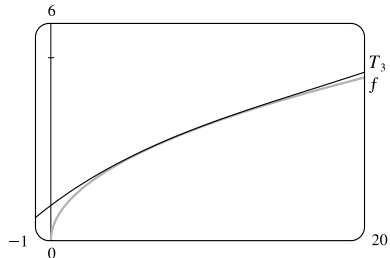


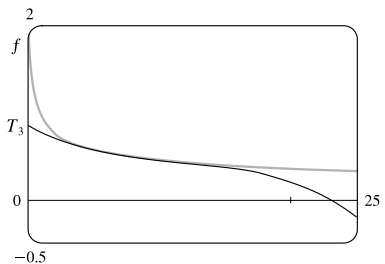
Answers

E Click here for exercises.

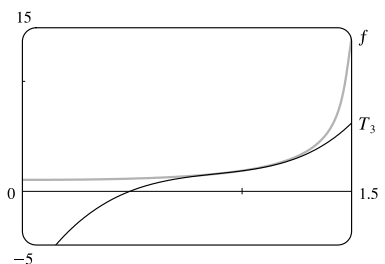
1. $3 + \frac{1}{6}(x-9) - \frac{1}{216}(x-9)^2 + \frac{1}{3888}(x-9)^3$



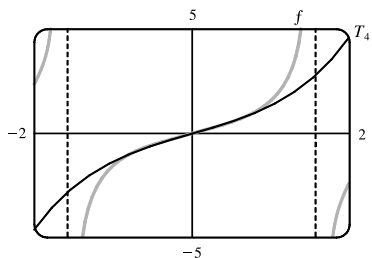
2. $\frac{1}{2} - \frac{1}{48}(x-8) + \frac{1}{576}(x-8)^2 - \frac{7}{41,472}(x-8)^3$



3. $2 + 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + 7\left(x - \frac{\pi}{3}\right)^2 + \frac{23\sqrt{3}}{3}\left(x - \frac{\pi}{3}\right)^3$

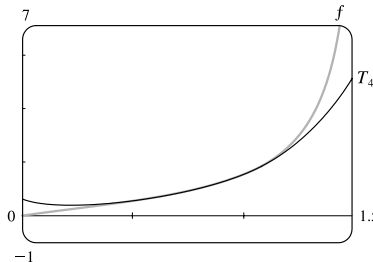


4. $x + \frac{x^3}{3}$

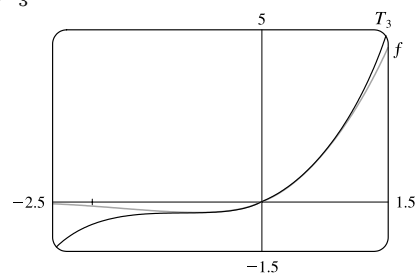


S Click here for solutions.

5. $1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3 + \frac{10}{3}\left(x - \frac{\pi}{4}\right)^4$



6. $x + x^2 + \frac{1}{3}x^3$



7. (a) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3 + \frac{\sqrt{2}}{48}\left(x - \frac{\pi}{4}\right)^4 + \frac{\sqrt{2}}{240}\left(x - \frac{\pi}{4}\right)^5$
 (b) 0.00033

8. (a) $1 + \frac{1}{3}x^2$
 (b) 0.014895

9. (a) $8 + \frac{3}{8}(x-16) - \frac{3}{1024}(x-16)^2 + \frac{5}{65,536}(x-16)^3$
 (b) 0.000003

10. (a) $\ln 4 + \frac{1}{4}(x-4) - \frac{1}{32}(x-4)^2 + \frac{1}{192}(x-4)^3$
 (b) 0.0031

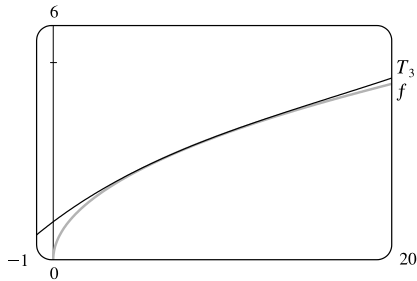
Solutions

E Click here for exercises.

