9.7 Cylindrical and Spherical Coordinates

A Click here for answers.

S Click here for solutions.

1–2 Plot the point whose cylindrical coordinates are given. Then find the rectangular coordinates of the point.

1. $(3, \pi/2, 1)$ **2.** $(\sqrt{2}, \pi/4, \sqrt{2})$

3–8 Change from rectangular to cylindrical coordinates.

3. $(-1, 0, 0)$ **4.** $(1, 1, 1)$

5. $(\sqrt{3}, 1, 4)$ **6.** $(-\sqrt{2}, \sqrt{2}, 0)$

7. $(4, 4, 4)$ **8.** $(-1, \sqrt{3}, 2)$

9–11 Change from spherical to rectangular coordinates.

9. $(2, \pi/2, 3\pi/4)$ **10.** $(4, \pi/4, \pi/6)$

11. $(2, \pi/4, \pi/4)$

12–15 Change from rectangular to spherical coordinates.

12. $(-3, 0, 0)$ **13.** $(1, 1, \sqrt{2})$

14. $(\sqrt{3}, 0, 1)$ **15.** $(-\sqrt{3}, -3, -2)$

16–19 Change from cylindrical to spherical coordinates.

16. $(\sqrt{2}, \pi/4, 0)$ **17.** $(1, \pi/2, 1)$

18. $(4, \pi/3, 4)$ **19.** $(12, \pi, 5)$

20–23 Write the equation (a) in cylindrical coordinates and (b) in spherical coordinates.

20. $x^2 + y^2 + z^2 = 16$ **21.** $x^2 + y^2 - z^2 = 16$

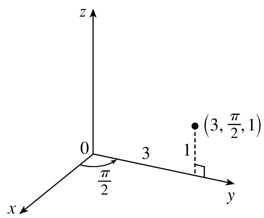
22. $x + 2y + 3z = 6$ **23.** $x^2 + y^2 = 2z$

Answers

E Click here for exercises.

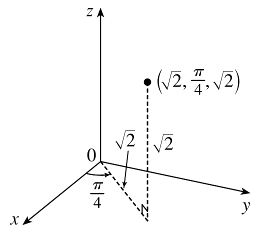
S Click here for solutions.

1.



$(0, 3, 1)$

2.



$(1, 1, \sqrt{2})$

3. $(1, \pi, 0)$

4. $\left(\sqrt{2}, \frac{\pi}{4}, 1\right)$

5. $\left(2, \frac{\pi}{6}, 4\right)$

6. $\left(2, \frac{3\pi}{4}, 0\right)$

7. $\left(4\sqrt{2}, \frac{\pi}{4}, 4\right)$

8. $\left(2, \frac{2\pi}{3}, 2\right)$

9. $\left(0, \sqrt{2}, -\sqrt{2}\right)$

10. $\left(\sqrt{2}, \sqrt{2}, 2\sqrt{3}\right)$

11. $\left(1, 1, \sqrt{2}\right)$

12. $\left(3, \pi, \frac{\pi}{2}\right)$

13. $\left(2, \frac{\pi}{4}, \frac{\pi}{4}\right)$

14. $\left(2, 0, \frac{\pi}{3}\right)$

15. $\left(4, \frac{4\pi}{3}, \frac{2\pi}{3}\right)$

16. $\left(\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2}\right)$

17. $\left(\sqrt{2}, \frac{\pi}{2}, \frac{\pi}{4}\right)$

18. $\left(4\sqrt{2}, \frac{\pi}{3}, \frac{\pi}{4}\right)$

19. $\left(13, \pi, \cos^{-1}\left(\frac{5}{13}\right)\right)$

20. (a) $r^2 + z^2 = 16$

(b) $\rho = 4$

21. (a) $r^2 - z^2 = 16$

(b) $\rho^2\left(1 - 2\cos^2\phi\right) = 16$

22. (a) $r\cos\theta + 2r\sin\theta + 3z = 6$

(b) $\rho\left(\sin\phi\cos\theta + 2\sin\phi\sin\theta + 3\cos\phi\right) = 6$

23. (a) $r^2 = 2z$

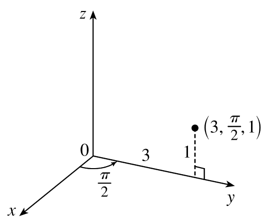
(b) $\rho\sin^2\phi = 2\cos\phi$

Solutions

E Click here for exercises.

