

9.5 Equations of Lines and Planes

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1–4 Find a vector equation and parametric equations for the line passing through the given point and parallel to the vector \mathbf{a} .

- $(3, -1, 8)$, $\mathbf{a} = \langle 2, 3, 5 \rangle$
- $(-2, 4, 5)$, $\mathbf{a} = \langle 3, -1, 6 \rangle$
- $(0, 1, 2)$, $\mathbf{a} = 6\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- $(1, -1, -2)$, $\mathbf{a} = 2\mathbf{i} - 7\mathbf{k}$

5–10 Find parametric equations and symmetric equations for the line through the given points.

- $(2, 1, 8)$, $(6, 0, 3)$
- $(-1, 0, 5)$, $(4, -3, 3)$
- $(3, 1, -1)$, $(3, 2, -6)$
- $(3, 1, \frac{1}{2})$, $(-1, 4, 1)$
- $(-\frac{1}{3}, 1, 1)$, $(0, 5, -8)$
- $(2, -7, 5)$, $(-4, 2, 5)$

11. Show that the line through the points $(0, 1, 1)$ and $(1, -1, 6)$ is perpendicular to the line through the points $(-4, 2, 1)$ and $(-1, 6, 2)$.

12–14 Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

12. $L_1: \frac{x-4}{2} = \frac{y+5}{4} = \frac{z-1}{-3},$

$L_2: \frac{x-2}{1} = \frac{y+1}{3} = \frac{z}{2}$

13. $L_1: \frac{x-1}{2} = \frac{y}{1} = \frac{z-1}{4},$

$L_2: \frac{x}{1} = \frac{y+2}{2} = \frac{z+2}{3}$

14. $L_1: x = 1 + t, y = 2 - t, z = 3t$

$L_2: x = 2 - s, y = 1 + 2s, z = 4 + s$

15–18 Find an equation of the plane passing through the given point and with normal vector \mathbf{n} .

- $(1, 4, 5)$, $\mathbf{n} = \langle 7, 1, 4 \rangle$
- $(-5, 1, 2)$, $\mathbf{n} = \langle 3, -5, 2 \rangle$
- $(1, 2, 3)$, $\mathbf{n} = 15\mathbf{i} + 9\mathbf{j} - 12\mathbf{k}$
- $(-1, -6, -4)$, $\mathbf{n} = -5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

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19–22 Find an equation of the plane passing through the given point and parallel to the specified plane.

- $(6, 5, -2)$, $x + y - z + 1 = 0$
- $(3, 0, 8)$, $2x + 5y + 8z = 17$
- $(-1, 3, -8)$, $3x - 4y - 6z = 9$
- $(2, -4, 5)$, $z = 2x + 3y$

23–26 Find an equation of the plane passing through the three given points.

- $(0, 0, 0)$, $(1, 1, 1)$, $(1, 2, 3)$
- $(-1, 1, -1)$, $(1, -1, 2)$, $(4, 0, 3)$
- $(1, 0, -3)$, $(0, -2, -4)$, $(4, 1, 6)$
- $(2, 1, -3)$, $(5, -1, 4)$, $(2, -2, 4)$

27–30 Find an equation of the plane that passes through the given point and contains the specified line.

- $(1, 6, -4)$; $x = 1 + 2t, y = 2 - 3t, z = 3 - t$
- $(-1, -3, 2)$; $x = -1 - 2t, y = 4t, z = 2 + t$
- $(0, 1, 2)$; $x = y = z$
- $(-1, 0, 1)$; $x = 5t, y = 1 + t, z = -t$

31–34 Find the point at which the line intersects the given plane.

- $x = 1 + t, y = 2t, z = 3t$; $x + y + z = 1$
- $x = 5, y = 4 - t, z = 2t$; $2x - y + z = 5$
- $x = 1 + 2t, y = -1, z = t$; $2x + y - z + 5 = 0$
- $x = 1 - t, y = t, z = 1 + t$; $z = 1 - 2x + y$

35–40 Determine whether the planes are parallel, perpendicular, or neither. If neither, find the angle between them.

