

 Answers

[E Click here for exercises.](#)

1. $\frac{50}{3} C$

2. $\frac{3\pi}{2} C$

3. $\frac{2}{3}, (0, \frac{1}{2})$

4. 9, (1, 2)

5. $\frac{1}{6}, (\frac{4}{7}, \frac{3}{4})$

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6. $\frac{648}{5}, (0, \frac{36}{7})$

7. $3\pi, (0, \frac{5}{6})$

8. $\frac{\pi}{4}, (\frac{\pi}{2}, \frac{16}{9\pi})$

9. $\frac{\pi - 2}{2}, (\frac{\pi^2 - 8}{2(\pi - 2)}, \frac{\pi}{4})$

10. $\frac{1}{10}, \frac{1}{16}, \frac{13}{80}$

Solutions

E Click here for exercises.

$$\begin{aligned} 1. Q &= \iint_D (x^2 + 3y^2) dA = \int_0^2 \int_1^2 (x^2 + 3y^2) dy dx \\ &= \int_0^2 [x^2 y + y^3]_{y=1}^{y=2} dx = \int_0^2 (x^2 + 7) dx \\ &= [\frac{1}{3}x^3 + 7x]_0^2 = \frac{8}{3} + 14 = \frac{50}{3} \text{ C} \end{aligned}$$

$$\begin{aligned} 2. Q &= \iint_{0 \leq x^2 + y^2 \leq 1} (1 + x^2 + y^2) dA \\ &= \int_0^{2\pi} \int_0^1 (1 + r^2) r dr d\theta = \int_0^{2\pi} d\theta \int_0^1 (r + r^3) dr \\ &= 2\pi [\frac{1}{2}r^2 + \frac{1}{4}r^4]_0^1 = \frac{3\pi}{2} \text{ C} \end{aligned}$$

$$\begin{aligned} 3. m &= \iint_D \rho(x, y) dA = \int_{-1}^1 \int_0^1 x^2 dy dx \\ &= \int_{-1}^1 x^2 dx \int_0^1 dy = [\frac{1}{3}x^3]_{-1}^1 [y]_0^1 = \frac{2}{3} \\ \bar{x} &= \frac{1}{m} \iint_D x\rho(x, y) dA = \frac{3}{2} \int_{-1}^1 \int_0^1 x^3 dy dx \\ &= \frac{3}{2} \int_{-1}^1 x^3 dx \int_0^1 dy = \frac{3}{2} [\frac{1}{4}x^4]_{-1}^1 [y]_0^1 = 0 \\ \bar{y} &= \frac{1}{m} \iint_D y\rho(x, y) dA = \frac{3}{2} \int_{-1}^1 \int_0^1 x^2 y dy dx \\ &= \frac{3}{2} \int_{-1}^1 x^2 dx \int_0^1 y dy = \frac{3}{2} [\frac{1}{3}x^3]_{-1}^1 [\frac{1}{2}y^2]_0^1 = \frac{1}{2} \\ \text{Hence } (\bar{x}, \bar{y}) &= (0, \frac{1}{2}). \end{aligned}$$

$$\begin{aligned} 4. m &= \int_0^3 \int_0^2 y dx dy = \int_0^2 dx \int_0^3 y dy = 9, \\ M_y &= \int_0^3 \int_0^2 xy dx dy = \int_0^2 x dx \int_0^3 y dy = 9 \text{ and} \\ M_x &= \int_0^3 \int_0^2 y^2 dx dy = \int_0^2 dx \int_0^3 y^2 dy = 18. \\ \text{Hence } m &= 9, (\bar{x}, \bar{y}) = (M_y/m, M_x/m) = (1, 2). \end{aligned}$$

$$\begin{aligned} 5. m &= \int_0^1 \int_{x^2}^1 xy dy dx = \int_0^1 (\frac{1}{2}x - \frac{1}{2}x^5) dx = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}, \\ M_y &= \int_0^1 \int_{x^2}^1 x^2 y dy dx = \int_0^1 (\frac{1}{2}x^2 - \frac{1}{2}x^6) dx = \\ \frac{1}{6} - \frac{1}{14} &= \frac{2}{21} \text{ and } M_x = \int_0^1 \int_{x^2}^1 xy^2 dy dx = \\ \int_0^1 (\frac{1}{3}x - \frac{1}{3}x^7) dx &= \frac{1}{6} - \frac{1}{24} = \frac{1}{8}. \\ \text{Hence } m &= \frac{1}{6}, (\bar{x}, \bar{y}) = (\frac{4}{7}, \frac{3}{4}). \end{aligned}$$

$$\begin{aligned} 6. m &= \int_{-3}^3 \int_0^{9-x^2} y dy dx = \int_{-3}^3 \frac{1}{2} (81 - 18x^2 + x^4) dx \\ &= 243 - 162 + \frac{243}{5} = \frac{648}{5} = 3^4 \cdot \frac{8}{5} \\ M_y &= 0 \text{ since } \rho \text{ is independent of } x \text{ and the region is} \\ &\text{symmetric about the } y\text{-axis, and} \\ M_x &= \int_{-3}^3 \int_0^{9-x^2} y^2 dy dx = \int_{-3}^3 \frac{1}{3} (9 - x^2)^3 dx \\ &= 2 \int_0^3 (243 - 81x^2 + 9x^4 - \frac{1}{3}x^6) dx \\ &= 2 [3^6 - 3^6 + \frac{1}{5}3^7 - \frac{1}{21}3^7] \\ &= 2 [3^6 \cdot \frac{21-5}{35}] = 3^6 \cdot \frac{32}{35} \\ \text{Hence } m &= \frac{648}{5}, (\bar{x}, \bar{y}) = (0, \frac{36}{7}). \end{aligned}$$

