

## 5.3 Evaluating Definite Integrals

**A** Click here for answers.

**1–2** ■ Verify by differentiation that the formula is correct.

$$1. \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$2. \int x^2 \sin x \, dx = -x^2 \cos x + 2 \int x \cos x \, dx$$

**3–4** ■ Find the general indefinite integral.

$$3. \int \sqrt{x}(x^2 - 1/x) \, dx$$

$$4. \int (2x + \sec x \tan x) \, dx$$

**5–29** ■ Evaluate the integral.

$$5. \int_0^1 (1 - 2x - 3x^2) \, dx$$

$$6. \int_1^2 (5x^2 - 4x + 3) \, dx$$

$$7. \int_0^1 (y^9 - 2y^5 + 3y) \, dy$$

$$8. \int_1^3 \left( \frac{1}{t^2} - \frac{1}{t^4} \right) dt$$

$$9. \int_{-2}^4 (3x - 5) \, dx$$

$$10. \int_1^2 x^{-2} \, dx$$

$$11. \int_0^4 \sqrt{x} \, dx$$

$$12. \int_0^1 x^{3/7} \, dx$$

$$13. \int_{-1}^4 \pi \, dx$$

$$14. \int_{-4}^{-1} \sqrt{3} \, dx$$

$$15. \int_3^6 (4 - 7x) \, dx$$

$$16. \int_1^4 (2x^2 - 3x + 1) \, dx$$

$$17. \int_1^3 (x - 2)(x + 3) \, dx$$

$$18. \int_0^{\pi/3} (1 - 2 \cos x) \, dx$$

$$19. \int_0^1 (5 \cos x + 4x) \, dx$$

$$20. \int_3^3 \sqrt{x^5 + 2} \, dx$$

$$21. \int_{\pi/4}^{\pi/3} \sin t \, dt$$

$$22. \int_1^2 \left( x + \frac{1}{x} \right)^2 dx$$

$$23. \int_0^1 (\sqrt[4]{x^5} + \sqrt[5]{x^4}) \, dx$$

$$24. \int_1^8 \frac{x - 1}{\sqrt[3]{x^2}} \, dx$$

**S** Click here for solutions.

$$25. \int_{\ln 3}^{\ln 6} 8e^x \, dx$$

$$26. \int_8^9 2^t \, dt$$

$$27. \int_{-e^2}^{-e} \frac{3}{x} \, dx$$

$$28. \int_1^{\sqrt{3}} \frac{6}{1 + x^2} \, dx$$

$$29. \int_0^{0.5} \frac{dx}{\sqrt{1 - x^2}}$$

$$30. \int_1^2 \frac{t^6 - t^2}{t^4} \, dt$$

$$31. \int_1^2 \frac{x^2 + 1}{\sqrt{x}} \, dx$$

$$32. \int_0^2 (x^3 - 1)^2 \, dx$$

$$33. \int_0^1 u(\sqrt{u} + \sqrt[3]{u}) \, du$$

$$34. \int_{-1}^2 |x - x^2| \, dx$$

$$35. \int_{-2}^3 |x^2 - 1| \, dx$$

$$36. \int_1^{-1} (x - 1)(3x + 2) \, dx$$

$$37. \int_1^4 \left( \sqrt{t} - \frac{2}{\sqrt{t}} \right) dt$$

$$38. \int_1^8 \left( \sqrt[3]{r} + \frac{1}{\sqrt[3]{r}} \right) dr$$

$$39. \int_{-1}^0 (x + 1)^3 \, dx$$

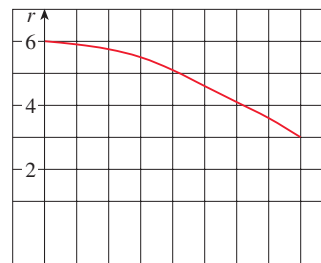
$$40. \int_{-5}^{-2} \frac{x^4 - 1}{x^2 + 1} \, dx$$

$$41. \int_{\pi/6}^{\pi/3} \csc^2 \theta \, d\theta$$

$$42. \int_{\pi/3}^{\pi/2} \csc x \cot x \, dx$$

$$43. \int_0^2 (x^2 - |x - 1|) \, dx$$

**44.** Water leaked from a tank at a rate of  $r(t)$  liters per hour, where the graph of  $r$  is as shown. Express the total amount of water that leaked out during the first four hours as a definite integral. Then use the Midpoint Rule to estimate that amount.



