

5.10 Improper Integrals

A Click here for answers.

1–33 **|||** Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

1. $\int_2^{\infty} \frac{1}{(x+3)^{3/2}} dx$
2. $\int_0^{\infty} e^{-x} dx$
3. $\int_{-\infty}^{\infty} x^3 dx$
4. $\int_{-\infty}^{\infty} \frac{1}{\sqrt[3]{w-5}} dw$
5. $\int_2^{\infty} \frac{1}{\sqrt{x+3}} dx$
6. $\int_{-\infty}^1 \frac{1}{(2x-3)^2} dx$
7. $\int_{-\infty}^{-1} \frac{1}{\sqrt[3]{x-1}} dx$
8. $\int_{-\infty}^{\infty} x dx$
9. $\int_{-\infty}^{\infty} (2x^2 - x + 3) dx$
10. $\int_{-\infty}^0 e^{3x} dx$
11. $\int_1^{\infty} \sin \pi x dx$
12. $\int_0^{\infty} \frac{5}{2x+3} dx$
13. $\int_{-\infty}^3 \frac{1}{x^2+9} dx$
14. $\int_e^{\infty} \frac{1}{x(\ln x)^2} dx$
15. $\int_0^{\infty} \frac{1}{(x+2)(x+3)} dx$
16. $\int_0^{\infty} \frac{x}{(x+2)(x+3)} dx$
17. $\int_0^{\infty} \cos x dx$
18. $\int_{-\infty}^{\pi/2} \sin 2\theta d\theta$
19. $\int_{-\infty}^1 xe^{2x} dx$
20. $\int_0^{\infty} xe^{-x} dx$
21. $\int_{-\infty}^{\infty} \frac{dx}{x^2+4x+6}$
22. $\int_0^{\infty} \frac{1}{2^x} dx$
23. $\int_0^2 \frac{1}{4x-5} dx$
24. $\int_4^5 \frac{1}{(5-x)^{2/5}} dx$

S Click here for solutions.

25. $\int_{\pi/4}^{\pi/2} \sec^2 x dx$
26. $\int_{\pi/4}^{\pi/2} \tan^2 x dx$
27. $\int_0^{\pi/4} \csc^2 t dt$
28. $\int_0^{\pi/4} \frac{\cos x}{\sqrt{\sin x}} dx$
29. $\int_{-2}^2 \frac{1}{x^2-1} dx$
30. $\int_0^2 \frac{x}{\sqrt{4-x^2}} dx$
31. $\int_0^9 \frac{dx}{(x+9)\sqrt{x}}$
32. $\int_{\pi/4}^{3\pi/4} \tan x dx$
33. $\int_1^e \frac{1}{x^4 \sqrt{\ln x}} dx$

 **34–37** **|||** Sketch the region and find its area (if the area is finite).

34. $S = \{(x, y) \mid x \geq 1, 0 \leq y \leq (\ln x)/x^2\}$
35. $S = \{(x, y) \mid x \geq 0, 0 \leq y \leq 1/\sqrt{x+1}\}$
36. $S = \{(x, y) \mid 0 \leq x \leq \pi, 0 \leq y \leq \tan x \sec x\}$
37. $S = \{(x, y) \mid 3 < x \leq 7, 0 \leq y \leq 1/\sqrt{x-3}\}$

38–40 **|||** Use the Comparison Theorem to determine whether the integral is convergent or divergent.

