 Answers

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[E Click here for exercises.](#)[S Click here for solutions.](#)

- $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$
- $x \sin x + \cos x + C$
- $-\frac{1}{4}x \cos 4x + \frac{1}{16} \sin 4x + C$
- $\frac{1}{3}x^2 \sin 3x + \frac{2}{9}x \cos 3x - \frac{2}{27} \sin 3x + C$
- $-\frac{x^2}{a} \cos ax + \frac{2x}{a^2} \sin ax + \frac{2}{a^3} \cos ax + C$
- $\frac{1}{8} (\sin 2\theta - 2\theta \cos 2\theta) + C$
- $\frac{1}{9}t^3 (3 \ln t - 1) + C$
- $\frac{1}{10}e^{-\theta} (3 \sin 3\theta - \cos 3\theta) + C$
- $1 - 2/e$
- $2 \ln 4 - \frac{3}{2}$
- $-\frac{1}{2}$
- $2 - 5/e$
- $\frac{1}{2}e^{x^2} (x^2 - 1) + C$
- $\frac{1}{2}x [\sin (\ln x) - \cos (\ln x)] + C$
- $\frac{1}{2} (x^2 \tan^{-1} x + \tan^{-1} x - x) + C$
- $\frac{1}{3}x^3 \sin (x^3) + \frac{1}{3} \cos (x^3) + C$
- $\frac{2}{3}x^{3/2} \ln x - \frac{4}{9}x^{3/2} + C$
- $\frac{1}{12} (\pi + 6\sqrt{3} - 12)$
- 0.080
- 7.10

## Solutions

**E** Click here for exercises.

1. Let  $u = x$ ,  $dv = e^{2x} dx \Rightarrow$   
 $du = dx$ ,  $v = \frac{1}{2}e^{2x}$ . Then by Equation 2,  
 $\int x e^{2x} dx = \frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x} dx = \frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x} + C$ .

2. Let  $u = x$ ,  $dv = \cos x dx \Rightarrow du = dx$ ,  
 $v = \sin x$ . Then by Equation 2,  
 $\int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x + C$ .

3. Let  $u = x$ ,  $dv = \sin 4x dx \Rightarrow du = dx$ ,  $v = -\frac{1}{4} \cos 4x$ .  
 Then  
 $\int x \sin 4x dx = -\frac{1}{4}x \cos 4x - \int (-\frac{1}{4} \cos 4x) dx$   
 $= -\frac{1}{4}x \cos 4x + \frac{1}{16} \sin 4x + C$

4. Let  $u = x^2$ ,  $dv = \cos 3x dx \Rightarrow$   
 $du = 2x dx$ ,  $v = \frac{1}{3} \sin 3x$ . Then  
 $I = \int x^2 \cos 3x dx = \frac{1}{3}x^2 \sin 3x - \frac{2}{3} \int x \sin 3x dx$  by  
 Equation 2. Next let  $U = x$ ,  $dV = \sin 3x dx \Rightarrow$   
 $dU = dx$ ,  $V = -\frac{1}{3} \cos 3x$  to get  
 $\int x \sin 3x dx = -\frac{1}{3}x \cos 3x + \frac{1}{3} \int \cos 3x dx$   
 $= -\frac{1}{3}x \cos 3x + \frac{1}{9} \sin 3x + C_1$

Substituting for  $\int x \sin 3x dx$ , we get

$$I = \frac{1}{3}x^2 \sin 3x - \frac{2}{3} \left( -\frac{1}{3}x \cos 3x + \frac{1}{9} \sin 3x + C_1 \right)$$

$$= \frac{1}{3}x^2 \sin 3x + \frac{2}{9}x \cos 3x - \frac{2}{27} \sin 3x + C$$

where  $C = -\frac{2}{3}C_1$ .

5. Let  $u = x^2$ ,  $dv = \sin ax dx \Rightarrow du = 2x dx$ ,  
 $v = -\frac{1}{a} \cos ax$ . Then

$$I = \int x^2 \sin ax dx$$

$$= -\frac{x^2}{a} \cos ax - \int \left( -\frac{1}{a} \right) \cos ax (2x dx)$$

$$= -\frac{x^2}{a} \cos ax + \frac{2}{a} \int x \cos ax dx$$

by Equation 2. Let  $U = x$ ,  $dV = \cos ax dx \Rightarrow$   
 $dU = dx$ ,  $V = \frac{1}{a} \sin ax$ . Then

$$\int x \cos ax dx = \frac{x}{a} \sin ax - \int \frac{1}{a} \sin ax dx$$

$$= \frac{x}{a} \sin ax + \frac{1}{a^2} \cos ax + C_1$$

So

$$I = -\frac{x^2}{a} \cos ax + \frac{2}{a} \left( \frac{x}{a} \sin ax + \frac{1}{a^2} \cos ax + C_1 \right)$$

$$= -\frac{x^2}{a} \cos ax + \frac{2x}{a^2} \sin ax + \frac{2}{a^3} \cos ax + C$$

