 Answers

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1. $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$
2. $x \sin x + \cos x + C$
3. $-\frac{1}{4}x \cos 4x + \frac{1}{16} \sin 4x + C$
4. $\frac{1}{3}x^2 \sin 3x + \frac{2}{9}x \cos 3x - \frac{2}{27} \sin 3x + C$
5. $-\frac{x^2}{a} \cos ax + \frac{2x}{a^2} \sin ax + \frac{2}{a^3} \cos ax + C$
6. $\frac{1}{8} (\sin 2\theta - 2\theta \cos 2\theta) + C$
7. $\frac{1}{9}t^3 (3 \ln t - 1) + C$
8. $\frac{1}{10}e^{-\theta} (3 \sin 3\theta - \cos 3\theta) + C$
9. $1 - 2/e$
10. $2 \ln 4 - \frac{3}{2}$
11. $-\frac{1}{2}$
12. $2 - 5/e$
13. $\frac{1}{2}e^{x^2} (x^2 - 1) + C$
14. $\frac{1}{2}x [\sin (\ln x) - \cos (\ln x)] + C$
15. $\frac{1}{2} (x^2 \tan^{-1} x + \tan^{-1} x - x) + C$
16. $\frac{1}{3}x^3 \sin (x^3) + \frac{1}{3} \cos (x^3) + C$
17. $\frac{2}{3}x^{3/2} \ln x - \frac{4}{9}x^{3/2} + C$
18. $\frac{1}{12} (\pi + 6\sqrt{3} - 12)$
19. 0.080
20. 7.10

Solutions

E Click here for exercises.

1. Let $u = x$, $dv = e^{2x} dx \Rightarrow$
 $du = dx$, $v = \frac{1}{2}e^{2x}$. Then by Equation 2,
 $\int x e^{2x} dx = \frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x} dx = \frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x} + C$.

2. Let $u = x$, $dv = \cos x dx \Rightarrow du = dx$,
 $v = \sin x$. Then by Equation 2,
 $\int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x + C$.

3. Let $u = x$, $dv = \sin 4x dx \Rightarrow du = dx$, $v = -\frac{1}{4} \cos 4x$.
 Then
 $\int x \sin 4x dx = -\frac{1}{4}x \cos 4x - \int (-\frac{1}{4} \cos 4x) dx$
 $= -\frac{1}{4}x \cos 4x + \frac{1}{16} \sin 4x + C$

4. Let $u = x^2$, $dv = \cos 3x dx \Rightarrow$
 $du = 2x dx$, $v = \frac{1}{3} \sin 3x$. Then
 $I = \int x^2 \cos 3x dx = \frac{1}{3}x^2 \sin 3x - \frac{2}{3} \int x \sin 3x dx$ by
 Equation 2. Next let $U = x$, $dV = \sin 3x dx \Rightarrow$
 $dU = dx$, $V = -\frac{1}{3} \cos 3x$ to get
 $\int x \sin 3x dx = -\frac{1}{3}x \cos 3x + \frac{1}{3} \int \cos 3x dx$
 $= -\frac{1}{3}x \cos 3x + \frac{1}{9} \sin 3x + C_1$

Substituting for $\int x \sin 3x dx$, we get

$$I = \frac{1}{3}x^2 \sin 3x - \frac{2}{3} \left(-\frac{1}{3}x \cos 3x + \frac{1}{9} \sin 3x + C_1 \right)$$

$$= \frac{1}{3}x^2 \sin 3x + \frac{2}{9}x \cos 3x - \frac{2}{27} \sin 3x + C$$

where $C = -\frac{2}{3}C_1$.

5. Let $u = x^2$, $dv = \sin ax dx \Rightarrow du = 2x dx$,
 $v = -\frac{1}{a} \cos ax$. Then

$$I = \int x^2 \sin ax dx$$

$$= -\frac{x^2}{a} \cos ax - \int \left(-\frac{1}{a} \right) \cos ax (2x dx)$$

$$= -\frac{x^2}{a} \cos ax + \frac{2}{a} \int x \cos ax dx$$

by Equation 2. Let $U = x$, $dV = \cos ax dx \Rightarrow$
 $dU = dx$, $V = \frac{1}{a} \sin ax$. Then

$$\int x \cos ax dx = \frac{x}{a} \sin ax - \int \frac{1}{a} \sin ax dx$$

$$= \frac{x}{a} \sin ax + \frac{1}{a^2} \cos ax + C_1$$

So

$$I = -\frac{x^2}{a} \cos ax + \frac{2}{a} \left(\frac{x}{a} \sin ax + \frac{1}{a^2} \cos ax + C_1 \right)$$

$$= -\frac{x^2}{a} \cos ax + \frac{2x}{a^2} \sin ax + \frac{2}{a^3} \cos ax + C$$

A Click here for answers.