38. $\int_1^{\infty} \frac{\sin^2 x}{x^2} dx$
39. $\int_1^{\infty} \frac{1}{\sqrt{x^3+1}} dx$
40. $\int_1^{\infty} \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$

Solutions

E Click here for exercises.

$$\begin{aligned} 1. \int_2^{\infty} \frac{dx}{(x+3)^{3/2}} &= \lim_{t \rightarrow \infty} \int_2^t \frac{dx}{(x+3)^{3/2}} \\ &= \lim_{t \rightarrow \infty} \left[\frac{-2}{\sqrt{x+3}} \right]_2^t \\ &= \lim_{t \rightarrow \infty} \left[\frac{-2}{\sqrt{t+3}} + \frac{2}{\sqrt{5}} \right] = \frac{2}{\sqrt{5}} \end{aligned}$$

$$\begin{aligned} 2. \int_0^{\infty} e^{-x} dx &= \lim_{t \rightarrow \infty} \int_0^t e^{-x} dx = \lim_{t \rightarrow \infty} [-e^{-x}]_0^t \\ &= \lim_{t \rightarrow \infty} (-e^{-t} + 1) = 1 \end{aligned}$$

$$\begin{aligned} 3. \int_{-\infty}^{\infty} x^3 dx &= \int_{-\infty}^0 x^3 dx + \int_0^{\infty} x^3 dx, \text{ but} \\ \int_{-\infty}^0 x^3 dx &= \lim_{t \rightarrow -\infty} \left[\frac{1}{4} x^4 \right]_t^0 = \lim_{t \rightarrow -\infty} \left(-\frac{1}{4} t^4 \right) \\ &= -\infty \end{aligned}$$

Divergent

$$\begin{aligned} 4. I &= \int_{-\infty}^{\infty} \frac{1}{\sqrt[3]{w-5}} dw \\ &= \lim_{a \rightarrow -\infty} \int_a^0 \frac{dw}{\sqrt[3]{w-5}} + \lim_{b \rightarrow 5^-} \int_0^b \frac{dw}{\sqrt[3]{w-5}} \\ &\quad + \lim_{c \rightarrow 5^+} \int_c^{10} \frac{dw}{\sqrt[3]{w-5}} + \lim_{d \rightarrow \infty} \int_{10}^d \frac{dw}{\sqrt[3]{w-5}} \end{aligned}$$

(The values 0 and 10 could be any pair of values surrounding 5.) If any one of these four integrals diverges, then I diverges, and (for example)

$$\begin{aligned} \lim_{d \rightarrow \infty} \int_{10}^d \frac{dw}{\sqrt[3]{w-5}} &= \lim_{d \rightarrow \infty} \left[\frac{3}{2} (w-5)^{2/3} \right]_{10}^d \\ &= \lim_{d \rightarrow \infty} \left[\frac{3}{2} (d-5)^{2/3} - \frac{3}{2} (5)^{2/3} \right] = \infty \end{aligned}$$

Thus, I is divergent.

$$\begin{aligned} 5. \int_2^{\infty} \frac{dx}{\sqrt{x+3}} &= \lim_{t \rightarrow \infty} \int_2^t \frac{dx}{\sqrt{x+3}} = \lim_{t \rightarrow \infty} [2\sqrt{x+3}]_2^t \\ &= \lim_{t \rightarrow \infty} (2\sqrt{t+3} - 2\sqrt{5}) = \infty \end{aligned}$$

Divergent

$$\begin{aligned} 6. \int_{-\infty}^1 \frac{dx}{(2x-3)^2} &= \lim_{t \rightarrow -\infty} \frac{1}{2} \int_t^1 \frac{2 dx}{(2x-3)^2} \\ &= \lim_{t \rightarrow -\infty} \frac{1}{2} \left[-\frac{1}{2x-3} \right]_t^1 \\ &= \lim_{t \rightarrow -\infty} \left[\frac{1}{2} + \frac{1}{2(2t-3)} \right] = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} 7. \int_{-\infty}^{-1} \frac{dx}{\sqrt[3]{x-1}} &= \lim_{t \rightarrow -\infty} \int_t^{-1} (x-1)^{-1/3} dx \\ &= \lim_{t \rightarrow -\infty} \left[\frac{3}{2} (x-1)^{2/3} \right]_t^{-1} \\ &= \lim_{t \rightarrow -\infty} \left[\frac{3}{2} \sqrt[3]{4} - \frac{3}{2} (t-1)^{2/3} \right] = -\infty \end{aligned}$$