## Answers

**E** [Click here for exercises.](#)

3.  $\frac{2}{7}x^{7/2} - 2x^{1/2} + C$

5.  $-1$

7.  $\frac{19}{15}$

9.  $-12$

11.  $\frac{16}{3}$

13.  $5\pi$

15.  $-\frac{165}{2}$

17.  $\frac{2}{3}$

19.  $5 \sin 1 + 2$

21.  $\frac{\sqrt{2}-1}{2}$

23.  $1$

25.  $24$

4.  $x^2 + \sec x + C$

6.  $\frac{26}{3}$

8.  $\frac{28}{81}$

10.  $\frac{1}{2}$

12.  $\frac{7}{10}$

14.  $3\sqrt{3}$

16.  $\frac{45}{2}$

18.  $\frac{\pi}{3} - \sqrt{3}$

20.  $0$

22.  $\frac{29}{6}$

24.  $\frac{33}{4}$

26.  $\frac{2^8}{\ln 2}$

**S** [Click here for solutions.](#)

27.  $-3$

29.  $\frac{\pi}{6}$

31.  $\frac{6}{5}(3\sqrt{2}-2)$

33.  $\frac{29}{35}$

35.  $\frac{28}{3}$

37.  $\frac{2}{3}$

38.  $\frac{63}{4}$

39.  $\frac{1}{4}$

40.  $36$

41.  $\frac{2}{3}\sqrt{3}$

42.  $-1 + \frac{2}{3}\sqrt{3}$

43.  $\frac{5}{3}$

44.  $\int_0^4 r(t) dt \approx 19.6 \text{ L}$

28.  $\frac{\pi}{2}$

30.  $\frac{11}{6}$

32.  $\frac{86}{7}$

34.  $\frac{11}{6}$

36.  $2$

## Solutions

E Click here for exercises.

