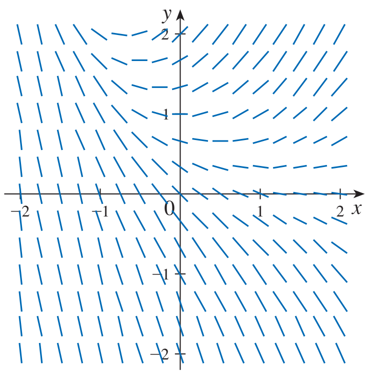
7.2 Direction Fields and Euler's Method

A Click here for answers.

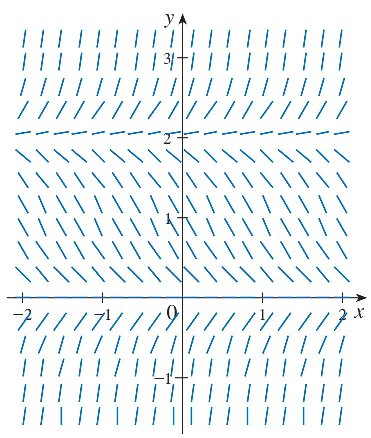
S Click here for solutions.

1. A direction field for the differential equation $y' = y - e^{-x}$ is shown. Sketch the graphs of the solutions that satisfy the given initial conditions.
(a) $y(0) = 0$ (b) $y(0) = 1$ (c) $y(0) = -1$



2. (a) A direction field for the differential equation $y' = 2y(y - 2)$ is shown. Sketch the graphs of the solutions that satisfy the given initial conditions.
(i) $y(0) = 1$ (ii) $y(0) = 2.5$ (iii) $y(0) = -1$

(b) Suppose the initial condition is $y(0) = c$. For what values of c is $\lim_{t\to\infty} y(t)$ finite? What are the equilibrium solutions?



3–4 Sketch a direction field for the differential equation. Then use it to sketch three solution curves.

3. $y' = x - y$

4. $y' = xy + y^2$

5–8 Sketch the direction field of the given differential equation. Then use it to sketch a solution curve that passes through the given point.

5. $y' = y^2$, $(0, 1)$

6. $y' = x^2 + y$, $(1, 1)$

7. $y' = x^2 + y^2$, $(0, 0)$

8. $y' = y(4 - y)$, $(0, 1)$

9. Use Euler's method with step size 0.5 to compute the approximate y-values y_1, y_2, y_3, and y_4 of the solution of the initial-value problem $y' = 1 + 3x - 2y$, $y(1) = 2$.

10. Use Euler's method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution of the initial-value problem $y' = x + y^2$, $y(0) = 0$.

11. Use Euler's method with step size 0.1 to estimate $y(0.5)$, where $y(x)$ is the solution of the initial-value problem $y' = x^2 + y^2$, $y(0) = 1$.

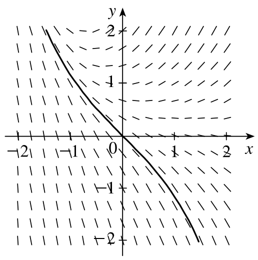
12. (a) Use Euler's method with step size 0.2 to estimate $y(0.4)$, where $y(x)$ is the solution of the initial-value problem $y' = 2xy^2$, $y(0) = 1$.
(b) Repeat part (a) with step size 0.1.

Answers

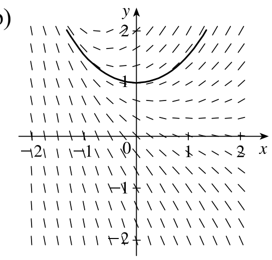
E Click here for exercises.

S Click here for solutions.

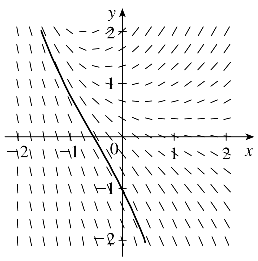
1. (a)



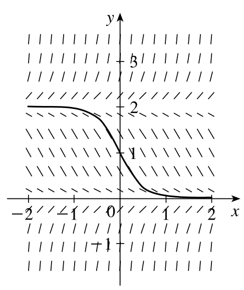
(b)



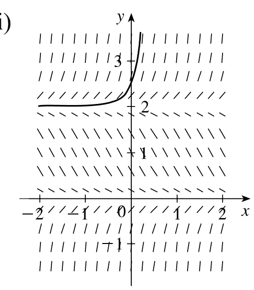
(c)



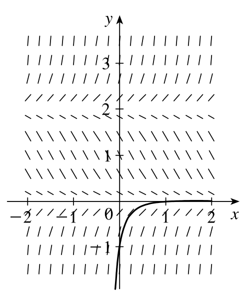
2. (a) (i)



(ii)

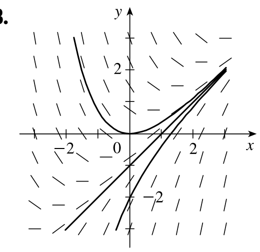


(iii)



(b) $c \leq 2$; $y = 0$, $y = 2$

3.