Divergent

A Click here for answers.

$$\begin{aligned} 8. \int_{-\infty}^{\infty} x dx &= \int_{-\infty}^0 x dx + \int_0^{\infty} x dx \text{ and} \\ \int_{-\infty}^0 x dx &= \lim_{t \rightarrow -\infty} \left[\frac{1}{2} x^2 \right]_t^0 = \lim_{t \rightarrow -\infty} \left(-\frac{1}{2} t^2 \right) = -\infty. \end{aligned}$$

Divergent

$$\begin{aligned} 9. \int_{-\infty}^{\infty} (2x^2 - x + 3) dx &= \int_{-\infty}^0 (2x^2 - x + 3) dx + \int_0^{\infty} (2x^2 - x + 3) dx \\ &\text{but} \\ \int_{-\infty}^0 (2x^2 - x + 3) dx &= \lim_{t \rightarrow -\infty} \left[\frac{2}{3} x^3 - \frac{1}{2} x^2 + 3x \right]_t^0 \\ &= \lim_{t \rightarrow -\infty} \left[-\frac{2}{3} t^3 + \frac{1}{2} t^2 - 3t \right] = \infty \end{aligned}$$

Divergent

$$\begin{aligned} 10. \int_{-\infty}^0 e^{3x} dx &= \lim_{t \rightarrow -\infty} \int_t^0 e^{3x} dx = \lim_{t \rightarrow -\infty} \left[\frac{1}{3} e^{3x} \right]_t^0 \\ &= \lim_{t \rightarrow -\infty} \left[\frac{1}{3} - \frac{1}{3} e^{3t} \right] = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} 11. \int_1^{\infty} \sin \pi x dx &= \lim_{t \rightarrow \infty} -\frac{1}{\pi} [\cos \pi x]_1^t \\ &= -\frac{1}{\pi} \lim_{t \rightarrow \infty} (\cos \pi t + 1) \end{aligned}$$

which does not exist. Divergent

$$\begin{aligned} 12. \int_0^{\infty} \frac{5 dx}{2x+3} &= \frac{5}{2} \lim_{t \rightarrow \infty} \int_0^t \frac{2 dx}{2x+3} \\ &= \frac{5}{2} \lim_{t \rightarrow \infty} [\ln(2x+3)]_0^t \\ &= \frac{5}{2} \lim_{t \rightarrow \infty} [\ln(2t+3) - \ln 3] = \infty \end{aligned}$$

Divergent

$$\begin{aligned} 13. \int_{-\infty}^3 \frac{dx}{x^2+9} &= \lim_{t \rightarrow -\infty} \left[\frac{1}{3} \tan^{-1} \left(\frac{1}{3} x \right) \right]_t^3 \\ &= \lim_{t \rightarrow -\infty} \frac{1}{3} \left[\frac{\pi}{4} - \tan^{-1} \left(\frac{1}{3} t \right) \right] \\ &= \frac{1}{3} \left(\frac{\pi}{4} + \frac{\pi}{2} \right) = \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} 14. \int_e^{\infty} \frac{dx}{x(\ln x)^2} &= \lim_{t \rightarrow \infty} \int_e^t \frac{dx}{x(\ln x)^2} = \lim_{t \rightarrow \infty} \left[-\frac{1}{\ln x} \right]_e^t \\ &= \lim_{t \rightarrow \infty} \left[1 - \frac{1}{\ln t} \right] = 1 \end{aligned}$$

$$\begin{aligned} 15. \int_0^{\infty} \frac{dx}{(x+2)(x+3)} &= \lim_{t \rightarrow \infty} \int_0^t \left[\frac{1}{x+2} - \frac{1}{x+3} \right] dx \\ &= \lim_{t \rightarrow \infty} \left[\ln \left(\frac{x+2}{x+3} \right) \right]_0^t \\ &= \lim_{t \rightarrow \infty} \left[\ln \left(\frac{t+2}{t+3} \right) - \ln \frac{2}{3} \right] \\ &= \ln 1 - \ln \frac{2}{3} = -\ln \frac{2}{3} \end{aligned}$$

