7.1 Modeling with Differential Equations

S Click here for solutions.

1. Show that $y = 2 + e^{-x^3}$ is a solution of the differential equation $y' + 3x^2y = 6x^2$.
2. Verify that $y = (2 + \ln x)/x$ is a solution of the initial-value problem

$$x^2y' + xy = 1 \qquad y(1) = 2$$

Solutions

E Click here for exercises.

1. $y = 2 + e^{-x^3} \quad \Rightarrow \quad y' = -3x^2 e^{-x^3}$.

$$\begin{aligned} \text{LHS} &= y' + 3x^2 y \\ &= -3x^2 e^{-x^3} + 3x^2 \left(2 + e^{-x^3}\right) \\ &= -3x^2 e^{-x^3} + 6x^2 + 3x^2 e^{-x^3} \\ &= 6x^2 \\ &= \text{RHS} \end{aligned}$$

2. $y = \dfrac{2 + \ln x}{x} \quad \Rightarrow$

$y' = \dfrac{x\,(1/x) - (2 + \ln x)\,(1)}{(x)^2} = \dfrac{-1 - \ln x}{x^2}$ and

$y\,(1) = \dfrac{2 + \ln 1}{1} = 2.$

$$\begin{aligned} \text{LHS} &= x^2 y' + xy \\ &= x^2 \left(\frac{-1 - \ln x}{x^2}\right) + x \left(\frac{2 + \ln x}{x}\right) \\ &= (-1 - \ln x) + (2 + \ln x) \\ &= 1 \\ &= \text{RHS} \end{aligned}$$