A Click here for answers.

1.

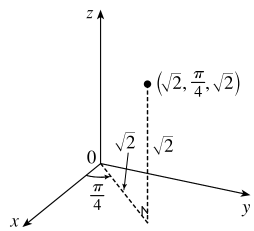


$x = 3\cos\frac{\pi}{2} = 0$,

$y = 3\sin\frac{\pi}{2} = 3$, and $z = 1$,

so the point is $(0, 3, 1)$ in rectangular coordinates.

2.



$x = \sqrt{2}\cos\frac{\pi}{4} = 1$,

$y = \sqrt{2}\sin\frac{\pi}{4} = 1$, $z = \sqrt{2}$,

so the point is $\left(1, 1, \sqrt{2}\right)$ in rectangular coordinates.

3. $r^2 = (-1)^2 + (0)^2 = 1$ so $r = 1$; $z = 0$; $\tan\theta = 0$ so $\theta = 0$ or π. But $x = -1$ so $\theta = \pi$ and the point is $(1, \pi, 0)$.

4. $r^2 = 1^2 + 1^2 = 2$ or $r = \sqrt{2}$, $\tan\theta = \frac{1}{1}$ so $\theta = \frac{\pi}{4}$ and $z = 1$. Thus in cylindrical coordinates the point is $\left(\sqrt{2}, \frac{\pi}{4}, 1\right)$.

5. $r^2 = 4$ so $r = 2$, $\tan\theta = \dfrac{1}{\sqrt{3}}$ so $\theta = \dfrac{\pi}{6}$ and $z = 4$. Thus the point in cylindrical coordinates is $\left(2, \frac{\pi}{6}, 4\right)$.

6. $r^2 = 4$ so $r = 2$; $\tan\theta = \sqrt{2}/\left(-\sqrt{2}\right) = -1$ and the point $\left(-\sqrt{2}, \sqrt{2}\right)$ is in the second quadrant of the xy-plane so $\theta = \frac{3\pi}{4}$; $z = 0$. The point is $\left(2, \frac{3\pi}{4}, 0\right)$.

7. $r = \sqrt{4^2 + 4^2} = 4\sqrt{2}$; $z = 4$; $\tan\theta = \frac{4}{4}$, so $\theta = \frac{\pi}{4}$ or $\theta = \frac{5\pi}{4}$, but both x and y are positive, so $\theta = \frac{\pi}{4}$ and the point is $\left(4\sqrt{2}, \frac{\pi}{4}, 4\right)$.

8. $r = \sqrt{1 + 3} = 2$; $\tan\theta = -\dfrac{\sqrt{3}}{1}$, so $\theta = \frac{2\pi}{3}$ or $\theta = \frac{5\pi}{3}$, but x is negative and y is positive, so $\theta = \frac{2\pi}{3}$ and the point is $\left(2, \frac{2\pi}{3}, 2\right)$.

9. $x = 2\sin\frac{3\pi}{4}\cos\frac{\pi}{2} = 0$, $y = 2\sin\frac{3\pi}{4}\sin\frac{\pi}{2} = \sqrt{2}$ and $z = 2\cos\frac{3\pi}{4} = -\sqrt{2}$ so the point is $\left(0, \sqrt{2}, -\sqrt{2}\right)$.

10. $x = 4\sin\frac{\pi}{6}\cos\frac{\pi}{4} = 4\left(\frac{1}{2}\right)\frac{1}{\sqrt{2}} = \sqrt{2}$,

$y = 4\sin\frac{\pi}{6}\sin\frac{\pi}{4} = \sqrt{2}$ and $z = 4\cos\frac{\pi}{6} = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

so the point is $\left(\sqrt{2}, \sqrt{2}, 2\sqrt{3}\right)$.

11. $x = 2\sin\frac{\pi}{4}\cos\frac{\pi}{4} = 1$, $y = 2\sin\frac{\pi}{4}\sin\frac{\pi}{4} = 1$ and $z = 2\cos\frac{\pi}{4} = \sqrt{2}$ so the point is $\left(1, 1, \sqrt{2}\right)$ in rectangular coordinates.

12. $\rho = \sqrt{9 + 0 + 0} = 3$, $\cos\phi = \frac{0}{3} = 0$ so $\phi = \frac{\pi}{2}$, and

$\cos\theta = \dfrac{-3}{3\sin\frac{\pi}{2}} = -1$ so $\theta = \pi$, thus spherical coordinates are $\left(3, \pi, \frac{\pi}{2}\right)$.