$$\begin{aligned}
 16. \int_0^\infty \frac{x \, dx}{(x+2)(x+3)} &= \lim_{t \rightarrow \infty} \int_0^t \left[\frac{-2}{x+2} + \frac{3}{x+3} \right] dx \\
 &= \lim_{t \rightarrow \infty} [3 \ln(x+3) - 2 \ln(x+2)]_0^t \\
 &= \lim_{t \rightarrow \infty} \left[\ln \frac{(t+3)^3}{(t+2)^2} - \ln \frac{27}{4} \right] = \infty
 \end{aligned}$$

Divergent

$$17. \int_0^\infty \cos x \, dx = \lim_{t \rightarrow \infty} [\sin x]_0^t = \lim_{t \rightarrow \infty} \sin t, \text{ which does not exist. Divergent}$$

$$\begin{aligned}
 18. \int_{-\infty}^{\pi/2} \sin 2\theta \, d\theta &= \lim_{t \rightarrow -\infty} \int_t^{\pi/2} \sin 2\theta \, d\theta \\
 &= \lim_{t \rightarrow -\infty} \left[-\frac{1}{2} \cos 2\theta \right]_t^{\pi/2} \\
 &= \lim_{t \rightarrow -\infty} \left(\frac{1}{2} + \frac{1}{2} \cos 2t \right)
 \end{aligned}$$

This limit does not exist, so the integral is divergent.

$$\begin{aligned}
 19. \int_{-\infty}^1 x e^{2x} \, dx &= \lim_{t \rightarrow -\infty} \int_t^1 x e^{2x} \, dx \\
 &= \lim_{t \rightarrow -\infty} \left[\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right]_t^1 \quad (\text{by parts}) \\
 &= \lim_{t \rightarrow -\infty} \left[\frac{1}{2} e^2 - \frac{1}{4} e^2 - \frac{1}{2} t e^{2t} + \frac{1}{4} e^{2t} \right] \\
 &= \frac{1}{4} e^2 - 0 + 0 = \frac{1}{4} e^2 \text{ since}
 \end{aligned}$$

$$\begin{aligned}
 \lim_{t \rightarrow -\infty} t e^{2t} &= \lim_{t \rightarrow -\infty} \frac{t}{e^{-2t}} \stackrel{H}{=} \lim_{t \rightarrow -\infty} \frac{1}{-2e^{-2t}} \\
 &= \lim_{t \rightarrow -\infty} -\frac{1}{2} e^{2t} = 0
 \end{aligned}$$

$$\begin{aligned}
 20. \int_0^\infty x e^{-x} \, dx &= \lim_{t \rightarrow \infty} [-x e^{-x} - e^{-x}]_0^t \\
 &= \lim_{t \rightarrow \infty} [1 - (t+1)e^{-t}] \\
 &= 1 - \lim_{t \rightarrow \infty} \frac{t+1}{e^t} \stackrel{H}{=} 1 - \lim_{t \rightarrow \infty} \frac{1}{e^t} \\
 &= 1 - 0 = 1
 \end{aligned}$$

$$\begin{aligned}
 21. \int_{-\infty}^\infty \frac{dx}{x^2 + 4x + 6} \\
 &= \int_{-\infty}^0 \frac{dx}{(x+2)^2 + 2} + \int_0^\infty \frac{dx}{(x+2)^2 + 2}
 \end{aligned}$$