6. $I = \int \theta \sin \theta \cos \theta d\theta = \frac{1}{4} \int 2\theta \sin 2\theta d\theta$
 $= \frac{1}{8} \int t \sin t dt$ (Put $t = 2\theta \Rightarrow dt = d\theta/2$).
 Let $u = t$, $dv = \sin t dt \Rightarrow du = dt$, $v = -\cos t$. Then
 $I = \frac{1}{8} (-t \cos t + \int \cos t dt)$
 $= \frac{1}{8} (-t \cos t + \sin t) + C$
 $= \frac{1}{8} (\sin 2\theta - 2\theta \cos 2\theta) + C$

7. Let $u = \ln t$, $dv = t^2 dt \Rightarrow du = dt/t$, $v = \frac{1}{3}t^3$. Then
 $\int t^2 \ln t dt = \frac{1}{3}t^3 \ln t - \int \frac{1}{3}t^3 (1/t) dt =$
 $\frac{1}{3}t^3 \ln t - \frac{1}{9}t^3 + C = \frac{1}{9}t^3 (3 \ln t - 1) + C$.

8. Let $u = \cos 3\theta$, $dv = e^{-\theta} d\theta \Rightarrow$
 $du = -3 \sin 3\theta d\theta$, $v = -e^{-\theta}$. Then
 $I = \int e^{-\theta} \cos 3\theta d\theta = -e^{-\theta} \cos 3\theta - 3 \int e^{-\theta} \sin 3\theta d\theta$.
 Integrate by parts again:
 $I = -e^{-\theta} \cos 3\theta + 3e^{-\theta} \sin 3\theta - \int e^{-\theta} 9 \cos 3\theta d\theta$, so
 $10 \int e^{-\theta} \cos 3\theta d\theta = e^{-\theta} (3 \sin 3\theta - \cos 3\theta) + C_1$ and
 $I = \frac{1}{10} e^{-\theta} (3 \sin 3\theta - \cos 3\theta) + C$, where $C = C_1/10$.

9. Let $u = t$, $dv = e^{-t} dt \Rightarrow du = dt$, $v = -e^{-t}$. By
 Formula 6,
 $\int_0^1 t e^{-t} dt = [-t e^{-t}]_0^1 + \int_0^1 e^{-t} dt$
 $= -1/e + [-e^{-t}]_0^1 = -1/e - 1/e + 1$
 $= 1 - 2/e$

10. $I = \int_1^4 \ln \sqrt{x} dx = \frac{1}{2} \int_1^4 \ln x dx = \frac{1}{2} [x \ln x - x]_1^4$ as in
 Example 2. So $I = \frac{1}{2} [(4 \ln 4 - 4) - (0 - 1)] = 2 \ln 4 - \frac{3}{2}$.

11. Let $u = x$, $dv = \cos 2x dx \Rightarrow du = dx$,
 $v = \frac{1}{2} \sin 2x dx$. Then
 $\int_0^{\pi/2} x \cos 2x dx = [\frac{1}{2} x \sin 2x]_0^{\pi/2} - \frac{1}{2} \int_0^{\pi/2} \sin 2x dx$
 $= 0 + [\frac{1}{4} \cos 2x]_0^{\pi/2}$
 $= \frac{1}{4} (-1 - 1) = -\frac{1}{2}$

12. Let $u = x^2$, $dv = e^{-x} dx \Rightarrow du = 2x dx$, $v = -e^{-x}$.
 Then
 $I = \int_0^1 x^2 e^{-x} dx = [-x^2 e^{-x}]_0^1 + \int_0^1 2x e^{-x} dx$
 $= -1/e + \int_0^1 2x e^{-x} dx$

Now use parts again with $u = 2x$, $dv = e^{-x}$. Then

$$I = -1/e - [2x e^{-x}]_0^1 + \int_0^1 2 e^{-x} dx$$

$$= -1/e - 2/e - [2e^{-x}]_0^1 = -3/e - 2/e + 2$$

$$= 2 - 5/e$$

13. Substitute $t = x^2 \Rightarrow dt = 2x dx$. Then use parts with $u = t, dv = e^t dt \Rightarrow du = dt, v = e^t$. Thus,

$$\begin{aligned} \int x^3 e^{x^2} dx &= \frac{1}{2} \int t e^t dt = \frac{1}{2} t e^t - \frac{1}{2} \int e^t dt \\ &= \frac{1}{2} t e^t - \frac{1}{2} e^t + C = \frac{1}{2} e^{x^2} (x^2 - 1) + C \end{aligned}$$

14. Let $w = \ln x$, so that $x = e^w$ and $dx = e^w dw$. Then

$$\begin{aligned} \int \sin(\ln x) dx &= \int e^w \sin w dw \\ &= \frac{1}{2} e^w (\sin w - \cos w) + C \text{ (by Example 4)} \\ &= \frac{1}{2} x [\sin(\ln x) - \cos(\ln x)] + C \end{aligned}$$

15. Let $u = \tan^{-1} x, dv = x dx \Rightarrow du = dx/(1+x^2), v = \frac{1}{2}x^2$.

$$\text{Then } \int x \tan^{-1} x dx = \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx.$$