- $\frac{d}{dx} \left( \frac{x}{2} - \frac{\sin 2x}{4} + C \right) = \frac{1}{2} - \frac{1}{4} (\cos 2x) (2) + 0$   
 $= \frac{1}{2} - \frac{1}{2} \cos 2x = \frac{1}{2} - \frac{1}{2} (1 - 2 \sin^2 x) = \sin^2 x$
- $\frac{d}{dx} (-x^2 \cos x + 2 \int x \cos x dx)$   
 $= -x^2 (-\sin x) - 2x \cos x + 2x \cos x = x^2 \sin x$
- $\int \sqrt{x} (x^2 - 1/x) dx = \int (x^{5/2} - x^{-1/2}) dx$   
 $= \frac{2}{7} x^{7/2} - 2x^{1/2} + C$
- $\int (2x + \sec x \tan x) dx = x^2 + \sec x + C$
- $\int_0^1 (1 - 2x - 3x^2) dx = [x - 2 \cdot \frac{1}{2} x^2 - 3 \cdot \frac{1}{3} x^3]_0^1$   
 $= [x - x^2 - x^3]_0^1 = (1 - 1 - 1) - 0 = -1$
- $\int_1^2 (5x^2 - 4x + 3) dx = [5 \cdot \frac{1}{3} x^3 - 4 \cdot \frac{1}{2} x^2 + 3x]_1^2$   
 $= 5 \cdot \frac{8}{3} - 4 \cdot 2 + 6 - (\frac{5}{3} - 2 + 3) = \frac{26}{3}$
- $\int_0^1 (y^9 - 2y^5 + 3y) dy = [\frac{1}{10} y^{10} - 2(\frac{1}{6} y^6) + 3(\frac{1}{2} y^2)]_0^1$   
 $= (\frac{1}{10} - \frac{1}{3} + \frac{3}{2}) - 0 = \frac{19}{15}$
- $\int_1^3 \left( \frac{1}{t^2} - \frac{1}{t^4} \right) dt = \int_1^3 (t^{-2} - t^{-4}) dt$   
 $= \left[ \frac{t^{-1}}{-1} - \frac{t^{-3}}{-3} \right]_1^3 = \left[ \frac{1}{3t^3} - \frac{1}{t} \right]_1^3$   
 $= \left( \frac{1}{81} - \frac{1}{3} \right) - \left( \frac{1}{3} - 1 \right) = \frac{28}{81}$
- $\int_{-2}^4 (3x - 5) dx = (3 \cdot \frac{1}{2} x^2 - 5x) \Big|_{-2}^4$   
 $= (3 \cdot 8 - 5 \cdot 4) - [3 \cdot 2 - (-10)] = -12$
- $\int_1^2 x^{-2} dx = [-x^{-1}]_1^2 = [-1/x]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$
- $\int_0^4 \sqrt{x} dx = \int_0^4 x^{1/2} dx = \left[ \frac{x^{3/2}}{3/2} \right]_0^4 = \left[ \frac{2x^{3/2}}{3} \right]_0^4$   
 $= \frac{2(4)^{3/2}}{3} - 0 = \frac{16}{3}$
- $\int_0^1 x^{3/7} dx = \left[ \frac{x^{10/7}}{10/7} \right]_0^1 = \left[ \frac{7}{10} x^{10/7} \right]_0^1 = \frac{7}{10} - 0 = \frac{7}{10}$
- $\int_{-1}^4 \pi dx = \pi [4 - (-1)] = 5\pi$
- $\int_{-4}^{-1} \sqrt{3} dx = \sqrt{3} (-1 + 4) = 3\sqrt{3}$
- $\int_3^6 (4 - 7x) dx = \int_3^6 4 dx - \int_3^6 7x dx$   
 $= 4(6 - 3) - 7 \int_3^6 x dx$   
 $= 12 - 7 \cdot \frac{1}{2} (6^2 - 3^2)$   
 $= 12 - \frac{7}{2} (27) = -\frac{165}{2}$

A Click here for answers.