Now

$$\begin{aligned}
 \int_{-\infty}^0 \frac{dx}{(x+2)^2 + 2} &= \lim_{t \rightarrow -\infty} \frac{1}{\sqrt{2}} \left[\tan^{-1} \left(\frac{x+2}{\sqrt{2}} \right) \right]_t^0 \\
 &= \lim_{t \rightarrow -\infty} \frac{1}{\sqrt{2}} \left[\tan^{-1} \sqrt{2} - \tan^{-1} \left(\frac{t+2}{\sqrt{2}} \right) \right] \\
 &= \frac{1}{\sqrt{2}} \left(\tan^{-1} \sqrt{2} + \frac{\pi}{2} \right)
 \end{aligned}$$

and

$$\begin{aligned}
 \int_0^\infty \frac{dx}{(x+2)^2 + 2} &= \lim_{t \rightarrow \infty} \frac{1}{\sqrt{2}} \left[\tan^{-1} \left(\frac{x+2}{\sqrt{2}} \right) \right]_0^t \\
 &= \frac{1}{\sqrt{2}} \left(\frac{\pi}{2} - \tan^{-1} \sqrt{2} \right)
 \end{aligned}$$

$$\text{Therefore, } \int_{-\infty}^\infty \frac{dx}{x^2 + 4x + 6} = \frac{\pi}{\sqrt{2}}.$$

$$\begin{aligned}
 22. \int_0^\infty \frac{dx}{2^x} &= \lim_{t \rightarrow \infty} \int_0^t 2^{-x} \, dx = \lim_{t \rightarrow \infty} \left[-\frac{1}{\ln 2} 2^{-x} \right]_0^t \\
 &= \frac{1}{\ln 2} \lim_{t \rightarrow \infty} (1 - 2^{-t}) = \frac{1}{\ln 2}
 \end{aligned}$$

$$\begin{aligned}
 23. \int_0^2 \frac{dx}{4x-5} &= \int_0^{5/4} \frac{dx}{4x-5} + \int_{5/4}^2 \frac{dx}{4x-5}. \text{ But} \\
 \int_0^{5/4} \frac{dx}{4x-5} &= \lim_{t \rightarrow 5/4^-} \left[\frac{1}{4} \ln |4x-5| \right]_0^t \\
 &= \lim_{t \rightarrow 5/4^-} \frac{1}{4} [\ln |4t-5| - \ln 5] = -\infty
 \end{aligned}$$

Divergent

$$\begin{aligned}
 24. \int_4^5 \frac{dx}{(5-x)^{2/5}} &= \lim_{t \rightarrow 5^-} \left[-\frac{5}{3} (5-x)^{3/5} \right]_4^t \\
 &= \lim_{t \rightarrow 5^-} \left[-\frac{5}{3} (5-t)^{3/5} + \frac{5}{3} \right] \\
 &= 0 + \frac{5}{3} = \frac{5}{3}
 \end{aligned}$$

$$\begin{aligned}
 25. \int_{\pi/4}^{\pi/2} \sec^2 x \, dx &= \lim_{t \rightarrow \pi/2^-} [\tan x]_{\pi/4}^t \\
 &= \lim_{t \rightarrow \pi/2^-} (\tan t - 1) = \infty
 \end{aligned}$$

Divergent

$$\begin{aligned}
 26. \int_{\pi/4}^{\pi/2} \tan^2 x \, dx &= \lim_{t \rightarrow \pi/2^-} \int_{\pi/4}^t (\sec^2 x - 1) \, dx \\
 &= \lim_{t \rightarrow \pi/2^-} [\tan x - x]_{\pi/4}^t \\
 &= \frac{\pi}{4} - 1 + \lim_{t \rightarrow \pi/2^-} (\tan t - t) = \infty
 \end{aligned}$$

Divergent

$$\begin{aligned}
 27. \int_0^{\pi/4} \csc^2 t \, dt &= \lim_{s \rightarrow 0^+} \int_s^{\pi/4} \csc^2 t \, dt \\
 &= \lim_{s \rightarrow 0^+} [-\cot t]_s^{\pi/4} \\
 &= \lim_{s \rightarrow 0^+} \left[-\cot \frac{\pi}{4} + \cot s \right] = \infty
 \end{aligned}$$