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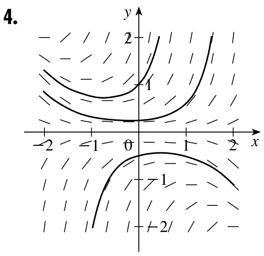
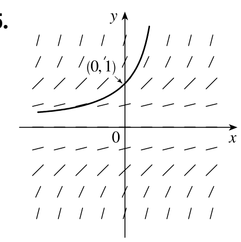
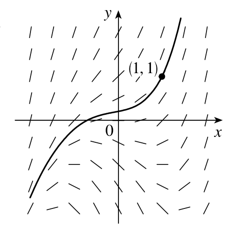


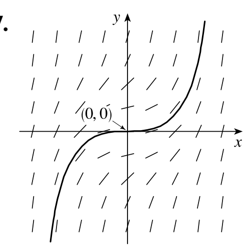
5.



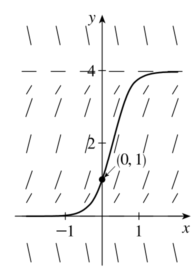
6.



7.



8.



9. 2, 2.75, 3.5, 4.25

10. 0.4150

11. 1.8371

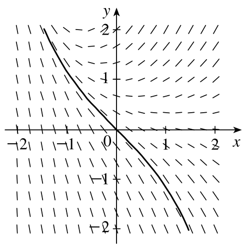
12. (a) 1.08 (b) 1.1292

Solutions

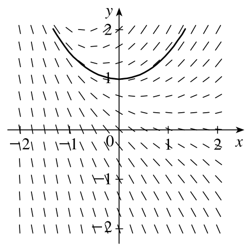
E Click here for exercises.

A Click here for answers.

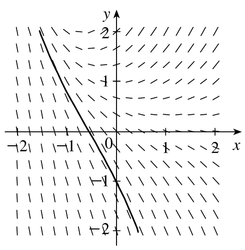
1. (a)



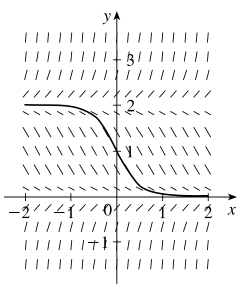
(b)



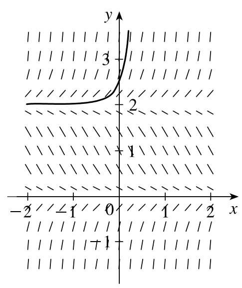
(c)



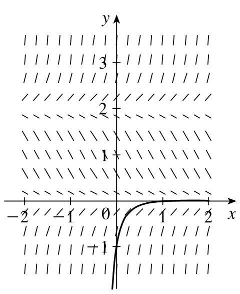
2. (a) (i)



(ii)

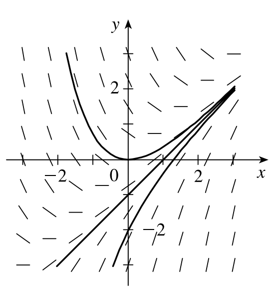


(iii)

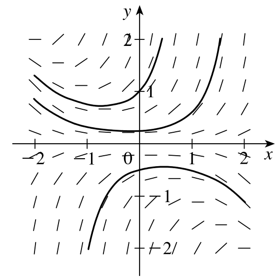


(b) For $c \leq 2$, $\lim_{t\to\infty} y(t)$ is finite. In fact, if $c = 2$ then $\lim_{t\to\infty} y(t) = 2$ and if $c < 2$ then $\lim_{t\to\infty} y(t) = 0$. The equilibrium solutions are $y = 0$ and $y = 2$.

3. $y' = x - y$

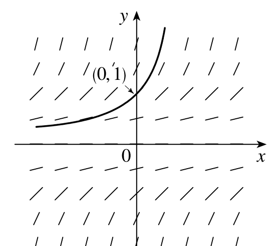


4. $y' = xy + y^2$



5.

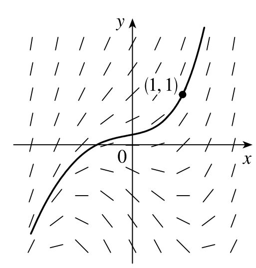
x	y	$y' = y^2$
0	0	0
0	1	1
0	−1	1
1	0	0
−1	0	0
1	−1	1
1	1	1
1	2	4
1	−2	4
−1	2	4
−1	−2	4



The solution curve through $(0, 1)$

6.