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$$\begin{aligned} 7. \text{ Working in polar coordinates,} \\ m &= \int_0^{2\pi} \int_0^{1+\sin\theta} 2r dr d\theta = \int_0^{2\pi} (1 + \sin\theta)^2 d\theta \\ &= \int_0^{2\pi} [1 + 2\sin\theta + \frac{1}{2}(1 - \cos 2\theta)] d\theta \\ &= [\frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta]_0^{2\pi} = 3\pi \\ M_y &= 0 \text{ since the lamina is homogeneous and symmetric} \\ &\text{with respect to the } y\text{-axis, and} \\ M_x &= \int_0^{2\pi} \int_0^{1+\sin\theta} (2r^2 \sin\theta) dr d\theta \\ &= \frac{2}{3} \int_0^{2\pi} (1 + \sin\theta)^3 \sin\theta d\theta \\ &= \frac{2}{3} \int_0^{2\pi} (\sin\theta + 3\sin^2\theta + 3\sin^3\theta + \sin^4\theta) d\theta \\ &= \frac{2}{3} (3\pi + \frac{3}{4}\pi) = \frac{5\pi}{2} \\ \text{Hence } m &= 3\pi, (\bar{x}, \bar{y}) = (0, \frac{5}{6}). \end{aligned}$$

$$\begin{aligned} 8. m &= \int_0^\pi \int_0^{\sin x} y dy dx = \int_0^\pi \frac{1}{2} \sin^2 x dx \\ &= [\frac{1}{4}x - \frac{1}{8}\sin 2x]_0^\pi = \frac{1}{4}\pi \\ M_y &= \int_0^\pi \int_0^{\sin x} xy dy dx = \int_0^\pi \frac{1}{2} x \sin^2 x dx \\ &= [\frac{1}{8}x^2 - \frac{1}{8}x \sin 2x - \frac{1}{16}\cos 2x]_0^\pi = \frac{1}{8}\pi^2 \end{aligned}$$

and

$$\begin{aligned} M_x &= \int_0^\pi \int_0^{\sin x} y^2 dy dx = \int_0^\pi \frac{1}{3} \sin^3 x dx \\ &= \frac{1}{3} [-\cos x + \frac{1}{3}\cos^3 x]_0^\pi = \frac{4}{9} \\ \text{Hence } m &= \frac{\pi}{4}, (\bar{x}, \bar{y}) = (\frac{\pi}{2}, \frac{16}{9\pi}). \end{aligned}$$

$$\begin{aligned} 9. m &= \int_0^{\pi/2} \int_0^{\cos x} x dy dx = \int_0^{\pi/2} x \cos x dx \\ &= [x \sin x + \cos x]_0^{\pi/2} = \frac{\pi}{2} - 1 \\ M_y &= \int_0^{\pi/2} \int_0^{\cos x} x^2 dy dx = \int_0^{\pi/2} x^2 \cos x dx \\ &= [x^2 \sin x + 2x \cos x - 2 \sin x]_0^{\pi/2} = \frac{\pi^2}{4} - 2 \\ \text{and} \\ M_x &= \int_0^{\pi/2} \int_0^{\cos x} xy dy dx = \int_0^{\pi/2} \frac{1}{2} x \cos^2 x dx \\ &= \frac{1}{2} [x^2 - x + (x-1)\sin x \cos x]_0^{\pi/2} = \frac{\pi^2}{8} - \frac{\pi}{4} \\ \text{Hence } m &= \frac{\pi-2}{2}, (\bar{x}, \bar{y}) = (\frac{\pi^2-8}{2(\pi-2)}, \frac{\pi}{4}). \end{aligned}$$

$$\begin{aligned} 10. I_x &= \iint_D y^2 \rho(x, y) dA = \int_0^1 \int_{x^2}^1 y^2 (xy) dy dx \\ &= \int_0^1 [\frac{1}{4}xy^4]_{y=x^2}^{y=1} dx = \int_0^1 \frac{1}{4} (x - x^9) dx \\ &= \frac{1}{8} - \frac{1}{40} = \frac{1}{10} \\ I_y &= \iint_D x^2 \rho(x, y) dA = \int_0^1 \int_{x^2}^1 x^3 y dy dx \\ &= \int_0^1 [\frac{1}{2}x^3 y^2]_{y=x^2}^{y=1} dx = \int_0^1 \frac{1}{2} (x^3 - x^7) dx \\ &= \frac{1}{8} - \frac{1}{16} = \frac{1}{16} \\ I_0 &= I_x + I_y = \frac{13}{80}. \end{aligned}$$