

## 12.1 Double Integrals over Rectangles

**A** [Click here for answers.](#)

**S** [Click here for solutions.](#)

- Find approximations to  $\iint_R (x - 3y^2) dA$  using the same subrectangles as in Example 3 but choosing the sample point to be the (a) upper left corner, (b) upper right corner, (c) lower left corner, (d) lower right corner of each subrectangle.
- Find the approximation to the volume in Example 1 if the Midpoint Rule is used.
- (a) Estimate the volume of the solid that lies below the surface  $z = x^2 + 4y$  and above the rectangle  $R = \{(x, y) \mid 0 \leq x \leq 2, 0 \leq y \leq 3\}$ . Use a Riemann sum with  $m = 2, n = 3$ , and take the sample point to be the upper right corner of each subrectangle.  
(b) Use the Midpoint Rule to estimate the volume of the solid in part (a).
- If  $R = [-2, 2] \times [-1, 1]$ , use a Riemann sum with  $m = n = 4$  to estimate the value of  $\iint_R (2x + x^2y) dA$ . Take the sample points to be the lower left corners of the subrectangles.

||| **Answers**

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**E** [Click here for exercises.](#)

**S** [Click here for solutions.](#)

1. (a)  $-17.75$   
(b)  $-15.75$   
(c)  $-8.75$   
(d)  $-6.75$
2. 49
3. (a) 63  
(b) 43.5
4.  $-11$

## Solutions

[E Click here for exercises.](#)

1. (a)  $\sum_{i=1}^2 \sum_{j=1}^2 f(x_{ij}^*, y_{ij}^*) \Delta A$
- $$= f\left(0, \frac{3}{2}\right) \Delta A + f(0, 2) \Delta A$$
- $$+ f\left(1, \frac{3}{2}\right) \Delta A + f(1, 2) \Delta A$$
- $$= \left(-\frac{27}{4}\right) \frac{1}{2} + (-12) \frac{1}{2}$$
- $$+ \left(1 - \frac{27}{4}\right) \frac{1}{2} + (1 - 12) \frac{1}{2}$$
- $$= -17.75$$
- (b)  $\frac{1}{2} [f(1, \frac{3}{2}) + f(1, 2) + f(2, \frac{3}{2}) + f(2, 2)]$
- $$= \frac{1}{2} \left[-\frac{23}{4} + (-11) + \left(-\frac{19}{4}\right) + (-10)\right]$$
- $$= \frac{1}{2} \left(-\frac{63}{2}\right) = -15.75$$
- (c)  $\frac{1}{2} [f(0, 1) + f(0, \frac{3}{2}) + f(1, 1) + f(1, \frac{3}{2})]$
- $$= \frac{1}{2} \left[-3 - \frac{27}{4} - 2 - \frac{23}{4}\right] = -8.75$$
- (d)  $\frac{1}{2} [f(1, 1) + f(1, \frac{3}{2}) + f(2, 1) + f(2, \frac{3}{2})]$
- $$= \frac{1}{2} \left[-2 - \frac{23}{4} - 1 - \frac{19}{4}\right] = -6.75$$
2.  $V \approx (1) [f(\frac{1}{2}, \frac{1}{2}) + f(\frac{1}{2}, \frac{3}{2}) + f(\frac{3}{2}, \frac{1}{2}) + f(\frac{3}{2}, \frac{3}{2})]$
- $$= \left[\frac{61}{4} + \frac{45}{4} + \frac{53}{4} + \frac{37}{4}\right] = 49$$

3. (a) The subrectangles are shown in the figure. The surface is the graph of  $f(x, y) = x^2 + 4y$  and  $\Delta A = 1$ , so we estimate

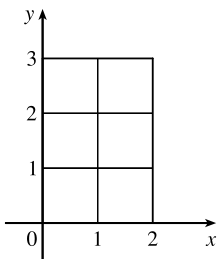
$$V \approx \sum_{i=1}^2 \sum_{j=1}^3 f(x_i, y_j) \Delta A$$

$$= f(1, 1) \Delta A + f(1, 2) \Delta A + f(1, 3) \Delta A$$

$$+ f(2, 1) \Delta A + f(2, 2) \Delta A + f(2, 3) \Delta A$$

$$= 5(1) + 9(1) + 13(1) + 8(1) + 12(1) + 16(1)$$

$$= 63$$



(b)  $V \approx \sum_{i=1}^2 \sum_{j=1}^3 f(\bar{x}_i, \bar{y}_j) \Delta A$

$$= f\left(\frac{1}{2}, \frac{1}{2}\right) \Delta A + f\left(\frac{1}{2}, \frac{3}{2}\right) \Delta A + f\left(\frac{1}{2}, \frac{5}{2}\right) \Delta A$$

$$+ f\left(\frac{3}{2}, \frac{1}{2}\right) \Delta A + f\left(\frac{3}{2}, \frac{3}{2}\right) \Delta A + f\left(\frac{3}{2}, \frac{5}{2}\right) \Delta A$$

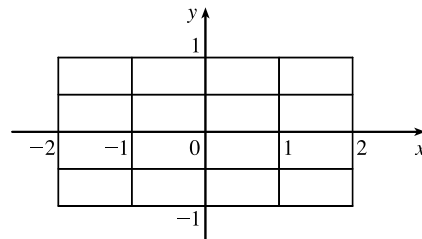
$$= \frac{9}{4}(1) + \frac{25}{4}(1) + \frac{41}{4}(1)$$

$$+ \frac{17}{4}(1) + \frac{33}{4}(1) + \frac{49}{4}(1)$$

$$= \frac{87}{2} = 43.5$$

[A Click here for answers.](#)

4. The subrectangles are shown in the figure.



Since  $\Delta A = \frac{1}{2}$ , we estimate

$$\iint_R (2x + x^2y) dA \approx \sum_{i=1}^4 \sum_{j=1}^4 f(x_{ij}^*, y_{ij}^*) \Delta A$$

$$= \frac{1}{2} [f(-2, -1) + f(-2, -\frac{1}{2}) + f(-2, 0) + f(-2, \frac{1}{2})$$

$$+ f(-1, -1) + f(-1, -\frac{1}{2}) + f(-1, 0) + f(-1, \frac{1}{2})$$

$$+ f(0, -1) + f(0, -\frac{1}{2}) + f(0, 0) + f(0, \frac{1}{2}) + f(1, -1)$$

$$+ f(1, -\frac{1}{2}) + f(1, 0) + f(1, \frac{1}{2})]$$

$$= \frac{1}{2} (-22) = -11$$