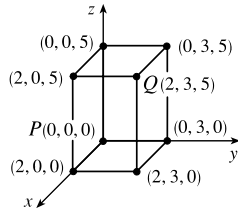




Answers

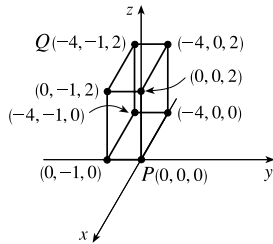
**E** Click here for exercises.

1. (a)



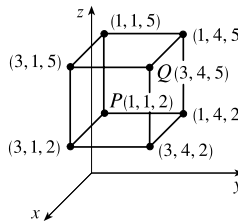
(b)  $\sqrt{38}$

2. (a)



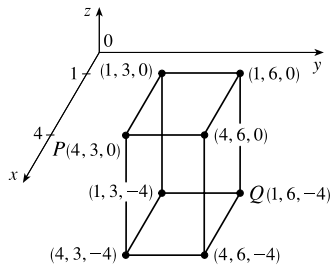
(b)  $\sqrt{21}$

3. (a)



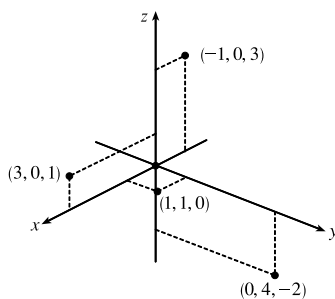
(b)  $\sqrt{22}$

4. (a)



(b)  $\sqrt{34}$

5.

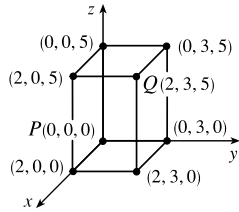


**S** Click here for solutions.

6.  $|AB| = \sqrt{6}$ ,  $|BC| = \sqrt{33}$ ,  $|CA| = 3\sqrt{3}$ ; right triangle
7.  $|AB| = \sqrt{21}$ ,  $|BC| = \sqrt{6}$ ,  $|CA| = 3\sqrt{3}$ ; right triangle
8.  $|AB| = 3$ ,  $|BC| = 3$ ,  $|CA| = \sqrt{26}$ ; isosceles
9.  $|AB| = \sqrt{69}$ ,  $|BC| = \sqrt{158}$ ,  $|CA| = 13$ ; neither
10.  $x^2 + (y-1)^2 + (z+1)^2 = 16$ ;  $(y-1)^2 + (z+1)^2 = 16$ ,  $x = 0$
11.  $(x+1)^2 + (y-2)^2 + (z-4)^2 = \frac{1}{4}$
12.  $(x+6)^2 + (y+1)^2 + (z-2)^2 = 12$
13.  $(x-1)^2 + (y-2)^2 + (z+3)^2 = 49$
14.  $(-1, -4, 2)$ , 7
15.  $(0, -1, \frac{1}{2})$ ,  $\frac{\sqrt{7}}{2}$
16.  $(3, 2, 5)$ ,  $\sqrt{38}$
17.  $(-\frac{1}{2}, 1, -3)$ ,  $\frac{7}{2}$
18.  $(\frac{1}{2}, 0, 0)$ ,  $\frac{1}{2}$
19.  $(-\frac{1}{2}a, -\frac{1}{2}b, -\frac{1}{2}c)$ ,  $\sqrt{\frac{1}{4}(a^2 + b^2 + c^2) - d}$
20.  $(x-1)^2 + (y-2)^2 + (z-3)^2 = 30$
21. A plane parallel to the  $yz$ -plane and 9 units in front of it
22. A plane parallel to the  $xy$ -plane and 8 units below it
23. A half-space containing all points to the right of the plane  $y = 2$
24. A half-space containing all points on and below the  $xy$ -plane
25. All points on and between the two horizontal planes  $z = 2$  and  $z = -2$
26. A plane perpendicular to the  $xz$ -plane and intersecting the  $xz$ -plane in the line  $x = z$ ,  $y = 0$
27. The two planes  $x = 0$  and  $y = 0$
28. A hyperbolic cylinder

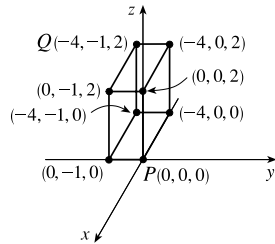

**Solutions**
[Click here for exercises.](#)

1. (a)



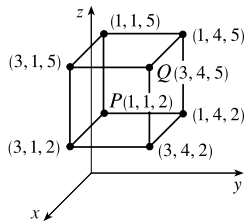
$$(b) |PQ| = \sqrt{(2-0)^2 + (3-0)^2 + (5-0)^2} = \sqrt{38}$$