**A** Click here for answers.

6.  $I = \int \theta \sin \theta \cos \theta d\theta = \frac{1}{4} \int 2\theta \sin 2\theta d\theta$   
 $= \frac{1}{8} \int t \sin t dt$  (Put  $t = 2\theta \Rightarrow dt = d\theta/2$ ).  
 Let  $u = t$ ,  $dv = \sin t dt \Rightarrow du = dt$ ,  $v = -\cos t$ . Then  
 $I = \frac{1}{8} (-t \cos t + \int \cos t dt)$   
 $= \frac{1}{8} (-t \cos t + \sin t) + C$   
 $= \frac{1}{8} (\sin 2\theta - 2\theta \cos 2\theta) + C$

7. Let  $u = \ln t$ ,  $dv = t^2 dt \Rightarrow du = dt/t$ ,  $v = \frac{1}{3}t^3$ . Then  
 $\int t^2 \ln t dt = \frac{1}{3}t^3 \ln t - \int \frac{1}{3}t^3 (1/t) dt =$   
 $\frac{1}{3}t^3 \ln t - \frac{1}{9}t^3 + C = \frac{1}{9}t^3 (3 \ln t - 1) + C$ .

8. Let  $u = \cos 3\theta$ ,  $dv = e^{-\theta} d\theta \Rightarrow$   
 $du = -3 \sin 3\theta d\theta$ ,  $v = -e^{-\theta}$ . Then  
 $I = \int e^{-\theta} \cos 3\theta d\theta = -e^{-\theta} \cos 3\theta - 3 \int e^{-\theta} \sin 3\theta d\theta$ .  
 Integrate by parts again:  
 $I = -e^{-\theta} \cos 3\theta + 3e^{-\theta} \sin 3\theta - \int e^{-\theta} 9 \cos 3\theta d\theta$ , so  
 $10 \int e^{-\theta} \cos 3\theta d\theta = e^{-\theta} (3 \sin 3\theta - \cos 3\theta) + C_1$  and  
 $I = \frac{1}{10} e^{-\theta} (3 \sin 3\theta - \cos 3\theta) + C$ , where  $C = C_1/10$ .

9. Let  $u = t$ ,  $dv = e^{-t} dt \Rightarrow du = dt$ ,  $v = -e^{-t}$ . By  
 Formula 6,  
 $\int_0^1 t e^{-t} dt = [-t e^{-t}]_0^1 + \int_0^1 e^{-t} dt$   
 $= -1/e + [-e^{-t}]_0^1 = -1/e - 1/e + 1$   
 $= 1 - 2/e$

10.  $I = \int_1^4 \ln \sqrt{x} dx = \frac{1}{2} \int_1^4 \ln x dx = \frac{1}{2} [x \ln x - x]_1^4$  as in  
 Example 2. So  $I = \frac{1}{2} [(4 \ln 4 - 4) - (0 - 1)] = 2 \ln 4 - \frac{3}{2}$ .

11. Let  $u = x$ ,  $dv = \cos 2x dx \Rightarrow du = dx$ ,  
 $v = \frac{1}{2} \sin 2x dx$ . Then  
 $\int_0^{\pi/2} x \cos 2x dx = [\frac{1}{2} x \sin 2x]_0^{\pi/2} - \frac{1}{2} \int_0^{\pi/2} \sin 2x dx$   
 $= 0 + [\frac{1}{4} \cos 2x]_0^{\pi/2}$   
 $= \frac{1}{4} (-1 - 1) = -\frac{1}{2}$

12. Let  $u = x^2$ ,  $dv = e^{-x} dx \Rightarrow du = 2x dx$ ,  $v = -e^{-x}$ .  
 Then  
 $I = \int_0^1 x^2 e^{-x} dx = [-x^2 e^{-x}]_0^1 + \int_0^1 2x e^{-x} dx$   
 $= -1/e + \int_0^1 2x e^{-x} dx$

Now use parts again with  $u = 2x$ ,  $dv = e^{-x}$ . Then

$$I = -1/e - [2x e^{-x}]_0^1 + \int_0^1 2 e^{-x} dx$$

$$= -1/e - 2/e - [2e^{-x}]_0^1 = -3/e - 2/e + 2$$

$$= 2 - 5/e$$

13. Substitute  $t = x^2 \Rightarrow dt = 2x dx$ . Then use parts with  $u = t, dv = e^t dt \Rightarrow du = dt, v = e^t$ . Thus,

$$\begin{aligned}\int x^3 e^{x^2} dx &= \frac{1}{2} \int t e^t dt = \frac{1}{2} t e^t - \frac{1}{2} \int e^t dt \\ &= \frac{1}{2} t e^t - \frac{1}{2} e^t + C = \frac{1}{2} e^{x^2} (x^2 - 1) + C\end{aligned}$$

14. Let  $w = \ln x$ , so that  $x = e^w$  and  $dx = e^w dw$ . Then

$$\begin{aligned}\int \sin(\ln x) dx &= \int e^w \sin w dw \\ &= \frac{1}{2} e^w (\sin w - \cos w) + C \quad (\text{by Example 4}) \\ &= \frac{1}{2} x [\sin(\ln x) - \cos(\ln x)] + C\end{aligned}$$