- $x + z = 1$, $y + z = 1$
- $-8x - 6y + 2z = 1$, $z = 4x + 3y$
- $x + 4y - 3z = 1$, $-3x + 6y + 7z = 0$
- $2x + 2y - z = 4$, $6x - 3y + 2z = 5$
- $2x + 4y - 2z = 1$, $-3x - 6y + 3z = 10$
- $2x - 5y + z = 3$, $4x + 2y + 2z = 1$

Answers

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- $\mathbf{r} = (3 + 2t)\mathbf{i} + (-1 + 3t)\mathbf{j} + (8 + 5t)\mathbf{k};$
 $x = 3 + 2t, y = -1 + 3t, z = 8 + 5t$
- $\mathbf{r} = (-2 + 3t)\mathbf{i} + (4 - t)\mathbf{j} + (5 + 6t)\mathbf{k};$
 $x = -2 + 3t, y = 4 - t, z = 5 + 6t$
- $\mathbf{r} = (6t)\mathbf{i} + (1 + 3t)\mathbf{j} + (2 + 2t)\mathbf{k};$
 $x = 6t, y = 1 + 3t, z = 2 + 2t$
- $\mathbf{r} = (1 + 2t)\mathbf{i} - \mathbf{j} + (-2 - 7t)\mathbf{k};$
 $x = 1 + 2t, y = -1, z = -(2 + 7t)$
- $x = 2 + 4t, y = 1 - t, z = 8 - 5t;$
 $\frac{x-2}{4} = \frac{y-1}{-1} = \frac{z-8}{-5}$
- $x = -1 + 5t, y = -3t, z = 5 - 2t; \frac{x+1}{5} = \frac{y}{-3} = \frac{z-5}{-2}$
- $x = 3, y = 1 + t, z = -1 - 5t; x = 3, y - 1 = \frac{z+1}{-5}$
- $x = -1 - 4t, y = 4 + 3t, z = 1 + \frac{1}{2}t;$
 $\frac{x+1}{-4} = \frac{y-4}{3} = \frac{z-1}{1/2}$
- $x = -\frac{1}{3} + \frac{1}{3}t, y = 1 + 4t, z = 1 - 9t;$
 $\frac{x+1/3}{1/3} = \frac{y-1}{4} = \frac{z-1}{-9}$
- $x = 2 - 6t, y = -7 + 9t, z = 5; \frac{x-2}{-6} = \frac{y+7}{9}, z = 5$

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- Skew
- Skew
- $3x - 5y + 2z = -16$
- $-5x + 2y - 2z = 1$
- $2x + 5y + 8z = 70$
- $2x + 3y - z = -13$
- $-5x + 7y + 8z = 4$
- $7x - 21y - 9z = 20$
- $x + 2z = 3$
- $y + z = 1$
- $(5, \frac{13}{3}, -\frac{2}{3})$
- $(0, 1, 2)$
- Parallel
- Neither, 79°
- Perpendicular
- Intersecting, $(1, 0, 1)$
- $7x + y + 4z = 31$
- $5x + 3y - 4z = -1$
- $x + y - z = 13$
- $3x - 4y - 6z = 33$
- $x - 2y + z = 0$
- $-17x + 6y + 5z = -32$
- $25x + 14y + 8z = 77$
- $x - 2y + z = 0$
- $(1, 0, 0)$
- $(-3, -1, -2)$
- Neither, 60°
- Perpendicular
- Parallel


Solutions

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- $\mathbf{r}_0 = 3\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ and $\mathbf{v} = \mathbf{a}$ so a vector equation is

$$\mathbf{r} = (3\mathbf{i} - \mathbf{j} + 8\mathbf{k}) + t(2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$$

$$= (3 + 2t)\mathbf{i} + (-1 + 3t)\mathbf{j} + (8 + 5t)\mathbf{k}$$

and parametric equations are $x = 3 + 2t$, $y = -1 + 3t$, $z = 8 + 5t$.
- $\mathbf{r}_0 = -2\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$, $\mathbf{v} = \mathbf{a}$ so a vector equation is

$$\mathbf{r} = (-2\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) + t(3\mathbf{i} - \mathbf{j} + 6\mathbf{k})$$

$$= (-2 + 3t)\mathbf{i} + (4 - t)\mathbf{j} + (5 + 6t)\mathbf{k}$$

while parametric equations are: $x = -2 + 3t$, $y = 4 - t$, $z = 5 + 6t$.
- $\mathbf{r} = (\mathbf{j} + 2\mathbf{k}) + t(6\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$

