5.8 Integration Using Tables and Computer Algebra Systems

A Click here for answers.

S Click here for solutions.

1–13 Use the Table of Integrals [on Reference Pages 6–10] to evaluate the integral.

1. $\int x \sin^{-1}(x^2)\,dx$

2. $\int x^3 \sin^{-1}(x^2)\,dx$

3. $\int \frac{\sin x \cos x}{\sqrt{1 + \sin x}}\,dx$

4. $\int e^x \cos(3x + 4)\,dx$

5. $\int e^x \ln(1 + e^x)\,dx$

6. $\int \frac{\sqrt{9x^2 - 1}}{x^2}\,dx$

7. $\int \frac{\sqrt{4 - 3x^2}}{x}\,dx$

8. $\int \frac{\sin\theta}{1 + 2\cos\theta}\,d\theta$

9. $\int_{-2}^{1} \sqrt{5 - 4x - x^2}\,dx$

10. $\int \sqrt{2 + 3\cos x}\,\tan x\,dx$

11. $\int \frac{x}{\sqrt{x^2 - 4x}}\,dx$

12. $\int_0^{\pi/2} \cos^5 x\,dx$

13. $\int x^2 \tan^{-1} x\,dx$

14. Find the volume of the solid obtained when the region under the curve $y = 1/(1 + 5x)^2$ from 0 to 1 is rotated about the y-axis.

Answers

E Click here for exercises.

S Click here for solutions.

1. $\frac{1}{2}\left(x^2\sin^{-1}\left(x^2\right)+\sqrt{1-x^4}\right)+C$

2. $\dfrac{2x^4-1}{8}\sin^{-1}\left(x^2\right)+\dfrac{x^2\sqrt{1-x^4}}{8}+C$

3. $-\frac{2}{3}(2-\sin x)\sqrt{1+\sin x}+C$

4. $\frac{3}{10}e^x\left[\frac{1}{3}\cos(3x+4)+\sin(3x+4)\right]+C$

5. $(1+e^x)\ln(1+e^x)-e^x+C$

6. $-\dfrac{\sqrt{9x^2-1}}{x}+3\ln\left|3x+\sqrt{9x^2-1}\right|+C$

7. $\sqrt{4-3x^2}-2\ln\left|\dfrac{2+\sqrt{4-3x^2}}{x}\right|+C$

8. $-\frac{1}{2}\ln|1+2\cos\theta|+C$

9. $\frac{9\pi}{4}$

10. $-2\sqrt{2+3\cos x}-\sqrt{2}\ln\left|\dfrac{\sqrt{2+3\cos x}-\sqrt{2}}{\sqrt{2+3\cos x}+\sqrt{2}}\right|+C$

11. $\sqrt{x^2-4x}+2\ln\left|x-2+\sqrt{x^2-4x}\right|+C$

12. $\frac{8}{15}$

13. $\frac{1}{3}x^3\tan^{-1}x-\frac{1}{6}x^2+\frac{1}{6}\ln\left(1+x^2\right)+C$

14. $\frac{2\pi}{25}\left(\ln 6-\frac{5}{6}\right)$

Solutions

E Click here for exercises.

A Click here for answers.

1. Let $u = x^2$. Then $du = 2x\,dx$, so

$$\int x\sin^{-1}\left(x^2\right)dx = \tfrac{1}{2}\int \sin^{-1}u\,du \overset{87}{=} \tfrac{1}{2}\left(u\sin^{-1}u + \sqrt{1-u^2}\right) + C = \tfrac{1}{2}\left(x^2\sin^{-1}\left(x^2\right) + \sqrt{1-x^4}\right) + C$$

2. Let $u = x^2$. Then $du = 2x\,dx$, so

$$\int x^3\sin^{-1}\left(x^2\right)dx = \tfrac{1}{2}\int u\sin^{-1}u\,du \overset{90}{=} \frac{2u^2-1}{8}\sin^{-1}u + \frac{u\sqrt{1-u^2}}{8} + C = \frac{2x^4-1}{8}\sin^{-1}\left(x^2\right) + \frac{x^2\sqrt{1-x^4}}{8} + C$$

