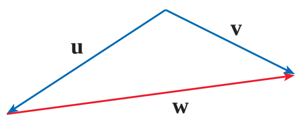
9.2 Vectors

A Click here for answers.

S Click here for solutions.

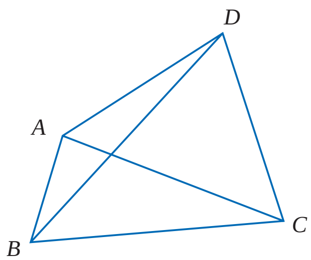
1. Express **w** in terms of the vectors **u** and **v** in the figure.



2. Write each combination of vectors as a single vector.

(a) $\overrightarrow{AB} + \overrightarrow{BC}$ (b) $\overrightarrow{CD} + \overrightarrow{DA}$

(c) $\overrightarrow{BC} - \overrightarrow{DC}$ (d) $\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}$



3–5 Find a vector **a** with representation given by the directed line segment $\overrightarrow{AB}$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

3. $A(1, 3)$, $B(4, 4)$

4. $A(4, -1)$, $B(1, 2)$

5. $A(1, -2, 0)$, $B(1, -2, 3)$

6–9 Find the sum of the given vectors and illustrate geometrically.

6. $\langle 2, 3 \rangle$, $\langle 3, -4 \rangle$

7. $\langle -1, 2 \rangle$, $\langle 5, 3 \rangle$

8. $\langle 1, 0, 1 \rangle$, $\langle 0, 0, 1 \rangle$

9. $\langle 0, 3, 2 \rangle$, $\langle 1, 0, -3 \rangle$

10–15 Find a unit vector that has the same direction as the given vector.

10. $\langle 1, 2 \rangle$

11. $\langle 3, -5 \rangle$

12. $\langle -2, 4, 3 \rangle$

13. $\langle 1, -4, 8 \rangle$

14. $\mathbf{i} + \mathbf{j}$

15. $2\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$

16. A quadrilateral has one pair of opposite sides parallel and of equal length. Use vectors to prove that the other pair of opposite sides is parallel and of equal length.

Answers

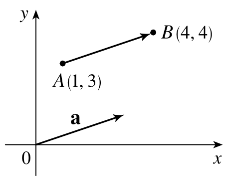
E Click here for exercises.

S Click here for solutions.

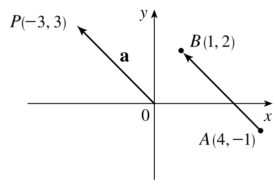
1. $\mathbf{w} = \mathbf{v} - \mathbf{u}$

2. (a) $\overrightarrow{AC}$ (b) $\overrightarrow{CA}$ (c) $\overrightarrow{BD}$ (d) $\overrightarrow{BA}$

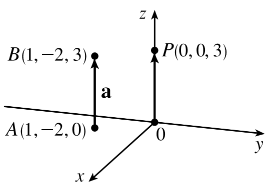
3. $\langle 3, 1 \rangle$



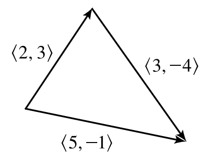
4. $\langle -3, 3 \rangle$



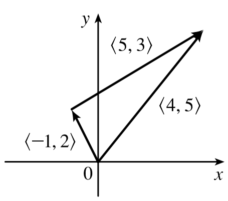
5. $\langle 0, 0, 3 \rangle$



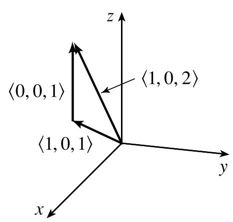
6. $\langle 5, -1 \rangle$



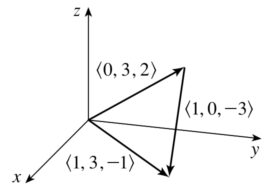
7. $\langle 4, 5 \rangle$



8. $\langle 1, 0, 2 \rangle$



9. $\langle 1, 3, -1 \rangle$



10. $\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

11. $\left\langle \frac{3}{\sqrt{34}}, -\frac{5}{\sqrt{34}} \right\rangle$

12. $\left\langle -\frac{2}{\sqrt{29}}, \frac{4}{\sqrt{29}}, \frac{3}{\sqrt{29}} \right\rangle$

13. $\left\langle \frac{1}{9}, -\frac{4}{9}, \frac{8}{9} \right\rangle$

14. $\frac{1}{\sqrt{2}}\,\mathbf{i} + \frac{1}{\sqrt{2}}\,\mathbf{j}$

15. $\frac{2}{\sqrt{69}}\,\mathbf{i} - \frac{4}{\sqrt{69}}\,\mathbf{j} + \frac{7}{\sqrt{69}}\,\mathbf{k}$

Solutions

E Click here for exercises.

A Click here for answers.

1. Geometrically, by the Triangle Law, we can see that $\mathbf{u}+\mathbf{w}=\mathbf{v}$, thus $\mathbf{w}=\mathbf{v}-\mathbf{u}$. Alternately, $\mathbf{w}$ can be visualized directly as the difference of $\mathbf{v}$ and $\mathbf{u}$ (see Figure 8 in the text).

2. (a) By the Triangle Law, $\overrightarrow{AB}+\overrightarrow{BC}$ is the vector with initial point A and terminal point C, namely $\overrightarrow{AC}$.

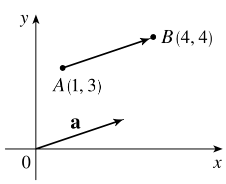
(b) By the Triangle Law, $\overrightarrow{CD}+\overrightarrow{DA}$ is the vector with initial point C and terminal point A, namely $\overrightarrow{CA}$.