$$= (6t)\mathbf{i} + (1 + 3t)\mathbf{j} + (2 + 2t)\mathbf{k}$$

is a vector equation, while $x = 6t$, $y = 1 + 3t$, $z = 2 + 2t$ are parametric equations.
- $\mathbf{r} = (\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + t(2\mathbf{i} - 7\mathbf{k})$

$$= (1 + 2t)\mathbf{i} - \mathbf{j} + (-2 - 7t)\mathbf{k}$$

is a vector equation, while $x = 1 + 2t$, $y = -1$, $z = -(2 + 7t)$ are parametric equations.
- The parallel vector is

$$\mathbf{v} = \langle 6 - 2, 0 - 1, 3 - 8 \rangle = \langle 4, -1, -5 \rangle$$

so the direction numbers are $a = 4$, $b = -1$, $c = -5$. Letting $P_0 = (2, 1, 8)$, parametric equations are $x = 2 + 4t$, $y = 1 - t$, $z = 8 - 5t$, and symmetric equations are

$$\frac{x - 2}{4} = \frac{y - 1}{-1} = \frac{z - 8}{-5}.$$
- $\mathbf{v} = \langle 5, -3, -2 \rangle$ and letting $P_0 = (-1, 0, 5)$, parametric equations are $x = -1 + 5t$, $y = -3t$, $z = 5 - 2t$ while symmetric equations are

$$\frac{x + 1}{5} = \frac{y}{-3} = \frac{z - 5}{-2}.$$
- $\mathbf{v} = \langle 0, 1, -5 \rangle$ and letting $P_0 = (3, 1, -1)$, parametric equations are $x = 3$, $y = 1 + t$, $z = -1 - 5t$ while symmetric equations are $x = 3$, $y - 1 = \frac{z + 1}{-5}$. Notice here that the direction number $a = 0$, so rather than writing $\frac{x - 3}{0}$ in the symmetric equation we write the equation $x = 3$ separately.
- $\mathbf{v} = \langle -4, 3, \frac{1}{2} \rangle$ and letting $P_0 = (-1, 4, 1)$, parametric equations are $x = -1 - 4t$, $y = 4 + 3t$, $z = 1 + \frac{1}{2}t$ while symmetric equations are

$$\frac{x + 1}{-4} = \frac{y - 4}{3} = \frac{z - 1}{1/2}.$$
- $\mathbf{v} = \langle \frac{1}{3}, 4, -9 \rangle$ and letting $P_0 = (-\frac{1}{3}, 1, 1)$, parametric equations are $x = -\frac{1}{3} + \frac{1}{3}t$, $y = 1 + 4t$, $z = 1 - 9t$ while symmetric equations are