2. (a)



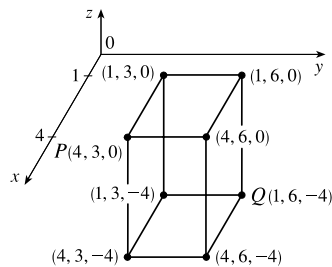
$$(b) |PQ| = \sqrt{(-4-0)^2 + (-1-0)^2 + (2-0)^2} = \sqrt{21}$$

3. (a)



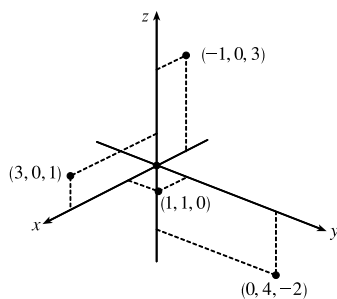
$$(b) |PQ| = \sqrt{(3-1)^2 + (4-1)^2 + (5-2)^2} = \sqrt{22}$$

4. (a)



$$(b) |PQ| = \sqrt{(1-4)^2 + (6-3)^2 + (-4-0)^2} = \sqrt{34}$$

5.


[Click here for answers.](#)

6. We can find the lengths of the sides of the triangle by using the distance formula between pairs of vertices:

$$|AB| = \sqrt{(5-3)^2 + [-3-(-4)]^2 + (0-1)^2} \\ = \sqrt{4+1+1} = \sqrt{6}$$

$$|BC| = \sqrt{(6-5)^2 + [-7-(-3)]^2 + (4-0)^2} \\ = \sqrt{1+16+16} = \sqrt{33}$$

$$|CA| = \sqrt{(3-6)^2 + [-4-(-7)]^2 + (1-4)^2} \\ = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

Since the Pythagorean Theorem is satisfied by  $|AB|^2 + |CA|^2 = |BC|^2$ ,  $ABC$  is a right triangle.  $ABC$  is not isosceles, as no two sides have the same length.

$$7. |AB| = \sqrt{(3-2)^2 + (3-1)^2 + (4-0)^2} = \sqrt{21}$$

$$|BC| = \sqrt{(5-3)^2 + (4-3)^2 + (3-4)^2} = \sqrt{6}$$

$$|CA| = \sqrt{(5-2)^2 + (4-1)^2 + (3-0)^2} = \sqrt{27} = 3\sqrt{3}$$

Since no two of the sides are equal in length the triangle isn't isosceles. But  $|AB|^2 + |BC|^2 = 27 = |CA|^2$ , so the triangle is a right triangle.

$$8. |AB| = \sqrt{(3-5)^2 + (3-5)^2 + (2-1)^2} = \sqrt{9} = 3$$

$$|BC| = \sqrt{(1-3)^2 + (4-3)^2 + (4-2)^2} = \sqrt{9} = 3$$

$$|CA| = \sqrt{(5-1)^2 + (5-4)^2 + (1-4)^2} = \sqrt{26}$$

Since  $|AB| = |BC|$  the triangle is isosceles. But the sum of the squares of the lengths of the shorter sides doesn't equal the square of the length of the longest side, so the triangle isn't a right triangle.

$$9. |AB| = \sqrt{[5-(-2)]^2 + (4-6)^2 + (-3-1)^2} \\ = \sqrt{49+4+16} = \sqrt{69}$$

$$|BC| = \sqrt{(2-5)^2 + (-6+4)^2 + [4-(-3)]^2} \\ = \sqrt{9+100+49} = \sqrt{158}$$

$$|CA| = \sqrt{(-2-2)^2 + [6-(-6)]^2 + (1-4)^2} \\ = \sqrt{16+144+9} = \sqrt{169} = 13$$

Since no two sides are of equal length and since  $|AB|^2 + |BC|^2 \neq |CA|^2$  the triangle is neither isosceles nor a right triangle.

10. An equation of the sphere with center  $(0, 1, -1)$  and radius 4 is  $(x - 0)^2 + (y - 1)^2 + [z - (-1)]^2 = 4^2$  or  $x^2 + (y - 1)^2 + (z + 1)^2 = 16$ . The intersection of this sphere with the  $yz$ -plane is the set of points on the sphere whose  $x$ -coordinate is 0. Putting  $x = 0$  in the equation, we have  $(y - 1)^2 + (z + 1)^2 = 16$ ,  $x = 0$ , which represents a circle in the  $yz$ -plane with center  $(0, 1, -1)$  and radius 4.