3. Let $u = \sin x$. Then $du = \cos x\,dx$, so

$$\int \frac{\sin x\cos x}{\sqrt{1+\sin x}}dx = \int \frac{u\,du}{\sqrt{1+u}} \overset{55}{=} \tfrac{2}{3}(u-2)\sqrt{1+u} + C = -\tfrac{2}{3}(2-\sin x)\sqrt{1+\sin x} + C$$

4. Let $u = 3x+4$. Then

$$\int e^x\cos(3x+4)\,dx = \tfrac{1}{3}\int e^{(u-4)/3}\cos u\,du = \tfrac{1}{3}e^{-4/3}\int e^{u/3}\cos u\,du \overset{99}{=} \frac{1}{3}e^{-4/3}\frac{e^{u/3}}{(1/3)^2+1^2}\left(\tfrac{1}{3}\cos u + \sin u\right) + C = \tfrac{3}{10}e^x\left[\tfrac{1}{3}\cos(3x+4) + \sin(3x+4)\right] + C$$

5. Let $u = 1 + e^x$, so $du = e^x\,dx$. Then

$$\int e^x\ln(1+e^x)\,dx \overset{100}{=} \int \ln u\,du = u\ln u - u + C = (1+e^x)\ln(1+e^x) - e^x - 1 + C = (1+e^x)\ln(1+e^x) - e^x + C_1$$

6. Let $u = 3x$. Then $du = 3\,dx$, so

$$\int \frac{\sqrt{9x^2-1}}{x^2}dx = \int \frac{\sqrt{u^2-1}}{u^2/9}\frac{du}{3} = 3\int \frac{\sqrt{u^2-1}}{u^2}du \overset{42}{=} -\frac{3\sqrt{u^2-1}}{u} + 3\ln\left|u+\sqrt{u^2-1}\right| + C = -\frac{\sqrt{9x^2-1}}{x} + 3\ln\left|3x+\sqrt{9x^2-1}\right| + C$$

7. By Formula 32,

$$\int \frac{\sqrt{4-3x^2}}{x}dx = \int \frac{\sqrt{4-u^2}}{u/\sqrt{3}}\frac{du}{\sqrt{3}} \quad \left[\begin{array}{l} u = \sqrt{3}\,x, \\ du = \sqrt{3}\,dx \end{array}\right]$$

$$= \int \frac{\sqrt{4-u^2}}{u}du = \sqrt{4-u^2} - 2\ln\left|\frac{2+\sqrt{4-u^2}}{u}\right| + C_1 = \sqrt{4-3x^2} - 2\ln\left|\frac{2+\sqrt{4-3x^2}}{\sqrt{3}x}\right| + C_1 = \sqrt{4-3x^2} - 2\ln\left|\frac{2+\sqrt{4-3x^2}}{x}\right| + C$$

8.

$$\int \frac{\sin\theta}{1+2\cos\theta}d\theta = -\frac{1}{2}\int \frac{1}{u}du \quad \left[\begin{array}{l} u = 1+2\cos\theta, \\ du = -2\sin\theta\,d\theta \end{array}\right]$$

$$= -\tfrac{1}{2}\ln|u| + C = -\tfrac{1}{2}\ln|1+2\cos\theta| + C$$

9.