Divergent

$$\begin{aligned}
 28. \int_0^{\pi/4} \frac{\cos x \, dx}{\sqrt{\sin x}} &= \lim_{t \rightarrow 0^+} \int_t^{\pi/4} \frac{\cos x \, dx}{\sqrt{\sin x}} \\
 &= \lim_{t \rightarrow 0^+} \left[2\sqrt{\sin x} \right]_t^{\pi/4} \\
 &= \lim_{t \rightarrow 0^+} \left(2\sqrt{\frac{1}{\sqrt{2}}} - 2\sqrt{\sin t} \right) \\
 &= 2\sqrt{\frac{1}{\sqrt{2}}} = \frac{2}{2^{1/4}} = 2^{3/4}
 \end{aligned}$$

$$29. \int_{-2}^2 \frac{dx}{x^2-1} = \int_{-2}^{-1} \frac{dx}{x^2-1} + \int_{-1}^0 \frac{dx}{x^2-1} \\ + \int_0^1 \frac{dx}{x^2-1} + \int_1^2 \frac{dx}{x^2-1}$$

and

$$\int \frac{dx}{x^2-1} = \int \frac{dx}{(x-1)(x+1)} = \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + C, \text{ so}$$

$$\int_0^1 \frac{dx}{x^2-1} = \lim_{t \rightarrow 1^-} \left[\frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| \right]_0^t = \lim_{t \rightarrow 1^-} \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| \\ = -\infty$$

Divergent

$$30. \int_0^2 \frac{x dx}{\sqrt{4-x^2}} = \lim_{t \rightarrow 2^-} \int_0^t \frac{x dx}{\sqrt{4-x^2}} \\ = \lim_{t \rightarrow 2^-} \left[-\sqrt{4-x^2} \right]_0^t = \lim_{t \rightarrow 2^-} (2 - \sqrt{4-t^2}) = 2$$

$$31. \int_0^9 \frac{dx}{(x+9)\sqrt{x}} = \lim_{t \rightarrow 0^+} \int_t^9 \frac{dx}{(x+9)\sqrt{x}} \\ = \lim_{t \rightarrow 0^+} \int_{\sqrt{t}}^3 \frac{2u du}{(u^2+9)u} \quad (u = \sqrt{x}, x = u^2, dx = 2u du) \\ = \lim_{t \rightarrow 0^+} \int_{\sqrt{t}}^3 \frac{2 du}{u^2+9} = \lim_{t \rightarrow 0^+} \left[\frac{2}{3} \tan^{-1} \left(\frac{1}{3}u \right) \right]_{\sqrt{t}}^3 \\ = \frac{2}{3} \left(\frac{\pi}{4} \right) = \frac{\pi}{6}$$

$$32. \int_{\pi/4}^{3\pi/4} \tan x dx = \int_{\pi/4}^{\pi/2} \tan x dx + \int_{\pi/2}^{3\pi/4} \tan x dx$$

But

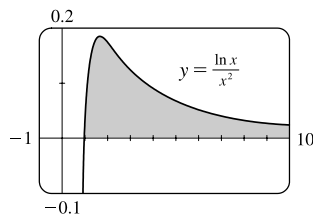
$$\int_{\pi/4}^{\pi/2} \tan x dx = \lim_{t \rightarrow (\pi/2)^-} [\ln |\sec x|]_{\pi/4}^t \\ = \lim_{t \rightarrow (\pi/2)^-} [\ln(\sec t) - \ln \sqrt{2}] = \infty$$