$$\frac{x + 1/3}{1/3} = \frac{y - 1}{4} = \frac{z - 1}{-9}.$$

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- $\mathbf{v} = \langle -6, 9, 0 \rangle$ and letting $P_0 = (2, -7, 5)$, parametric equations are $x = 2 - 6t$, $y = -7 + 9t$, $z = 5$ while symmetric equations are $\frac{x - 2}{-6} = \frac{y + 7}{9}$, $z = 5$.
- Direction vectors of the lines are $\mathbf{v}_1 = \langle 1, -2, 5 \rangle$ and $\mathbf{v}_2 = \langle 3, 4, 1 \rangle$. Since $\mathbf{v}_1 \cdot \mathbf{v}_2 = 3 - 8 + 5 = 0$, the direction vectors and thus the lines are perpendicular.
- The lines aren't parallel since the direction vectors $\langle 2, 4, -3 \rangle$ and $\langle 1, 3, 2 \rangle$ aren't parallel, so we check to see if the lines intersect. The parametric equations of the lines are L_1 : $x = 4 + 2t$, $y = -5 + 4t$, $z = 1 - 3t$ and L_2 : $x = 2 + s$, $y = -1 + 3s$, $z = 2s$. For the lines to intersect we must be able to find one value of t and one value of s satisfying the following three equations: $4 + 2t = 2 + s$, $-5 + 4t = -1 + 3s$, $1 - 3t = 2s$. Solving the first two equations we get $t = -5$, $s = -8$ and checking, we see that these values don't satisfy the third equation. Thus L_1 and L_2 aren't parallel and don't intersect, so they must be skew lines.
- Since the direction vectors $\langle 2, 1, 4 \rangle$ and $\langle 1, 2, 3 \rangle$ aren't parallel, the lines aren't parallel. Here the parametric equations are L_1 : $x = 1 + 2t$, $y = t$, $z = 1 + 4t$; L_2 : $x = s$, $y = -2 + 2s$, $z = -2 + 3s$. Thus, for the lines to intersect, the three equations $1 + 2t = s$, $t = -2 + 2s$ and $1 + 4t = -2 + 3s$ must be satisfied simultaneously. Solving the first two equations gives $t = 0$, $s = 1$ and, checking, we see these values do satisfy the third equation, so the lines intersect when $t = 0$ and $s = 1$, that is, at the point $(1, 0, 1)$.
- Since the direction vectors are $\langle 1, -1, 3 \rangle$ and $\langle -1, 2, 1 \rangle$, the lines aren't parallel. For the lines to intersect, the three equations $1 + t = 2 - s$, $2 - t = 1 + 2s$, $3t = 4 + s$ must be satisfied simultaneously. Solving the first two equations gives $t = 1$, $s = 0$ and, checking, we see these values don't satisfy the third equation. Thus L_1 and L_2 aren't parallel and don't intersect, so they must be skew lines.
- Setting $a = 7$, $b = 1$, $c = 4$, $x_0 = 1$, $y_0 = 4$, $z_0 = 5$ in Equation 7 gives $7(x - 1) + 1(y - 4) + 4(z - 5) = 0$ or $7x + y + 4z = 31$ to be an equation of the plane.
- Setting $a = 3$, $b = -5$, $c = 2$, $x_0 = -5$, $y_0 = 1$, $z_0 = 2$ in Equation 7 gives $3(x + 5) - 5(y - 1) + 2(z - 2) = 0$ or $3x - 5y + 2z = -16$ to be an equation of the plane.
- Setting $a = 15$, $b = 9$, $c = -12$, $x_0 = 1$, $y_0 = 2$, $z_0 = 3$ in Equation 7 gives $15(x - 1) + 9(y - 2) - 12(z - 3) = 0$ or $5x + 3y - 4z = -1$ to be an equation of the plane.