- $\int_1^4 (2x^2 - 3x + 1) dx = 2 \int_1^4 x^2 dx - 3 \int_1^4 x dx + \int_1^4 1 dx$   
 $= 2 \cdot \frac{1}{3} (4^3 - 1^3) - 3 \cdot \frac{1}{2} (4^2 - 1^2) + 1(4 - 1)$   
 $= \frac{45}{2} = 22.5$
- $\int_1^3 (x - 2)(x + 3) dx = \int_1^3 (x^2 + x - 6) dx$   
 $= \int_1^3 x^2 dx + \int_1^3 x dx + \int_1^3 (-6) dx$   
 $= \frac{1}{3} (3^3 - 1^3) + \frac{1}{2} (3^2 - 1^2) + (-6)(3 - 1) = \frac{2}{3}$
- $\int_0^{\pi/3} (1 - 2 \cos x) dx = \int_0^{\pi/3} 1 dx - 2 \int_0^{\pi/3} \cos x dx$   
 $= 1 \left( \frac{\pi}{3} - 0 \right) - 2 \sin \frac{\pi}{3} = \frac{\pi}{3} - \sqrt{3}$
- $\int_0^1 (5 \cos x + 4x) dx = 5 \int_0^1 \cos x dx + 4 \int_0^1 x dx$   
 $= 5 \sin 1 + 4 \cdot \frac{1}{2} (1^2 - 0^2) = 5 \sin 1 + 2$
- $\int_3^3 \sqrt{x^5 + 2} dx = 0$  since the lower and upper limits are equal.
- $\int_{\pi/4}^{\pi/3} \sin t dt = [-\cos t]_{\pi/4}^{\pi/3}$   
 $= -\cos \frac{\pi}{3} + \cos \frac{\pi}{4} = -\frac{1}{2} + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{2}$
- $\int_1^2 \left( x + \frac{1}{x} \right)^2 dx = \int_1^2 (x^2 + 2 + x^{-2}) dx$   
 $= \left[ \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} \right]_1^2 = \left[ \frac{x^3}{3} + 2x - \frac{1}{x} \right]_1^2$   
 $= \left( \frac{8}{3} + 4 - \frac{1}{2} \right) - \left( \frac{1}{3} + 2 - 1 \right) = \frac{29}{6}$
- $\int_0^1 \left[ \sqrt[4]{x^5} + \sqrt[5]{x^4} \right] dx = \int_0^1 (x^{5/4} + x^{4/5}) dx$   
 $= \left[ \frac{x^{9/4}}{9/4} + \frac{x^{9/5}}{9/5} \right]_0^1 = \left[ \frac{4}{9} x^{9/4} + \frac{5}{9} x^{9/5} \right]_0^1$   
 $= \frac{4}{9} + \frac{5}{9} - 0 = 1$
- $\int_1^8 \frac{x-1}{\sqrt[3]{x^2}} dx = \int_1^8 (x^{1/3} - x^{-2/3}) dx$   
 $= \left[ \frac{x^{4/3}}{4/3} - \frac{x^{1/3}}{1/3} \right]_1^8$   
 $= \left[ \frac{3}{4} x^{4/3} - 3x^{1/3} \right]_1^8$   
 $= \left( \frac{3}{4} \cdot 16 - 3 \cdot 2 \right) - \left( \frac{3}{4} - 3 \right) = \frac{33}{4}$
- $\int_{\ln 3}^{\ln 6} 8e^x dx = [8e^x]_{\ln 3}^{\ln 6} = 8(e^{\ln 6} - e^{\ln 3})$   
 $= 8(6 - 3) = 24$
- $\int_8^9 2^t dt = \left[ \frac{1}{\ln 2} 2^t \right]_8^9 = \frac{1}{\ln 2} (2^9 - 2^8) = \frac{2^8}{\ln 2}$
- $\int_{-e^2}^{-e} (3/x) dx = [3 \ln |x|]_{-e^2}^{-e} = 3 \ln e - 3 \ln (e^2)$   
 $= 3 \cdot 1 - 3 \cdot 2 = -3$

