2} [\frac{3}{2}r^2 - \frac{1}{3}r^3]_1^3 \\ &= \frac{\pi}{2} (\frac{27}{2} - \frac{27}{3} - \frac{3}{2} + \frac{1}{3}) = \frac{5\pi}{3} \end{aligned}$$

3. The region of integration is given in spherical coordinates by

$E = \{(\rho, \theta, \phi) \mid 0 \leq \rho \leq 1, 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq \phi \leq \frac{\pi}{2}\}$ . Thus

$E$  is the solid in the first octant bounded by the sphere  $\rho = x^2 + y^2 + z^2 = 1$  and the three coordinate planes.



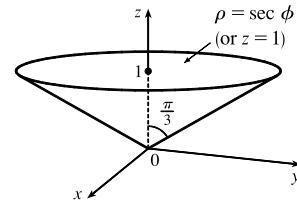
$$\begin{aligned} \int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi &= \int_0^{\pi/2} \int_0^{\pi/2} [\frac{1}{3}\rho^3 \sin \phi]_{\rho=0}^{\rho=1} \, d\theta \, d\phi \\ &= \int_0^{\pi/2} \int_0^{\pi/2} \frac{1}{3} \sin \phi \, d\theta \, d\phi = \int_0^{\pi/2} \frac{1}{3} \sin \phi [\theta]_{\theta=0}^{\theta=\pi/2} \, d\phi \\ &= \frac{1}{3} \int_0^{\pi/2} \frac{\pi}{2} \sin \phi \, d\phi = \frac{\pi}{6} [-\cos \phi]_0^{\pi/2} = \frac{\pi}{6} \end{aligned}$$

[Click here for answers.](#)

4. The region of integration is given in spherical coordinates by

$E = \{(\rho, \theta, \phi) \mid 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \frac{\pi}{3}, 0 \leq \rho \leq \sec \phi\}$ .

Since  $\rho = \sec \phi$  is equivalent to  $\rho \cos \phi = z = 1$ ,  $E$  is the solid bounded by the cone  $\phi = \frac{\pi}{3}$  and the plane  $z = 1$ .



$$\begin{aligned} \int_0^{\pi/3} \int_0^{2\pi} \int_0^{\sec \phi} \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi &= \int_0^{\pi/3} \int_0^{2\pi} [\frac{1}{3}\rho^3 \sin \phi]_{\rho=0}^{\rho=\sec \phi} \, d\theta \, d\phi \\ &= \frac{1}{3} \int_0^{\pi/3} \int_0^{2\pi} \frac{\sin \phi}{\cos^3 \phi} \, d\theta \, d\phi \\ &= \frac{2\pi}{3} \int_0^{\pi/3} (\tan \phi \sec^2 \phi) \, d\phi = \frac{2\pi}{3} [\frac{1}{2} \tan^2 \phi]_0^{\pi/3} = \pi \end{aligned}$$

5.  $\iiint_E (x^2 + y^2) \, dV = \int_{-1}^2 \int_0^{2\pi} \int_0^2 (r^2) r \, dr \, d\theta \, dz$

$$= (3) (2\pi) [\frac{1}{4}r^4]_0^2 = 24\pi$$

6.  $\iiint_E \sqrt{x^2 + y^2} \, dV = \int_0^{2\pi} \int_0^3 \int_0^{9-r^2} r^2 \, dz \, dr \, d\theta$

$$= 2\pi \int_0^3 (9r^2 - r^4) \, dr = 2\pi (81 - \frac{243}{5}) = \frac{324\pi}{5}$$

7. In cylindrical coordinates  $E$  is bounded by the cylinders

$r = 1$  and  $r = 2$ , the plane  $z = x + 2 = r \cos \theta + 2$ ,

and the  $xy$ -plane, so  $E$  is given by

$\{(r, \theta, z) \mid 0 \leq \theta \leq 2\pi, 1 \leq r \leq 2, 0 \leq z \leq r \cos \theta + 2\}$ .

Thus

$$\begin{aligned} \iiint_E y \, dV &= \int_0^{2\pi} \int_1^2 \int_0^{2+r \cos \theta} (r \sin \theta) r \, dz \, dr \, d\theta \\ &= \int_0^{2\pi} \int_1^2 r^2 \sin \theta [z]_{z=0}^{z=2+r \cos \theta} \, dr \, d\theta \\ &= \int_0^{2\pi} \int_1^2 (2r^2 + r^3 \cos \theta) \sin \theta \, dr \, d\theta \\ &= \int_0^{2\pi} [\frac{2}{3}r^3 + \frac{1}{4}r^4 \cos \theta]_{r=1}^{r=2} \sin \theta \, d\theta \\ &= \int_0^{2\pi} (\frac{14}{3} + \frac{15}{4} \cos \theta) \sin \theta \, d\theta \\ &= [-\frac{14}{3} \cos \theta - \frac{15}{8} \cos^2 \theta]_0^{2\pi} = 0 \end{aligned}$$