But

$$\begin{aligned} \int \frac{x^2}{1+x^2} dx &= \int \frac{(1+x^2) - 1}{1+x^2} dx \\ &= \int 1 dx - \int \frac{1}{1+x^2} dx \\ &= x - \tan^{-1} x + C_1 \end{aligned}$$

so

$$\begin{aligned} \int x \tan^{-1} x dx &= \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x + C_1) \\ &= \frac{1}{2} (x^2 \tan^{-1} x + \tan^{-1} x - x) + C \end{aligned}$$

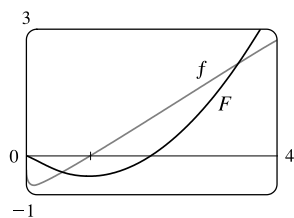
16. Substitute $t = x^3 \Rightarrow dt = 3x^2 dx$. Then use parts with $u = t, dv = \cos t dt$. Thus

$$\begin{aligned} \int x^5 \cos(x^3) dx &= \frac{1}{3} \int x^3 \cos(x^3) \cdot 3x^2 dx = \frac{1}{3} \int t \cos t dt \\ &= \frac{1}{3} t \sin t - \frac{1}{3} \int \sin t dt \\ &= \frac{1}{3} t \sin t + \frac{1}{3} \cos t + C \\ &= \frac{1}{3} x^3 \sin(x^3) + \frac{1}{3} \cos(x^3) + C \end{aligned}$$

17. Let $u = \ln x, dv = \sqrt{x} dx \Rightarrow du = dx/x, v = \int \sqrt{x} dx = \frac{2}{3}x^{3/2}$. Thus

$$\begin{aligned} \int \sqrt{x} \ln x dx &= \frac{2}{3} x^{3/2} \ln x - \int \frac{2}{3} x^{3/2} (1/x) dx \\ &= \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C \end{aligned}$$

We see from the graph that this is reasonable, since the antiderivative is increasing where the original function is positive.

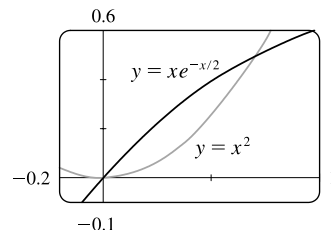


18. Let $u = \sin^{-1} x, dv = dx \Rightarrow du = \frac{dx}{\sqrt{1-x^2}}, v = x$.

Then

$$\begin{aligned} \text{area} &= \int_0^{1/2} \sin^{-1} x dx \\ &= [x \sin^{-1} x]_0^{1/2} - \int_0^{1/2} \frac{x}{\sqrt{1-x^2}} dx \\ &= \frac{1}{2} \left(\frac{\pi}{6}\right) + \left[\sqrt{1-x^2}\right]_0^{1/2} \\ &= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 = \frac{1}{12} (\pi + 6\sqrt{3} - 12) \end{aligned}$$

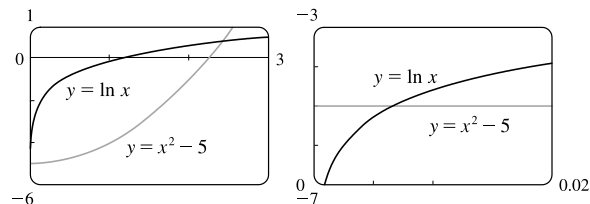
19.



From the graph, we see that the curves intersect at approximately $x = 0$ and $x = 0.70$, with $x e^{-x/2} > x^2$ on $(0, 0.70)$. So the area bounded by the curves is approximately $A = \int_0^{0.70} (x e^{-x/2} - x^2) dx$. We separate this into two integrals, and evaluate the first one by parts with $u = x, dv = e^{-x/2} dx \Rightarrow du = dx, v = -2e^{-x/2}$:

$$\begin{aligned} A &= \left[-2x e^{-x/2}\right]_0^{0.70} - \int_0^{0.70} (-2e^{-x/2}) dx - \left[\frac{1}{3}x^3\right]_0^{0.70} \\ &= [-2(0.70)e^{-0.35} - 0] - \left[4e^{-x/2}\right]_0^{0.70} - \frac{1}{3}[0.70^3 - 0] \\ &\approx 0.080 \end{aligned}$$

20.



From the graphs, we see that the curves intersect at approximately $x = 0.0067$ and $x = 2.43$, with $\ln x > x^2 - 5$ on $(0.0067, 2.43)$. So the area bounded by the curves is about

$$\begin{aligned} A &= \int_{0.0067}^{2.43} [\ln x - (x^2 - 5)] dx \\ &= \int_{0.0067}^{2.43} (\ln x - x^2 + 5) dx \\ &= \left[(x \ln x - x) - \frac{1}{3}x^3 + 5x\right]_{0.0067}^{2.43} \text{ (see Example 2)} \\ &\approx 7.10 \end{aligned}$$