$$\begin{aligned}
28. \int_1^{\sqrt{3}} \frac{6}{1+x^2} dx &= 6 [\tan^{-1} x]_1^{\sqrt{3}} \\
&= 6 \tan^{-1} \sqrt{3} - 6 \tan^{-1} 1 \\
&= 6 \cdot \frac{\pi}{3} - 6 \cdot \frac{\pi}{4} = \frac{\pi}{2} \\
29. \int_0^{0.5} \frac{dx}{\sqrt{1-x^2}} &= [\sin^{-1} x]_0^{0.5} = \sin^{-1} \frac{1}{2} - \sin^{-1} 0 = \frac{\pi}{6} \\
30. \int_1^2 \frac{t^6 - t^2}{t^4} dt &= \int_1^2 (t^2 - t^{-2}) dt = \left[ \frac{t^3}{3} - \frac{t^{-1}}{-1} \right]_1^2 \\
&= \left[ \frac{t^3}{3} + \frac{1}{t} \right]_1^2 = \left( \frac{8}{3} + \frac{1}{2} \right) - \left( \frac{1}{3} + 1 \right) = \frac{11}{6} \\
31. \int_1^2 \frac{x^2 + 1}{\sqrt{x}} dx &= \int_1^2 (x^{3/2} + x^{-1/2}) dx \\
&= \left[ \frac{x^{5/2}}{5/2} + \frac{x^{1/2}}{1/2} \right]_1^2 = \left[ \frac{2}{5} x^{5/2} + 2x^{1/2} \right]_1^2 \\
&= \left( \frac{2}{5} 4\sqrt{2} + 2\sqrt{2} \right) - \left( \frac{2}{5} + 2 \right) \\
&= \frac{18\sqrt{2} - 12}{5} = \frac{6}{5} (3\sqrt{2} - 2) \\
32. \int_0^2 (x^3 - 1)^2 dx &= \int_0^2 (x^6 - 2x^3 + 1) dx \\
&= \left[ \frac{1}{7} x^7 - 2 \left( \frac{1}{4} x^4 \right) + x \right]_0^2 = \left( \frac{128}{7} - 2 \cdot 4 + 2 \right) - 0 = \frac{86}{7} \\
33. \int_0^1 u (\sqrt{u} + \sqrt[3]{u}) du &= \int_0^1 (u^{3/2} + u^{4/3}) du = \left[ \frac{u^{5/2}}{5/2} + \frac{u^{7/3}}{7/3} \right]_0^1 \\
&= \left[ \frac{2}{5} u^{5/2} + \frac{3}{7} u^{7/3} \right]_0^1 = \frac{2}{5} + \frac{3}{7} = \frac{29}{35} \\
34. \int_{-1}^2 |x - x^2| dx &= \int_{-1}^0 (x^2 - x) dx + \int_0^1 (x - x^2) dx + \int_1^2 (x^2 - x) dx \\
&= \left[ \frac{x^3}{3} - \frac{x^2}{2} \right]_{-1}^0 + \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 + \left[ \frac{x^3}{3} - \frac{x^2}{2} \right]_1^2 \\
&= 0 - \left( -\frac{1}{3} - \frac{1}{2} \right) + \left( \frac{1}{2} - \frac{1}{3} \right) - 0 + \left( \frac{8}{3} - 2 \right) - \left( \frac{1}{3} - \frac{1}{2} \right) \\
&= \frac{11}{6} \\
35. \int_{-2}^3 |x^2 - 1| dx &= \int_{-2}^{-1} (x^2 - 1) dx + \int_{-1}^1 (1 - x^2) dx + \int_1^3 (x^2 - 1) dx \\
&= \left[ \frac{x^3}{3} - x \right]_{-2}^{-1} + \left[ x - \frac{x^3}{3} \right]_{-1}^1 + \left[ \frac{x^3}{3} - x \right]_1^3 \\
&= \left( -\frac{1}{3} + 1 \right) - \left( -\frac{8}{3} + 2 \right) + \left( 1 - \frac{1}{3} \right) \\
&\quad - \left( -1 + \frac{1}{3} \right) + \left( 9 - 3 \right) - \left( \frac{1}{3} - 1 \right) \\
&= \frac{28}{3} \\
36. \int_1^{-1} (x-1)(3x+2) dx &= -\int_{-1}^1 (3x^2 - x - 2) dx \\
&= - \left[ 3 \frac{x^3}{3} - \frac{x^2}{2} - 2x \right]_{-1}^1 \\
&= \left[ -x^3 + \frac{x^2}{2} + 2x \right]_{-1}^1 \\
&= \left( -1 + \frac{1}{2} + 2 \right) - \left( 1 + \frac{1}{2} - 2 \right) = 2
\end{aligned}$$