8. In cylindrical coordinates,  $E$  is bounded by the cylinder

$r = 1$  and the planes  $z = 0$ ,  $z = y = r \sin \theta$  with

$y \geq 0 \Rightarrow 0 \leq \theta \leq \pi$ , so  $E$  is given by

$\{(r, \theta, z) \mid 0 \leq \theta \leq \pi, 0 \leq r \leq 1, 0 \leq z \leq r \sin \theta\}$ . Thus

$$\begin{aligned} \iiint_E xz \, dV &= \int_0^{\pi} \int_0^1 \int_0^{r \sin \theta} r^2 z \cos \theta \, dz \, dr \, d\theta \\ &= \int_0^{\pi} \int_0^1 [\frac{1}{2}z^2]_{z=0}^{z=r \sin \theta} r^2 \cos \theta \, dr \, d\theta \\ &= \frac{1}{2} \int_0^{\pi} \int_0^1 r^4 \sin^2 \theta \cos \theta \, dr \, d\theta \\ &= \frac{1}{2} \int_0^{\pi} [\frac{1}{5}r^5]_{r=0}^{r=1} \sin^2 \theta \cos \theta \, d\theta \\ &= \frac{1}{10} \int_0^{\pi} (\sin^2 \theta \cos \theta) \, d\theta = \frac{1}{30} [\sin^3 \theta]_0^{\pi} = 0 \end{aligned}$$

$$\begin{aligned}
9. \iint\int_E x e^{(x^2+y^2+z^2)^2} dV &= \int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 (\rho \sin \phi \cos \theta) e^{\rho^4} (\rho^2 \sin \phi) d\rho d\phi d\theta \\
&= \int_0^{\pi/2} \cos \theta d\theta \int_0^{\pi/2} \sin^2 \phi d\phi \int_1^2 \rho^3 e^{\rho^4} d\rho \\
&= [\sin \theta]_0^{\pi/2} \left[ \frac{1}{2} \phi - \frac{1}{4} \sin 2\phi \right]_0^{\pi/2} \left[ \frac{1}{4} e^{\rho^4} \right]_1^2 \\
&= (1) \left( \frac{\pi}{4} \right) \left[ \frac{1}{4} (e^{16} - e) \right] = \frac{1}{16} \pi (e^{16} - e)
\end{aligned}$$

$$\begin{aligned}
10. \iint\int_E \sqrt{x^2+y^2+z^2} dV &= \int_0^{2\pi} \int_0^{\pi/6} \int_0^2 (\rho) \rho^2 \sin \phi d\rho d\phi d\theta \\
&= \int_0^{2\pi} d\theta \int_0^{\pi/6} \sin \phi d\phi \int_0^2 \rho^3 d\rho \\
&= [\theta]_0^{2\pi} [-\cos \phi]_0^{\pi/6} \left[ \frac{1}{4} \rho^4 \right]_0^2 = (2\pi) \left( 1 - \frac{\sqrt{3}}{2} \right) (4) \\
&= 8\pi \left( 1 - \frac{\sqrt{3}}{2} \right) = 4\pi (2 - \sqrt{3})
\end{aligned}$$

$$\begin{aligned}
11. \iint\int_E x^2 dV &= \int_0^{2\pi} \int_0^{\pi/4} \int_1^3 \rho^4 \sin^3 \phi \cos^2 \theta d\rho d\phi d\theta \\
&= \int_0^{2\pi} \cos^2 \theta d\theta \left( \int_0^{\pi/4} \sin^3 \phi d\phi \right) \int_1^3 \rho^4 d\rho \\
&= \pi \left[ -\cos \phi + \frac{1}{3} \cos^3 \phi \right]_0^{\pi/4} \left[ \frac{1}{5} \rho^5 \right]_1^3 \\
&= \pi \left( \frac{8-5\sqrt{2}}{12} \right) \left( \frac{242}{5} \right) = 121\pi \left( \frac{8-5\sqrt{2}}{30} \right)
\end{aligned}$$

12. By the symmetry of the problem  $M_{yz} = M_{xz} = 0$ . Then

$$\begin{aligned}
M_{xy} &= \int_0^{2\pi} \int_0^{\pi/3} \int_0^{4 \cos \phi} \rho^3 \cos \phi \sin \phi d\rho d\phi d\theta \\
&= \int_0^{2\pi} \int_0^{\pi/3} \cos \phi \sin \phi (64 \cos^4 \phi) d\phi d\theta \\
&= \int_0^{2\pi} 64 \left[ -\frac{1}{6} \cos^6 \phi \right]_{\phi=0}^{\phi=\pi/3} d\theta = \int_0^{2\pi} \frac{21}{2} d\theta = 21\pi
\end{aligned}$$

Hence  $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 2.1)$ .