# 10.1 Vector Functions and Space Curves

A Click here for answers.

S Click here for solutions.

**1.** Find the domain of the vector function

$$\mathbf{r}(t) = \ln t\,\mathbf{i} + \frac{t}{t-1}\,\mathbf{j} + e^{-t}\,\mathbf{k}$$

**2–5** |||| Find the limit.

**2.** $\lim_{t \to 0} \langle t, \cos t, 2 \rangle$

**3.** $\lim_{t \to 0} \left\langle \dfrac{1 - \cos t}{t}, t^3, e^{-1/t^2} \right\rangle$

**4.** $\lim_{t \to 1} \left\langle \sqrt{t+3}\,\mathbf{i} + \dfrac{t-1}{t^2-1}\,\mathbf{j} + \dfrac{\tan t}{t}\,\mathbf{k} \right\rangle$

**5.** $\lim_{t \to \infty} \left\langle e^{-t}\,\mathbf{i} + \dfrac{t-1}{t+1}\,\mathbf{j} + \tan^{-1} t\,\mathbf{k} \right\rangle$

**6–8** |||| Sketch the curve with the given vector equation. Indicate with an arrow the direction in which $t$ increases.

**6.** $\mathbf{r}(t) = \langle t^2, t, 2 \rangle$

**7.** $\mathbf{r}(t) = \langle t, -t, 2t \rangle$

**8.** $\mathbf{r}(t) = \langle \sin t, t, \cos t \rangle$

**9–10** |||| Use a computer to graph the curve with the given vector equation. Make sure you choose a parameter domain and viewpoints that reveal the true nature of the curve.

**9.** $\mathbf{r}(t) = \langle t^2, t^3 - t, t \rangle$

**10.** $\mathbf{r}(t) = \langle \sqrt{t}, t, t^2 - 2 \rangle$

# Answers

E Click here for exercises.

S Click here for solutions.

**1.** $(0,1) \cup (1,\infty)$

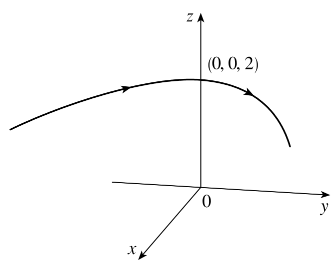
**2.** $\langle 0,1,2 \rangle$

**3.** $\langle 0,0,0 \rangle$

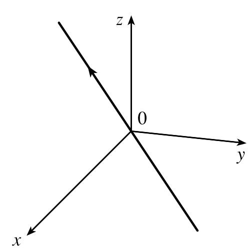
**4.** $\left\langle 2, \frac{1}{2}, \tan 1 \right\rangle$

**5.** $\left\langle 0, 1, \frac{\pi}{2} \right\rangle$

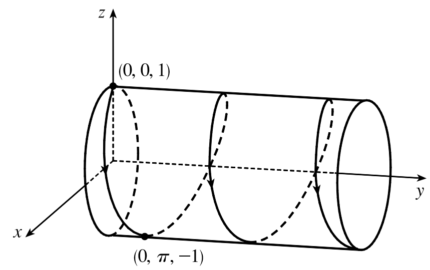
**6.**



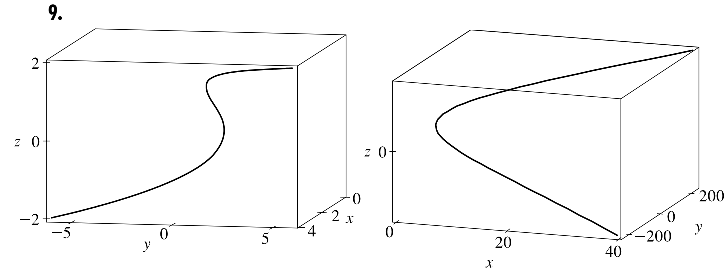

**7.**



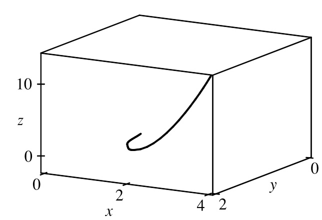

**8.**




**9.**




**10.**



# Solutions

E Click here for exercises.

A Click here for answers.