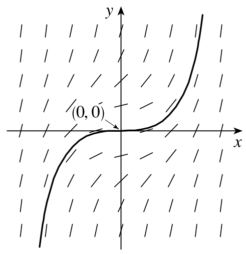
x	y	$y' = x^2 + y$
0	0	0
0	1	1
0	−1	−1
1	0	1
−1	0	1
1	1	2
−1	1	2
1	−1	0
−1	−1	0
2	0	4
2	1	5
2	−1	3



The solution curve through $(1, 1)$

7.

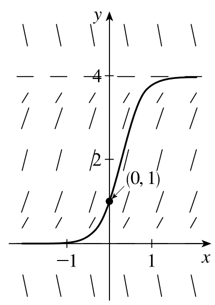
x	y	$y' = x^2 + y^2$
0	0	0
0	1	1
1	0	1
1	1	2
-1	1	2
0	2	4
2	0	4
2	2	8
2	1	5
-2	-1	5
1	2	5



The solution curve through $(0, 0)$

8.

x	y	$y' = y\,(4 - y)$
0	0	0
0	1	3
0	-1	-5
0	2	4
0	-2	-12
0	0.5	1.75
0	-0.5	-2.25
1	0	0
1	1	3
1	2	4
1	-1	-5



Note: The solution curve is asymptotic to $y = 0$ and $y = 4$.

9. $h = 0.5$, $x_0 = 1$, $y_0 = 2$, and $F(x, y) = 1 + 3x - 2y$. So

$$\begin{aligned} y_n &= y_{n-1} + hF(x_{n-1}, y_{n-1}) \\ &= y_{n-1} + 0.5\,(1 + 3x_{n-1} - 2y_{n-1}) \\ &= 0.5 + 1.5x_{n-1} \end{aligned}$$

Thus, $y_1 = 0.5 + 1.5 \cdot 1 = 2$, $y_2 = 0.5 + 1.5 \cdot 1.5 = 2.75$, $y_3 = 0.5 + 1.5 \cdot 2 = 3.5$, $y_4 = 0.5 + 1.5 \cdot 2.5 = 4.25$.

10. $h = 0.2$, $x_0 = 0$, $y_0 = 0$, and $F(x, y) = x + y^2$. We need to find y_5, because $x_5 = 1$. So

$y_n = y_{n-1} + 0.2\left(x_{n-1} + y_{n-1}^2\right)$.

$y_1 = 0 + 0.2\,(0 + 0) = 0$, $y_2 = 0 + 0.2\left(0.2 + 0^2\right) = 0.04$,

$y_3 = 0.04 + 0.2\left(0.4 + 0.04^2\right) = 0.12032$,

$y_4 = 0.12032 + 0.2\left(0.6 + 0.12032^2\right) \approx 0.24322$,

$y_5 = 0.24322 + 0.2\left(0.8 + 0.24322^2\right) \approx 0.4150 \approx y\,(1)$.

11. $h = 0.1$, $x_0 = 0$, $y_0 = 1$, and $F(x, y) = x^2 + y^2$. We need to find y_5, because $x_5 = 0.5$. So

$y_n = y_{n-1} + 0.1\left(x_{n-1}^2 + y_{n-1}^2\right)$.

$y_1 = 1 + 0.1\left(0^2 + 1^2\right) = 1.1$,

$y_2 = 1.1 + 0.1\left(0.1^2 + 1.1^2\right) = 1.222$,

$y_3 = 1.222 + 0.1\left(0.2^2 + 1.222^2\right) \approx 1.37533$,

$y_4 = 1.37533 + 0.1\left(0.3^2 + 1.37533^2\right) \approx 1.57348$,

$y_5 = 1.57348 + 0.1\left(0.4^2 + 1.57348^2\right) \approx 1.8371 \approx y\,(0.5)$.

12. (a) $h = 0.2$, $x_0 = 0$, $y_0 = 1$, and $F(x, y) = 2xy^2$. We need to find y_2, because $x_2 = 0.4$.

$y_1 = 1 + 0.2\left(2 \cdot 0 \cdot 1^2\right) = 1$,

$y_2 = 1 + 0.2\left(2 \cdot 0.2 \cdot 1^2\right) = 1.08 \approx y\,(0.4)$.

(b) $h = 0.1$ now, so we need to find y_4.

$y_1 = 1 + 0.1\left(2 \cdot 0 \cdot 1^2\right) = 1$,

$y_2 = 1 + 0.1\left(2 \cdot 0.1 \cdot 1^2\right) = 1.02$,

$y_3 = 1.02 + 0.1\left(2 \cdot 0.2 \cdot 1.02^2\right) \approx 1.06162$,

$y_4 = 1.06162 + 0.1\left(2 \cdot 0.3 \cdot 1.06162^2\right) \approx 1.1292 \approx y\,(0.4)$.