1.

n	$f^{(n)}(x)$	$f^{(n)}(9)$
0	$x^{1/2}$	3
1	$\frac{1}{2}x^{-1/2}$	$\frac{1}{6}$
2	$-\frac{1}{4}x^{-3/2}$	$-\frac{1}{108}$
3	$\frac{3}{8}x^{-5/2}$	$\frac{1}{648}$

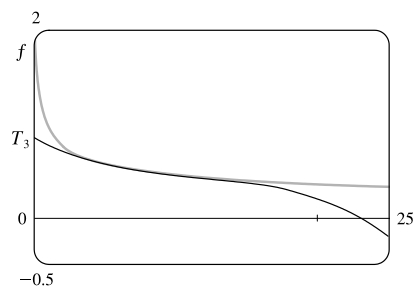
$$\begin{aligned} T_3(x) &= \sum_{n=0}^3 \frac{f^{(n)}(9)}{n!} (x-9)^n \\ &= 3 + \frac{1}{6}(x-9) - \frac{1}{216}(x-9)^2 + \frac{1}{3888}(x-9)^3 \end{aligned}$$



2.

n	$f^{(n)}(x)$	$f^{(n)}(8)$
0	$x^{-1/3}$	$\frac{1}{2}$
1	$-\frac{1}{3}x^{-4/3}$	$-\frac{1}{48}$
2	$\frac{4}{9}x^{-7/3}$	$\frac{1}{288}$
3	$-\frac{28}{27}x^{-10/3}$	$-\frac{7}{6912}$

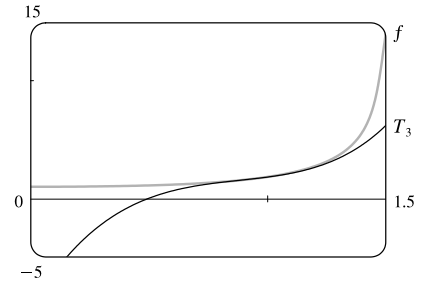
$$\begin{aligned} T_3(x) &= \sum_{n=0}^3 \frac{f^{(n)}(8)}{n!} (x-8)^n \\ &= \frac{1}{2} - \frac{1}{48}(x-8) + \frac{1}{576}(x-8)^2 - \frac{7}{41,472}(x-8)^3 \end{aligned}$$



3.

n	$f^{(n)}(x)$	$f^{(n)}(\frac{\pi}{3})$
0	$\sec x$	2
1	$\sec x \tan x$	$2\sqrt{3}$
2	$\sec x \tan^2 x + \sec^3 x$	14
3	$\sec x \tan^3 x + 5 \sec^3 x \tan x$	$46\sqrt{3}$

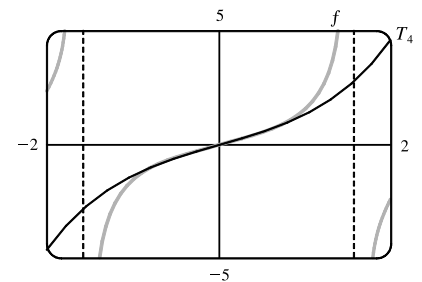
$$\begin{aligned} T_3(x) &= \sum_{n=0}^3 \frac{f^{(n)}(\frac{\pi}{3})}{n!} \left(x - \frac{\pi}{3}\right)^n \\ &= 2 + 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + 7\left(x - \frac{\pi}{3}\right)^2 + \frac{23\sqrt{3}}{3}\left(x - \frac{\pi}{3}\right)^3 \end{aligned}$$



4.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$\tan x$	0
1	$\sec^2 x$	1
2	$2 \sec^2 x \tan x$	0
3	$4 \sec^2 x \tan^2 x + 2 \sec^4 x$	2
4	$8 \sec^2 x \tan^3 x + 16 \sec^4 x \tan x$	0

$$T_4(x) = \sum_{n=0}^4 \frac{f^{(n)}(0)}{n!} x^n = x + \frac{2x^3}{3!} = x + \frac{x^3}{3}$$

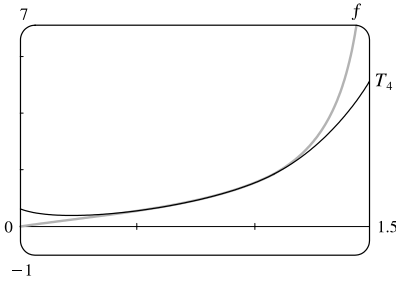


5.