11.  $[x - (-1)]^2 + (y - 2)^2 + (z - 4)^2 = (\frac{1}{2})^2$  or  
 $(x + 1)^2 + (y - 2)^2 + (z - 4)^2 = \frac{1}{4}$

12.  $(x + 6)^2 + (y + 1)^2 + (z - 2)^2 = 12$

13.  $(x - 1)^2 + (y - 2)^2 + (z + 3)^2 = 49$

14. Completing squares in the equation gives

$$(x^2 + 2x + 1) + (y^2 + 8y + 16) + (z^2 - 4z + 4) = 28 + 1 + 16 + 4 \Rightarrow$$

$$(x + 1)^2 + (y + 4)^2 + (z - 2)^2 = 49, \text{ which we recognize}$$

as an equation of a sphere with center  $(-1, -4, 2)$  and radius 7.

15. Completing squares in the equation gives

$$2x^2 + 2(y^2 + 2y + 1) + 2(z^2 - z + \frac{1}{4}) = 1 + 2 + \frac{1}{2} = \frac{7}{2} \Rightarrow$$

$$(x - 0)^2 + (y + 1)^2 + (z - \frac{1}{2})^2 = \frac{7}{4}, \text{ which we recognize}$$

as an equation of a sphere with center  $(0, -1, \frac{1}{2})$  and radius  $\frac{\sqrt{7}}{2}$ .

16.  $(x^2 - 6x + 9) + (y^2 - 4y + 4) + (z^2 - 10z + 25) = 9 + 4 + 25 \Rightarrow$   
 $(x - 3)^2 + (y - 2)^2 + (z - 5)^2 = 38 \Rightarrow C(3, 2, 5),$   
 and  $r = \sqrt{38}$ .

17.  $(x^2 + x + \frac{1}{4}) + (y^2 - 2y + 1) + (z^2 + 6z + 9) = 2 + \frac{1}{4} + 1 + 9 \Rightarrow$   
 $(x + \frac{1}{2})^2 + (y - 1)^2 + (z + 3)^2 = \frac{49}{4} \Rightarrow$   
 $C(-\frac{1}{2}, 1, -3), \text{ and } r = \frac{7}{2}.$

18.  $(x^2 - x + \frac{1}{4}) + y^2 + z^2 = 0 + \frac{1}{4} \Rightarrow$   
 $(x - \frac{1}{2})^2 + (y - 0)^2 + (z - 0)^2 = \frac{1}{4} \Rightarrow C(\frac{1}{2}, 0, 0),$   
 $r = \frac{1}{2}.$

19.  $(x^2 + ax + \frac{1}{4}a^2) + (y^2 + by + \frac{1}{4}b^2) + (z^2 + cz + \frac{1}{4}c^2) = -d + \frac{1}{4}(a^2 + b^2 + c^2) \Rightarrow$   
 $(x + \frac{1}{2}a)^2 + (y + \frac{1}{2}b)^2 + (z + \frac{1}{2}c)^2 = \frac{1}{4}(a^2 + b^2 + c^2) - d$   
 $\Rightarrow C(-\frac{1}{2}a, -\frac{1}{2}b, -\frac{1}{2}c), \text{ and } r = \sqrt{\frac{1}{4}(a^2 + b^2 + c^2) - d}.$

20.  $r^2$  is equal to the square of the distance between the center of the sphere,  $(1, 2, 3)$ , and the given point  $(-1, 1, -2)$ . That is,  $r^2 = (-1 - 1)^2 + (1 - 2)^2 + (-2 - 3)^2 = 30$ .

Therefore, an equation of the sphere is  $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 30$ .

21. The equation  $x = 9$  represents a plane parallel to the  $yz$ -plane and 9 units in front of it.

22. The equation  $z = -8$  represents a plane parallel to the  $xy$ -plane and 8 units below it.

23. The inequality  $y > 2$  represents a half-space containing all points to the right of the plane  $y = 2$ .

24. The inequality  $z \leq 0$  represents a half-space containing all points on and below the  $xy$ -plane.

25. The inequality  $|z| \leq 2$  is equivalent to  $-2 \leq z \leq 2$ , so it represents all points on and between the two horizontal planes  $z = 2$  and  $z = -2$ .

26. The equation  $z = x$  represents a plane perpendicular to the  $xz$ -plane and intersecting the  $xz$ -plane in the line  $x = z, y = 0$ .

27. Since either  $x$  or  $y$  must be zero, the region consists of the two perpendicular planes  $x = 0$  (the  $yz$ -plane) and  $y = 0$  (the  $xz$ -plane).

28. In the  $xy$ -plane the equation  $xy = 1$  represents a hyperbola with center at the origin. Since  $z$  can assume any value, the region in  $\mathbb{R}^3$  is a hyperbolic cylinder.