(c) First we consider $\overrightarrow{BC}-\overrightarrow{DC}$ as $\overrightarrow{BC}+\left(-\overrightarrow{DC}\right)$. Then since $-\overrightarrow{DC}$ has the same length as $\overrightarrow{CD}$ but points in the opposite direction, we have $-\overrightarrow{DC}=\overrightarrow{CD}$ and so $\overrightarrow{BC}-\overrightarrow{DC}=\overrightarrow{BC}+\overrightarrow{CD}=\overrightarrow{BD}$.

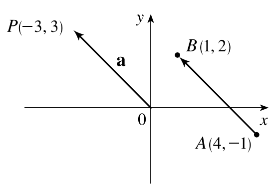
(d) We use the Triangle Law twice:

$$\begin{aligned}\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DA} &= \left(\overrightarrow{BC}+\overrightarrow{CD}\right)+\overrightarrow{DA}\\ &= \overrightarrow{BD}+\overrightarrow{DA}=\overrightarrow{BA}\end{aligned}$$

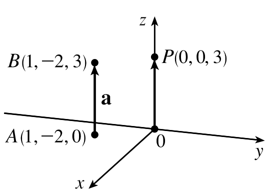
3. $\mathbf{a}=\langle 4-1, 4-3\rangle=\langle 3,1\rangle$



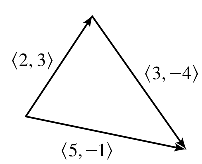
4. $\mathbf{a}=\langle 1-4, 2+1\rangle$
$=\langle -3,3\rangle$



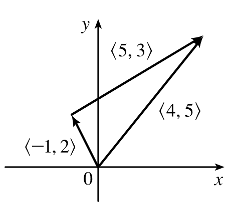
5. $\mathbf{a}=\langle 1-1, -2+2, 3-0\rangle$
$=\langle 0,0,3\rangle$



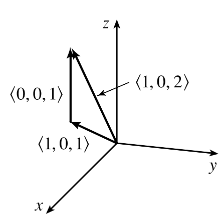
6. $\langle 2,3\rangle+\langle 3,-4\rangle=\langle 5,-1\rangle$ (using position vectors and the parallelogram law)



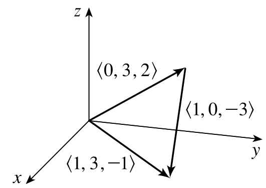
7. $\langle -1,2\rangle+\langle 5,3\rangle$
$=\langle -1+5, 2+3\rangle$
$=\langle 4,5\rangle$



8. $\langle 1,0,1\rangle+\langle 0,0,1\rangle$
$=\langle 1+0, 0+0, 1+1\rangle$
$=\langle 1,0,2\rangle$



9. $\langle 0,3,2\rangle+\langle 1,0,-3\rangle=\langle 1,3,-1\rangle$



10. $|\langle 1,2\rangle|=\sqrt{1^2+2^2}=\sqrt{5}$. Thus
$\mathbf{u}=\frac{1}{\sqrt{5}}\langle 1,2\rangle=\left\langle\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}}\right\rangle$.

11. $|\langle 3,-5\rangle|=\sqrt{3^2+(-5)^2}=\sqrt{34}$. Thus
$\mathbf{u}=\frac{1}{\sqrt{34}}\langle 3,-5\rangle=\left\langle\frac{3}{\sqrt{34}},-\frac{5}{\sqrt{34}}\right\rangle$.

12. $|\langle -2,4,3\rangle|=\sqrt{(-2)^2+4^2+3^2}=\sqrt{29}$. Thus
$\mathbf{u}=\frac{1}{\sqrt{29}}\langle -2,4,3\rangle=\left\langle -\frac{2}{\sqrt{29}},\frac{4}{\sqrt{29}},\frac{3}{\sqrt{29}}\right\rangle$.

13. $|\langle 1,-4,8\rangle|=\sqrt{1^2+(-4)^2+8^2}=\sqrt{81}=9$. Thus
$\mathbf{u}=\frac{1}{9}\langle 1,-4,8\rangle=\left\langle\frac{1}{9},-\frac{4}{9},\frac{8}{9}\right\rangle$.

14. $|\mathbf{i}+\mathbf{j}|=\sqrt{1^2+1^2}=\sqrt{2}$. Thus
$\mathbf{u}=\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})=\frac{1}{\sqrt{2}}\mathbf{i}+\frac{1}{\sqrt{2}}\mathbf{j}$.

15. $|2\,\mathbf{i}-4\,\mathbf{j}+7\,\mathbf{k}|=\sqrt{2^2+(-4)^2+7^2}=\sqrt{69}$. Thus
$\mathbf{u}=\frac{1}{\sqrt{69}}(2\,\mathbf{i}-4\,\mathbf{j}+7\,\mathbf{k})=\frac{2}{\sqrt{69}}\mathbf{i}-\frac{4}{\sqrt{69}}\mathbf{j}+\frac{7}{\sqrt{69}}\mathbf{k}$.

16. Consider quadrilateral $ABCD$ with sides AB and CD parallel and of equal length; that is, $\overrightarrow{AB}=\overrightarrow{DC}$. Thus

$$\begin{aligned}\overrightarrow{AD} &= \overrightarrow{AB}+\overrightarrow{BD}=\overrightarrow{DC}+\overrightarrow{BD}\quad\left(\text{since }\overrightarrow{AB}=\overrightarrow{DC}\right)\\ &= \overrightarrow{BD}+\overrightarrow{DC}=\overrightarrow{BC}\end{aligned}$$

This shows that sides AD and BC are parallel and have equal lengths.