n	$f^{(n)}(x)$	$f^{(n)}(\frac{\pi}{4})$
0	$\tan x$	1
1	$\sec^2 x$	2
2	$2 \sec^2 x \tan x$	4
3	$4 \sec^2 x \tan^2 x + 2 \sec^4 x$	16
4	$8 \sec^2 x \tan^3 x + 16 \sec^4 x \tan x$	80

$$T_4(x) = \sum_{n=0}^4 \frac{f^{(n)}(\frac{\pi}{4})}{n!} (x - \frac{\pi}{4})^n$$

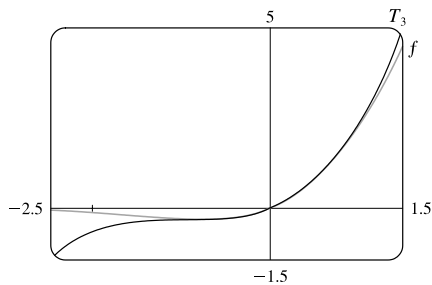
$$= 1 + 2(x - \frac{\pi}{4}) + 2(x - \frac{\pi}{4})^2 + \frac{8}{3}(x - \frac{\pi}{4})^3 + \frac{10}{3}(x - \frac{\pi}{4})^4$$



6.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$e^x \sin x$	0
1	$e^x (\sin x + \cos x)$	1
2	$2e^x \cos x$	2
3	$2e^x (\cos x - \sin x)$	2

$$T_3(x) = \sum_{n=0}^3 \frac{f^{(n)}(0)}{n!} x^n = x + x^2 + \frac{1}{3}x^3$$



7.

$$\begin{aligned} f(x) &= \sin x & f(\frac{\pi}{4}) &= \frac{\sqrt{2}}{2} \\ f'(x) &= \cos x & f'(\frac{\pi}{4}) &= \frac{\sqrt{2}}{2} \\ f''(x) &= -\sin x & f''(\frac{\pi}{4}) &= -\frac{\sqrt{2}}{2} \\ f'''(x) &= -\cos x & f'''(\frac{\pi}{4}) &= -\frac{\sqrt{2}}{2} \\ f^{(4)}(x) &= \sin x & f^{(4)}(\frac{\pi}{4}) &= \frac{\sqrt{2}}{2} \\ f^{(5)}(x) &= \cos x & f^{(5)}(\frac{\pi}{4}) &= \frac{\sqrt{2}}{2} \\ f^{(6)}(x) &= -\sin x \end{aligned}$$

(a) $\sin x \approx T_5(x)$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x - \frac{\pi}{4}) - \frac{\sqrt{2}}{4}(x - \frac{\pi}{4})^2 - \frac{\sqrt{2}}{12}(x - \frac{\pi}{4})^3 + \frac{\sqrt{2}}{48}(x - \frac{\pi}{4})^4 + \frac{\sqrt{2}}{240}(x - \frac{\pi}{4})^5$$

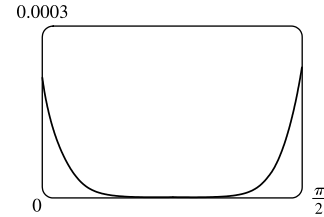
(b) $|R_5(x)| \leq \frac{M}{6!} |x - \frac{\pi}{4}|^6$, where $|f^{(6)}(x)| \leq M$. Now

$$0 \leq x \leq \frac{\pi}{2} \Rightarrow (x - \frac{\pi}{4})^6 \leq (\frac{\pi}{4})^6, \text{ and}$$

$$\text{letting } x = \frac{\pi}{2} \text{ gives } M = 1, \text{ so}$$

$$|R_5(x)| \leq \frac{1}{6!} (\frac{\pi}{4})^6 = \frac{1}{720} (\frac{\pi}{4})^6 \approx 0.00033.$$

(c)



From the graph of $|R_5(x)| = |\sin x - T_5(x)|$, it seems that the error is less than 0.00026 on $[0, \frac{\pi}{2}]$.