$$\begin{aligned}
37. \int_1^4 \left( \sqrt{t} - \frac{2}{\sqrt{t}} \right) dt &= \int_1^4 (t^{1/2} - 2t^{-1/2}) dt \\
&= \left[ \frac{t^{3/2}}{3/2} - 2 \frac{t^{1/2}}{1/2} \right]_1^4 = \left[ \frac{2}{3} t^{3/2} - 4t^{1/2} \right]_1^4 \\
&= \left( \frac{2}{3} \cdot 8 - 4 \cdot 2 \right) - \left( \frac{2}{3} - 4 \right) = \frac{2}{3} \\
38. \int_1^8 \left( \sqrt[3]{r} + \frac{1}{\sqrt[3]{r}} \right) dr &= \int_1^8 (r^{1/3} + r^{-1/3}) dr \\
&= \left[ \frac{r^{4/3}}{4/3} + \frac{r^{2/3}}{2/3} \right]_1^8 = \left[ \frac{3}{4} r^{4/3} + \frac{3}{2} r^{2/3} \right]_1^8 \\
&= \left( \frac{3}{4} \cdot 16 + \frac{3}{2} \cdot 4 \right) - \left( \frac{3}{4} + \frac{3}{2} \right) = \frac{63}{4} \\
39. \int_{-1}^0 (x+1)^3 dx &= \int_{-1}^0 (x^3 + 3x^2 + 3x + 1) dx \\
&= \left[ \frac{x^4}{4} + 3 \frac{x^3}{3} + 3 \frac{x^2}{2} + x \right]_{-1}^0 \\
&= 0 - \left[ \frac{1}{4} - 1 + \frac{3}{2} - 1 \right] = 2 - \frac{7}{4} = \frac{1}{4} \\
40. \int_{-5}^{-2} \frac{x^4 - 1}{x^2 + 1} dx &= \int_{-5}^{-2} (x^2 - 1) dx \\
&= \left[ \frac{x^3}{3} - x \right]_{-5}^{-2} = \left( -\frac{8}{3} + 2 \right) - \left( -\frac{125}{3} + 5 \right) = 36 \\
41. \int_{\pi/6}^{\pi/3} \csc^2 \theta d\theta &= [-\cot \theta]_{\pi/6}^{\pi/3} = -\cot \frac{\pi}{3} + \cot \frac{\pi}{6} \\
&= -\frac{1}{3} \sqrt{3} + \sqrt{3} = \frac{2}{3} \sqrt{3} \\
42. \int_{\pi/3}^{\pi/2} \csc x \cot x dx &= [-\csc x]_{\pi/3}^{\pi/2} \\
&= -\csc \frac{\pi}{2} + \csc \frac{\pi}{3} = -1 + \frac{2}{3} \sqrt{3} \\
43. \int_0^2 (x^2 - |x - 1|) dx &= \int_0^1 (x^2 + x - 1) dx + \int_1^2 (x^2 - x + 1) dx \\
&= \left[ \frac{x^3}{3} + \frac{x^2}{2} - x \right]_0^1 + \left[ \frac{x^3}{3} - \frac{x^2}{2} + x \right]_1^2 \\
&= \left( \frac{1}{3} + \frac{1}{2} - 1 \right) - 0 + \left( \frac{8}{3} - 2 + 2 \right) - \left( \frac{1}{3} - \frac{1}{2} + 1 \right) = \frac{5}{3} \\
44. Let  $w$  be the amount of water in the tank. We are given that the rate of water leaving the tank is  $r(t) = -dw/dt$ . So by the Net Change Theorem, the total loss of water from the tank after four hours is
$$\begin{aligned}
w(0) - w(4) &= -[w(4) - w(0)] \\
&= -\int_0^4 w'(t) dt \\
&= \int_0^4 r(t) dt
\end{aligned}$$
We use the Midpoint Rule with  $n = 4$  and  $\Delta t = 1$ :
$$\begin{aligned}
\int_0^4 r(t) dt &\approx \sum_{i=1}^4 r(\bar{t}_i) \Delta t \\
&= r(0.5) + r(1.5) + r(2.5) + r(3.5) \\
&\approx 5.9 + 5.4 + 4.7 + 3.6 \\
&= 19.6 \text{ L}
\end{aligned}$$$$