18. Setting $a = -5$, $b = 2$, $c = -2$, $x_0 = -1$,
 $y_0 = -6$, $z_0 = -4$ in Equation 7 gives
 $-5(x + 1) + 2(y + 6) - 2(z + 4) = 0$ or
 $-5x + 2y - 2z = 1$ to be an equation of the plane.
19. Since the two planes are parallel, they have the same normal vector. Thus $\mathbf{n} = \langle 1, 1, -1 \rangle$ and an equation of the plane is
 $1(x - 6) + 1(y - 5) + 1(z - 2) = 0$ or $x + y - z = 13$.
20. Since the two planes are parallel, they have the same normal vector. Thus $\mathbf{n} = \langle 2, 5, 8 \rangle$ and an equation of the plane is
 $2(x - 3) + 5(y - 0) + 8(z - 8) = 0$ or
 $2x + 5y + 8z = 70$.
21. An equation is $3(x + 1) - 4(y - 3) - 6(z + 8) = 0$ or
 $3x - 4y - 6z = 33$.
22. An equation is $2(x - 2) + 3(y + 4) - (z - 5) = 0$ or
 $2x + 3y - z = -13$.
23. Here the vectors $\mathbf{a} = \langle 1, 1, 1 \rangle$ and $\mathbf{b} = \langle 1, 2, 3 \rangle$ lie in the plane, so $\mathbf{a} \times \mathbf{b}$ is a normal vector to the plane. Thus
 $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 3 - 2, 1 - 3, 2 - 1 \rangle = \langle 1, -2, 1 \rangle$ and an equation of the plane is $x - 2y + z = 0$.
24. Here the vectors $\mathbf{a} = \langle 2, -2, 3 \rangle$ and $\mathbf{b} = \langle 5, -1, 4 \rangle$ lie in the plane, so a normal vector to the plane is
 $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -8 + 3, 15 - 8, -2 + 10 \rangle = \langle -5, 7, 8 \rangle$
and an equation of the plane is
 $-5(x + 1) + 7(y - 1) + 8(z + 1) = 0$ or
 $-5x + 7y + 8z = 4$.
25. $\mathbf{a} = \langle -1, -2, -1 \rangle$ and $\mathbf{b} = \langle 3, 1, 9 \rangle$ so
a normal vector to the plane is
 $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -18 + 1, -3 + 9, -1 + 6 \rangle = \langle -17, 6, 5 \rangle$
and an equation of the plane is
 $-17(x - 1) + 6(y - 0) + 5(z + 3) = 0$ or
 $-17x + 6y + 5z = -32$.
26. $\mathbf{a} = \langle 3, -2, 7 \rangle$ and $\mathbf{b} = \langle -3, -1, 0 \rangle$ so
a normal vector to the plane is
 $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 0 + 7, -21 - 0, -3 - 6 \rangle = \langle 7, -21, -9 \rangle$
and an equation of the plane is
 $7(x - 2) - 21(y - 1) - 9(z + 3) = 0$ or
 $7x - 21y - 9z = 20$.
27. To find an equation of the plane we must first find two nonparallel vectors in the plane, then their cross product will be a normal vector to the plane. But since the given line lies in the plane, its direction vector $\mathbf{a} = \langle 2, -3, -1 \rangle$ is one vector in the plane. To find another nonparallel vector \mathbf{b} which lies in the plane pick any point on the line [say $(1, 2, 3)$, found by setting $t = 0$] and let \mathbf{b} be the vector connecting this point to the given point in the plane. (But beware; we should first check that the given point is not on the given line. If it were on the line, the plane wouldn't be uniquely determined. What would \mathbf{n} then be when we set $\mathbf{n} = \mathbf{a} \times \mathbf{b}$?) Here $\mathbf{b} = \langle 0, 4, -7 \rangle$ so
 $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 21 + 4, 0 + 14, 8 - 0 \rangle = \langle 25, 14, 8 \rangle$
and an equation of the plane is
 $25(x - 1) + 14(y - 7) + 8(z + 4) = 0$ or
 $25x + 14y + 8z = 77$.
28. As in Problem 27, set $\mathbf{a} = \langle -2, 4, 1 \rangle$ and let \mathbf{b} be the vector connecting, say $(-1, 0, 2)$ to $(-1, -3, 2)$. Thus $\mathbf{b} = \langle 0, -3, 0 \rangle$ and
 $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 0 + 3, 0 - 0, 6 - 0 \rangle = \langle 3, 0, 6 \rangle$ and an equation of the plane is $3(x + 1) + 6(z - 2) = 0$ or
 $x + 2z = 3$.
29. $(0, 0, 0)$ is a point on $x = y = z$. $\langle 1, 1, 1 \rangle$ is the direction of this line, and thus also of the plane.
 $\langle 0 - 0, 1 - 0, 2 - 0 \rangle = \langle 0, 1, 2 \rangle$ is also a vector in the plane. Therefore,
 $\mathbf{n} = \langle 1, 1, 1 \rangle \times \langle 0, 1, 2 \rangle = \langle 2 - 1, -2 + 0, 1 - 0 \rangle$
 $= \langle 1, -2, 1 \rangle$
Choosing $(x_0, y_0, z_0) = (0, 0, 0)$, an equation of the plane is, by Equation 8, $x - 2y + z = 1 \cdot 0 - 2 \cdot 0 + 1 \cdot 0 \Leftrightarrow$
 $x - 2y + z = 0$.
30. $\mathbf{a} = \langle 5, 1, -1 \rangle$ is the direction of the line and hence also a direction of the plane. $(0, 1, 0)$ is a point on the line (and in the plane) so that $\mathbf{b} = \langle 0 + 1, 1 - 0, 0 - 1 \rangle = \langle 1, 1, -1 \rangle$ is another direction of the plane.
 $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -1 + 1, 5 - 1, 5 - 1 \rangle = \langle 0, 4, 4 \rangle$
and an equation of the plane is
 $0(x + 1) + 4(y - 0) + 4(z - 1) = 0 \Leftrightarrow y + z = 1$.
31. Substituting the parametric equations of the line into the equation of the plane gives $x + y + z = 1 + t + 2t + 3t = 1 \Rightarrow t = 0$. This value of t corresponds to the point of intersection $(1, 0, 0)$, obtained by substitution of $t = 0$ into the equations of the line.
32. Substitute the parametric expressions for x , y and z into the equation of the plane:
 $2x - y + z = 2(5) - (4 - t) + 2t = 5 \Rightarrow t = -\frac{1}{3}$.
Therefore, the point of intersection of the line and the plane is given by $x = 5$, $y = 4 - (-\frac{1}{3}) = \frac{13}{3}$ and
 $z = 2(-\frac{1}{3}) = -\frac{2}{3}$, that is, the point $(5, \frac{13}{3}, -\frac{2}{3})$.
33. Substituting the parametric equations of the line into the equation of the plane gives
 $2x + y - z + 5 = 2(1 + 2t) + (-1) - t + 5 = 0 \Rightarrow$
 $3t + 6 = 0 \Rightarrow t = -2$. Therefore, the point of intersection is $x = 1 + 2(-2) = -3$, $y = -1$ and $z = -2$ and the point of intersection is $(-3, -1, -2)$.