15. Let  $u = \tan^{-1} x, dv = x dx \Rightarrow du = dx/(1+x^2), v = \frac{1}{2}x^2$ .

$$\text{Then } \int x \tan^{-1} x dx = \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx.$$

But

$$\begin{aligned}\int \frac{x^2}{1+x^2} dx &= \int \frac{(1+x^2) - 1}{1+x^2} dx \\ &= \int 1 dx - \int \frac{1}{1+x^2} dx \\ &= x - \tan^{-1} x + C_1\end{aligned}$$

so

$$\begin{aligned}\int x \tan^{-1} x dx &= \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x + C_1) \\ &= \frac{1}{2} (x^2 \tan^{-1} x + \tan^{-1} x - x) + C\end{aligned}$$

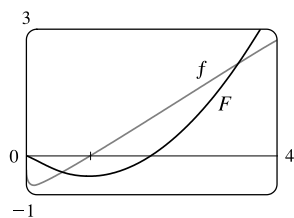
16. Substitute  $t = x^3 \Rightarrow dt = 3x^2 dx$ . Then use parts with  $u = t, dv = \cos t dt$ . Thus

$$\begin{aligned}\int x^5 \cos(x^3) dx &= \frac{1}{3} \int x^3 \cos(x^3) \cdot 3x^2 dx = \frac{1}{3} \int t \cos t dt \\ &= \frac{1}{3} t \sin t - \frac{1}{3} \int \sin t dt \\ &= \frac{1}{3} t \sin t + \frac{1}{3} \cos t + C \\ &= \frac{1}{3} x^3 \sin(x^3) + \frac{1}{3} \cos(x^3) + C\end{aligned}$$

17. Let  $u = \ln x, dv = \sqrt{x} dx \Rightarrow du = dx/x, v = \int \sqrt{x} dx = \frac{2}{3}x^{3/2}$ . Thus

$$\begin{aligned}\int \sqrt{x} \ln x dx &= \frac{2}{3} x^{3/2} \ln x - \int \frac{2}{3} x^{3/2} (1/x) dx \\ &= \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C\end{aligned}$$

We see from the graph that this is reasonable, since the antiderivative is increasing where the original function is positive.

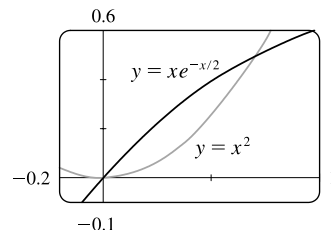


18. Let  $u = \sin^{-1} x, dv = dx \Rightarrow du = \frac{dx}{\sqrt{1-x^2}}, v = x$ .

Then

$$\begin{aligned}\text{area} &= \int_0^{1/2} \sin^{-1} x dx \\ &= [x \sin^{-1} x]_0^{1/2} - \int_0^{1/2} \frac{x}{\sqrt{1-x^2}} dx \\ &= \frac{1}{2} \left(\frac{\pi}{6}\right) + \left[\sqrt{1-x^2}\right]_0^{1/2} \\ &= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 = \frac{1}{12} (\pi + 6\sqrt{3} - 12)\end{aligned}$$

- 19.

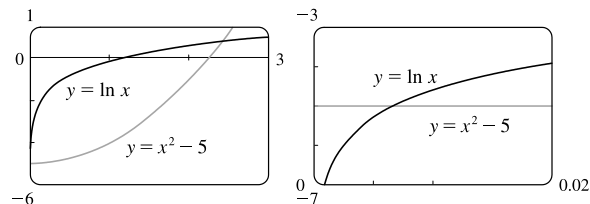


From the graph, we see that the curves intersect at approximately  $x = 0$  and  $x = 0.70$ , with  $x e^{-x/2} > x^2$  on  $(0, 0.70)$ . So the area bounded by the curves is

approximately  $A = \int_0^{0.70} (x e^{-x/2} - x^2) dx$ . We separate this into two integrals, and evaluate the first one by parts with  $u = x, dv = e^{-x/2} dx \Rightarrow du = dx, v = -2e^{-x/2}$ :

$$\begin{aligned}A &= \left[-2x e^{-x/2}\right]_0^{0.70} - \int_0^{0.70} (-2e^{-x/2}) dx - \left[\frac{1}{3}x^3\right]_0^{0.70} \\ &= [-2(0.70)e^{-0.35} - 0] - \left[4e^{-x/2}\right]_0^{0.70} - \frac{1}{3}[0.70^3 - 0] \\ &\approx 0.080\end{aligned}$$

- 20.



From the graphs, we see that the curves intersect at approximately  $x = 0.0067$  and  $x = 2.43$ , with  $\ln x > x^2 - 5$  on  $(0.0067, 2.43)$ . So the area bounded by the curves is about

$$\begin{aligned}A &= \int_{0.0067}^{2.43} [\ln x - (x^2 - 5)] dx \\ &= \int_{0.0067}^{2.43} (\ln x - x^2 + 5) dx \\ &= \left[(x \ln x - x) - \frac{1}{3}x^3 + 5x\right]_{0.0067}^{2.43} \quad (\text{see Example 2}) \\ &\approx 7.10\end{aligned}$$