8.

$$f(x) = (1 + x^2)^{1/3} \quad f(0) = 1$$

$$f'(x) = \frac{2}{3}x(1 + x^2)^{-2/3} \quad f'(0) = 0$$

$$f''(x) = \frac{2}{3}(1 - \frac{1}{3}x^2)(1 + x^2)^{-5/3} \quad f''(0) = \frac{2}{3}$$

$$f'''(x) = \frac{8x^3 - 72x}{27(1 + x^2)^{8/3}}$$

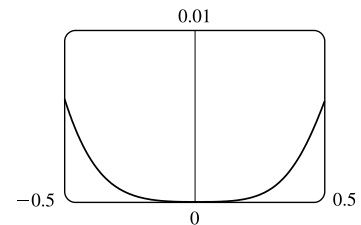
(a) $\sqrt[3]{1 + x^2} \approx T_2(x) = 1 + \frac{1}{3}x^2$

(b) $|R_2(x)| \leq \frac{M}{3!} |x|^3$, where $|f'''(x)| \leq M$. By

examining a graph of $|f'''(x)|$, we see that its maximum is approximately 0.71495314. Thus,

$$|R_2(x)| \leq \frac{0.71495314}{3!} (0.5)^3 \approx 0.014895.$$

(c)



It seems that the error is less than 0.0061 on $[-0.5, 0.5]$.

9.

$$f(x) = x^{3/4} \quad f(16) = 8$$

$$f'(x) = \frac{3}{4}x^{-1/4} \quad f'(16) = \frac{3}{8}$$

$$f''(x) = -\frac{3}{16}x^{-5/4} \quad f''(16) = -\frac{3}{512}$$

$$f'''(x) = \frac{15}{64}x^{-9/4} \quad f'''(16) = \frac{15}{32,768}$$

$$f^{(4)}(x) = -\frac{135}{256}x^{-13/4}$$

(a) $x^{3/4} \approx T_3(x)$

$$= 8 + \frac{3}{8}(x - 16) - \frac{3}{1024}(x - 16)^2 + \frac{5}{65,536}(x - 16)^3$$

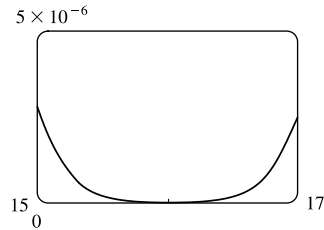
(b) $|R_3(x)| \leq \frac{M}{4!} |x - 16|^4$, where $|f^{(4)}(x)| \leq M$. Now

$$15 \leq x \leq 17 \Rightarrow |x - 16|^4 \leq 1^4 = 1, \text{ and letting}$$

$x = 15$ to minimize the denominator of $f^{(4)}(x)$ gives

$$|R_3(x)| \leq \frac{135 / [256 (15)^{13/4}]}{4!} (1) \approx 0.000003.$$

(c)



It appears that the error is less than 3×10^{-6} on $[15, 17]$.

10.

$f(x) = \ln x$	$f(4) = \ln 4$
$f'(x) = x^{-1}$	$f'(4) = \frac{1}{4}$
$f''(x) = -x^{-2}$	$f''(4) = -\frac{1}{16}$
$f'''(x) = 2x^{-3}$	$f'''(4) = \frac{1}{32}$
$f^{(4)}(x) = -6x^{-4}$	

(a) $\ln x \approx T_3(x)$

$$= \ln 4 + \frac{1}{4}(x - 4) - \frac{1}{32}(x - 4)^2 + \frac{1}{192}(x - 4)^3$$

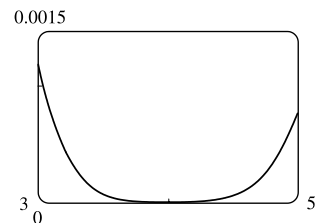
(b) $|R_3(x)| \leq \frac{M}{4!} |x - 4|^4$, where $|f^{(4)}(x)| \leq M$. Now

$$3 \leq x \leq 5 \Rightarrow (x - 4)^4 \leq 1^4 = 1,$$

and letting $x = 3$ gives $M = 6/3^4$, so

$$|R_3(x)| \leq \frac{6}{4! 3^4} \cdot 1 = \frac{1}{324} \approx 0.0031.$$

(c)



From the graph of $|R_3(x)| = |\ln x - T_3(x)|$, it appears that the error is less than 0.0013 on $[3, 5]$.