13. $\rho = \sqrt{1 + 1 + 2} = 2$, $\cos\phi = \dfrac{\sqrt{2}}{2}$ so $\phi = \frac{\pi}{4}$, and

$\cos\theta = \dfrac{1}{2\sin\frac{\pi}{4}} = \dfrac{1}{\sqrt{2}}$ so $\theta = \frac{\pi}{4}$, thus in spherical coordinates the point is $\left(2, \frac{\pi}{4}, \frac{\pi}{4}\right)$.

14. $\rho = \sqrt{3 + 1} = 2$, $\cos\phi = \frac{1}{2}$ so $\phi = \frac{\pi}{3}$, and

$\cos\theta = \dfrac{\sqrt{3}}{2\sin\frac{\pi}{3}} = \dfrac{\sqrt{3}\cdot 2}{2\cdot\sqrt{3}} = 1$ so $\theta = 0$, thus the point is $\left(2, 0, \frac{\pi}{3}\right)$ in spherical coordinates.

Note: It is also apparent that $\theta = 0$ since the point is in the xz-plane and $x > 0$.

15. $\rho = \sqrt{3 + 9 + 4} = 4$, $\cos\phi = -\frac{2}{4} = -\frac{1}{2}$ so $\phi = \frac{2\pi}{3}$, and

$\cos\theta = -\dfrac{\sqrt{3}}{4\sin\frac{5\pi}{6}} = -\dfrac{\sqrt{3}\cdot 2}{4\cdot\sqrt{3}} = -\frac{1}{2}$ and $y = -3$ so

$\theta = \frac{4\pi}{3}$. Thus in spherical coordinates the point is $\left(4, \frac{4\pi}{3}, \frac{2\pi}{3}\right)$.

16. $\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2} = \sqrt{2 + 0} = \sqrt{2}$; $\theta = \frac{\pi}{4}$;

$z = \rho\cos\phi = \sqrt{2}\cos\phi = 0$ so $\phi = \frac{\pi}{2}$ and the point is $\left(\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2}\right)$.

17. $\rho = \sqrt{r^2 + z^2} = \sqrt{1 + 1} = \sqrt{2}$, $z = 1 = \sqrt{2}\cos\phi$, so $\phi = \frac{\pi}{4}$, $\theta = \frac{\pi}{2}$ and the point is $\left(\sqrt{2}, \frac{\pi}{2}, \frac{\pi}{4}\right)$.

18. $\rho = \sqrt{r^2 + z^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2}$; $\theta = \frac{\pi}{3}$;

$z = 4 = 4\sqrt{2}\cos\phi$ so $\cos\phi = \frac{1}{\sqrt{2}} \quad\Rightarrow\quad \phi = \frac{\pi}{4}$ and the point is $\left(4\sqrt{2}, \frac{\pi}{3}, \frac{\pi}{4}\right)$.

19. $\rho = \sqrt{r^2 + z^2} = \sqrt{12^2 + 5^2} = 13$, $z = 5 = 13\cos\phi$, so $\phi = \cos^{-1}\left(\frac{5}{13}\right)$, $\theta = \pi$ and the point is $\left(13, \pi, \cos^{-1}\left(\frac{5}{13}\right)\right)$.

20. (a) $r^2 = x^2 + y^2$, so $r^2 + z^2 = 16$.

(b) $\rho^2 = x^2 + y^2 + z^2$, so $\rho^2 = 16$ or $\rho = 4$.

21. (a) $r^2 - z^2 = 16$

(b) $x^2 + y^2 - z^2 = x^2 + y^2 + z^2 - 2z^2$, so $\rho^2 - 2\rho^2\cos^2\phi = 16$ or $\rho^2\left(1 - 2\cos^2\phi\right) = 16$.

22. (a) $r\cos\theta + 2r\sin\theta + 3z = 6$

(b) $\rho\sin\phi\cos\theta + 2\rho\sin\phi\sin\theta + 3\rho\cos\phi = 6$ or $\rho\left(\sin\phi\cos\theta + 2\sin\phi\sin\theta + 3\cos\phi\right) = 6$.

23. (a) $r^2 = 2z$

(b) $\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta = 2\rho\cos\phi$ or $\rho^2\sin^2\phi = 2\rho\cos\phi$ or $\rho\sin^2\phi = 2\cos\phi$.