- 34.** Substitution into the equation of the plane of the parametric expressions for x , y and z gives $z = 1 - 2x + y \Rightarrow (1 + t) = 1 - 2(1 - t) + t \Rightarrow -2 + 2t = 0 \Rightarrow t = 1$. Thus, $x = 1 - 1$, $y = 1$ and $z = 1 + 1$ and the point of intersection is $(0, 1, 2)$.
- 35.** The normal vectors to the planes are $\mathbf{n}_1 = \langle 1, 0, 1 \rangle$ and $\mathbf{n}_2 = \langle 0, 1, 1 \rangle$. Thus the normal vectors (and consequently the planes) aren't parallel. Furthermore, $\mathbf{n}_1 \cdot \mathbf{n}_2 = 1 \neq 0$ so the planes aren't perpendicular. Letting θ be the angle between the two planes, we have $\cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1| |\mathbf{n}_2|} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$ and $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$.
- 36.** Here the normals are $\mathbf{n}_1 = \langle -8, -6, 2 \rangle$ and $\mathbf{n}_2 = \langle 4, 3, -1 \rangle$. Since $\mathbf{n}_1 = -2\mathbf{n}_2$, the normals (and thus the planes) are parallel.
- 37.** The normals are $\mathbf{n}_1 = \langle 1, 4, -3 \rangle$ and $\mathbf{n}_2 = \langle -3, 6, 7 \rangle$, so the normals (and thus the planes) aren't parallel. But $\mathbf{n}_1 \cdot \mathbf{n}_2 = -3 + 24 - 21 = 0$, so the normals (and thus the planes) are perpendicular.
- 38.** The normals are $\mathbf{n}_1 = \langle 2, 2, -1 \rangle$ and $\mathbf{n}_2 = \langle 6, -3, 2 \rangle$ so the planes aren't parallel. Furthermore, $\mathbf{n}_1 \cdot \mathbf{n}_2 = 12 - 6 - 2 = 4 \neq 0$, so the planes aren't perpendicular. Then $\cos \theta = \frac{4}{\sqrt{9}\sqrt{49}} = \frac{4}{21}$ and $\theta = \cos^{-1}\left(\frac{4}{21}\right) \approx 79^\circ$.
- 39.** The normals are $\mathbf{n}_1 = \langle 2, 4, -2 \rangle$ and $\mathbf{n}_2 = \langle -3, -6, 3 \rangle$. Since $\mathbf{n}_1 = -\frac{3}{2}\mathbf{n}_2$, the normals (and thus the planes) are parallel.
- 40.** The normals are $\mathbf{n}_1 = \langle 2, -5, 1 \rangle$ and $\mathbf{n}_2 = \langle 4, 2, 2 \rangle$. $\mathbf{n}_1 \cdot \mathbf{n}_2 = \langle 2, -5, 1 \rangle \cdot \langle 4, 2, 2 \rangle = 8 - 10 + 2 = 0$, so the normals (and thus the planes) are perpendicular.