Divergent

$$33. \text{ Let } u = \ln x. \text{ Then } du = \frac{dx}{x} \Rightarrow$$

$$\int_1^e \frac{dx}{x^4 \sqrt{\ln x}} = \lim_{t \rightarrow 1^+} \int_t^e \frac{dx}{x^4 \sqrt{\ln x}} = \lim_{t \rightarrow 1^+} \int_{\ln t}^1 \frac{du}{\sqrt{u}} \\ = \lim_{t \rightarrow 1^+} \left[\frac{4}{3} u^{3/4} \right]_{\ln t}^1 \\ = \lim_{t \rightarrow 1^+} \left\{ \frac{4}{3} \left[1 - (\ln t)^{3/4} \right] \right\} = \frac{4}{3}$$

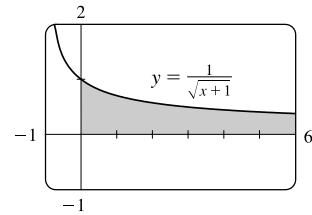
34.



We integrate by parts with $u = \ln x$, $dv = (1/x^2) dx$,
 $du = dx/x$, $v = -1/x$:

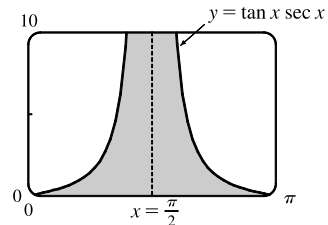
$$\int_1^\infty \frac{\ln x}{x^2} dx = \lim_{t \rightarrow \infty} \left(\left[-\frac{1}{x} \ln x \right]_1^t + \int_1^t \frac{1}{x^2} dx \right) \\ = \lim_{t \rightarrow \infty} \left(-\frac{\ln t}{t} - \frac{1}{t} + 1 \right) = 1$$

35.



Area = $\int_0^\infty \frac{dx}{\sqrt{x+1}} = \lim_{t \rightarrow \infty} [2\sqrt{x+1}]_0^t = \infty$, so the area is infinite.

36.



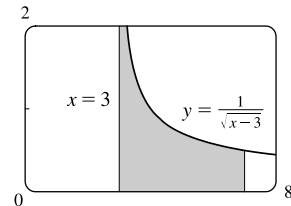
$$\int_0^\pi \tan x \sec x dx \\ = \int_0^{\pi/2} \tan x \sec x dx + \int_{\pi/2}^\pi \tan x \sec x dx$$

But

$$\int_0^{\pi/2} \tan x \sec x dx = \lim_{t \rightarrow (\pi/2)^-} \int_0^t \tan x \sec x dx \\ = \lim_{t \rightarrow (\pi/2)^-} [\sec x]_0^t = \lim_{t \rightarrow (\pi/2)^-} (\sec t - 1) = \infty$$

Divergent

37.



$$\text{Area} = \int_3^7 \frac{dx}{\sqrt{x-3}} = \lim_{t \rightarrow 3^+} [2\sqrt{x-3}]_t^7 \\ = 4 - \lim_{t \rightarrow 3^+} 2\sqrt{t-3} = 4 - 0 = 4$$

$$38. \frac{\sin^2 x}{x^2} \leq \frac{1}{x^2} \text{ on } [1, \infty). \int_1^\infty \frac{dx}{x^2} \text{ is convergent by}$$

Equation 2, so $\int_1^\infty \frac{\sin^2 x}{x^2} dx$ is convergent by the Comparison Theorem.

$$39. \frac{1}{\sqrt{x^3+1}} \leq \frac{1}{x^{3/2}} \text{ on } [1, \infty). \int_1^\infty \frac{dx}{x^{3/2}} \text{ converges by}$$

Equation 2, so $\int_1^\infty \frac{dx}{\sqrt{x^3+1}}$ is convergent by the Comparison Theorem.

$$40. \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} > \frac{1}{\sqrt{x}} \text{ on } [1, \infty). \int_1^\infty \frac{dx}{\sqrt{x}} \text{ is divergent by}$$

Equation 2 with $p = \frac{1}{2} \leq 1$, so $\int_1^\infty \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$ is divergent by the Comparison Theorem.