$$\int_{-2}^{1}\sqrt{5-4x-x^2}\,dx = \int_{-2}^{1}\sqrt{5-(x^2+4x)}\,dx = \int_{-2}^{1}\sqrt{5+4-(x^2+4x+4)}\,dx = \int_{-2}^{1}\sqrt{9-(x+2)^2}\,dx$$

$$= \int_0^3\sqrt{3^2-u^2}\,du \quad [u = x+2,\ du = dx]$$

$$\overset{30}{=} \left[\tfrac{u}{2}\sqrt{9-u^2} + \tfrac{9}{2}\sin^{-1}\tfrac{u}{3}\right]_0^3 = \left[\left(0 + \tfrac{9}{2}\cdot\tfrac{\pi}{2}\right) - (0+0)\right] = \tfrac{9\pi}{4}$$

10.

$$\int \sqrt{2+3\cos x}\tan x\,dx = -\int \frac{\sqrt{2+3\cos x}}{\cos x}(-\sin x\,dx)$$

$$= -\int \frac{\sqrt{2+3u}}{u}du \quad (u = \cos x,\ du = -\sin x\,dx)$$

$$\overset{58}{=} -2\sqrt{2+3u} - 2\int \frac{du}{u\sqrt{2+3u}} \overset{57}{=} -2\sqrt{2+3u} - 2\cdot\frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{2+3u}-\sqrt{2}}{\sqrt{2+3u}+\sqrt{2}}\right| + C$$

$$= -2\sqrt{2+3\cos x} - \sqrt{2}\ln\left|\frac{\sqrt{2+3\cos x}-\sqrt{2}}{\sqrt{2+3\cos x}+\sqrt{2}}\right| + C$$

11. Let $u = x-2$. Then

$$\int \frac{x\,dx}{\sqrt{x^2-4x}} = \int \frac{(x-2)+2}{\sqrt{(x-2)^2-4}}dx = \int \frac{u\,du}{\sqrt{u^2-4}} + 2\int \frac{du}{\sqrt{u^2-4}}$$

$$= \frac{1}{2}\int v^{-1/2}\,dv + 2\int \frac{du}{\sqrt{u^2-4}} \quad \left[\begin{array}{l} v = u^2-4, \\ dv = 2u\,du \end{array}\right]$$

$$\overset{2,43}{=} v^{1/2} + 2\ln\left|u+\sqrt{u^2-4}\right| + C = \sqrt{x^2-4x} + 2\ln\left|x-2+\sqrt{x^2-4x}\right| + C$$

12.

$$\int_0^{\pi/2}\cos^5 x\,dx \overset{74}{=} \tfrac{1}{5}\left[\cos^4 x\sin x\right]_0^{\pi/2} + \tfrac{4}{5}\int_0^{\pi/2}\cos^3 x\,dx \overset{68}{=} 0 + \tfrac{4}{5}\left[\tfrac{1}{3}\left(2+\cos^2 x\right)\sin x\right]_0^{\pi/2} = \tfrac{4}{15}(2-0) = \tfrac{8}{15}$$

13. Using Formula 95 with $n = 2$,

$$\int x^2\tan^{-1}x\,dx = \frac{1}{3}\left[x^3\tan^{-1}x - \int \frac{x^3\,dx}{1+x^2}\right] = \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int\left(x - \frac{x}{x^2+1}\right)dx$$

$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\frac{x^2}{2} + \frac{1}{6}\int\frac{du}{u} \quad \left[\begin{array}{l} u = x^2+1, \\ du = 2x\,dx \end{array}\right]$$

$$= \tfrac{1}{3}x^3\tan^{-1}x - \tfrac{1}{6}x^2 + \tfrac{1}{6}\ln\left(1+x^2\right) + C$$

14. Volume $= \displaystyle\int_0^1 \frac{2\pi x}{(1+5x)^2}\,dx$

$$\overset{51}{=} 2\pi\left[\frac{1}{25\,(1+5x)} + \tfrac{1}{25}\ln|1+5x|\right]_0^1$$

$$= \tfrac{2\pi}{25}\left(\tfrac{1}{6} + \ln 6 - 1 - \ln 1\right)$$

$$= \tfrac{2\pi}{25}\left(\ln 6 - \tfrac{5}{6}\right)$$