**1.** The component functions $\ln t$, $\dfrac{t}{t-1}$, and $e^{-t}$ are all defined when $t > 0$ and $t \neq 1$, so the domain of $\mathbf{r}(t)$ is $(0,1) \cup (1,\infty)$.

**2.** $\lim\limits_{t\to 0} \langle t, \cos t, 2\rangle = \left\langle \lim\limits_{t\to 0} t, \lim\limits_{t\to 0} \cos t, \lim\limits_{t\to 0} 2\right\rangle = \langle 0,1,2\rangle$

**3.** $\lim\limits_{t\to 0}\left\langle \dfrac{1-\cos t}{t}, t^3, e^{-1/t^2}\right\rangle =$

$\left\langle \lim\limits_{t\to 0}\dfrac{1-\cos t}{t}, \lim\limits_{t\to 0} t^3, \lim\limits_{t\to 0} e^{-1/t^2}\right\rangle = \langle 0,0,0\rangle$

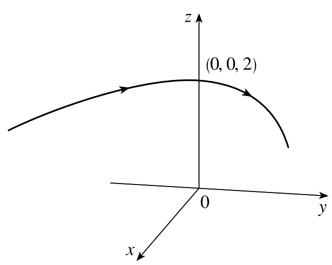
**4.** $\lim\limits_{t\to 1}\sqrt{t+3} = 2$, $\lim\limits_{t\to 1}\dfrac{t-1}{t^2-1} = \lim\limits_{t\to 1}\dfrac{1}{t+1} = \dfrac{1}{2}$,

$\lim\limits_{t\to 1}\left(\dfrac{\tan t}{t}\right) = \tan 1$.

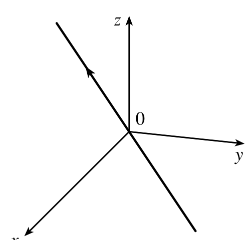
Thus the given limit equals $\left\langle 2, \frac{1}{2}, \tan 1\right\rangle$.

**5.** $\lim\limits_{t\to\infty} e^{-t} = 0$, $\lim\limits_{t\to\infty}\dfrac{t-1}{t+1} = 1$, $\lim\limits_{t\to\infty}\tan^{-1} t = \frac{\pi}{2}$, so the given limit equals $\left\langle 0,1,\frac{\pi}{2}\right\rangle$.

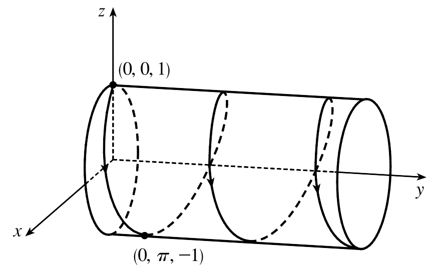
**6.** The parametric equations are $x = t^2, y = t, z = 2$ and the curve is thus given by $x = y^2, z = 2$, which is a parabola in the plane $z = 2$ with vertex $(0,0,2)$ and axis $z = 2, y = 0$.



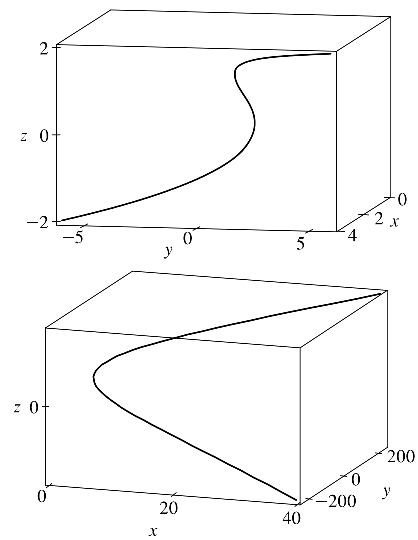

**7.** The corresponding parametric equations are $x = t, y = -t$, $z = 2t$, which are parametric equations of a line through the origin and with direction vector $\langle 1,-1,2\rangle$.




**8.** The parametric equations give $x^2 + z^2 = \sin^2 t + \cos^2 t = 1$, $y = t$, so the curve lies on the cylinder $x^2 + z^2 = 1$. Since $y = t$, the curve is a helix.




**9.** $\mathbf{r}(t) = \langle t^2, t^3 - t, t\rangle$




**10.** $\mathbf{r}(t) = \langle \sqrt{t}, t, t^2 - 2\rangle$