

## 5.2 The Definite Integral

**A** [Click here for answers.](#)

**1–7** ■ Use the Midpoint Rule with the given value of  $n$  to approximate the integral. Round the answer to four decimal places.

1.  $\int_0^5 x^3 dx, n = 5$       2.  $\int_1^3 \frac{1}{2x-7} dx, n = 4$

3.  $\int_1^2 \sqrt{1+x^2} dx, n = 10$       4.  $\int_0^{\pi/4} \tan x dx, n = 4$

5.  $\int_0^{10} \sin \sqrt{x} dx, n = 5$       6.  $\int_0^{\pi} \sec(x/3) dx, n = 6$

7.  $\int_2^4 x \ln x dx, n = 4$

**8–9** ■ Express the limit as a definite integral on the given interval.

8.  $\lim_{n \rightarrow \infty} \sum_{i=1}^n [2(x_i^*)^2 - 5x_i^*] \Delta x, [0, 1]$

9.  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{x_i^*} \Delta x, [1, 4]$

**10–19** ■ Use the form of the definition of the integral given in Equation 3 to evaluate the integral.

10.  $\int_a^b c dx$       11.  $\int_{-2}^7 (6 - 2x) dx$

12.  $\int_1^4 (x^2 - 2) dx$       13.  $\int_1^5 (2 + 3x - x^2) dx$

14.  $\int_0^1 (ax + b) dx$       15.  $\int_{-3}^0 (2x^2 - 3x - 4) dx$

16.  $\int_{-1}^1 (t^3 - t^2 + 1) dt$       17.  $\int_a^b (Px^2 + Qx + R) dx$

18.  $\int_0^b (x^3 + 4x) dx$       19.  $\int_2^5 (t^3 - 2t + 3) dt$

**S** [Click here for solutions.](#)

**20–23** ■ Evaluate the integral by interpreting it in terms of areas.

20.  $\int_1^3 (1 + 2x) dx$       21.  $\int_{-1}^3 (2 - x) dx$

22.  $\int_{-2}^2 (1 - |x|) dx$       23.  $\int_0^3 |3x - 5| dx$

**24–27** ■ Write the given sum or difference as a single integral in the form  $\int_a^b f(x) dx$ .

24.  $\int_1^3 f(x) dx + \int_3^6 f(x) dx + \int_6^{12} f(x) dx$

25.  $\int_5^8 f(x) dx + \int_0^5 f(x) dx$

26.  $\int_2^{10} f(x) dx - \int_2^7 f(x) dx$

27.  $\int_{-3}^5 f(x) dx - \int_{-3}^0 f(x) dx + \int_0^6 f(x) dx$

28. If  $\int_2^8 f(x) dx = 1.7$  and  $\int_5^8 f(x) dx = 2.5$ , find  $\int_2^5 f(x) dx$ .

29. If  $\int_0^1 f(t) dt = 2$ ,  $\int_0^4 f(t) dt = -6$ , and  $\int_3^4 f(t) dt = 1$ , find  $\int_1^3 f(t) dt$ .

**30–32** ■ Use Property 8 to estimate the value of the integral.

30.  $\int_{-3}^0 (x^2 + 2x) dx$       31.  $\int_{\pi/4}^{\pi/3} \cos x dx$

32.  $\int_{-1}^1 \sqrt{1+x^4} dx$


**Answers**


---

**E** [Click here for exercises.](#)

**S** [Click here for solutions.](#)

1. 153.125
2.  $-0.7873$
3. 1.8100
4. 0.3450
5. 6.4643
6. 3.9379
7. 6.6969
8.  $\int_0^1 (2x^2 - 5x) dx$
9.  $\int_1^4 \sqrt{x} dx$
10.  $(b - a)c$
11. 9
12. 15
13.  $\frac{8}{3}$
14.  $\frac{a}{2} + b$
15. 19.5
16.  $\frac{4}{3}$
17.  $P \left( \frac{b^3}{3} - \frac{a^3}{3} \right) + Q \left( \frac{b^2}{2} - \frac{a^2}{2} \right) + R(b - a)$
18.  $\frac{b^4}{4} + 2b^2$
19. 140.25
20. 10
21. 4
22. 0
23.  $\frac{41}{6}$
24.  $\int_1^{12} f(x) dx$
25.  $\int_0^8 f(x) dx$
26.  $\int_7^{10} f(x) dx$
27.  $\int_0^6 f(x) dx$
28.  $-0.8$
29.  $-9$
30.  $-3 \leq \int_{-3}^0 (x^2 + 2x) dx \leq 9.$
31.  $\frac{\pi}{24} \leq \int_{\pi/4}^{\pi/3} \cos x dx \leq \frac{\sqrt{2}\pi}{24}$
32.  $2 \leq \int_{-1}^1 \sqrt{1+x^4} dx \leq 2\sqrt{2}$

## Solutions

**E** Click here for exercises.

1. The width of the intervals is  $\Delta x = (5 - 0) / 5 = 1$  so the partition points are 0, 1, 2, 3, 4, 5 and the midpoints are 0.5, 1.5, 2.5, 3.5, 4.5. The Midpoint Rule gives

$$\begin{aligned}\int_0^5 x^3 dx &\approx \sum_{i=1}^5 f(\bar{x}_i) \Delta x \\ &= (0.5)^3 + (1.5)^3 + (2.5)^3 + (3.5)^3 + (4.5)^3 \\ &= 153.125\end{aligned}$$

2. The width of the interval  $\Delta x = (3 - 1) / 4 = 0.5$  so the partition points are 1.0, 1.5, 2.0, 2.5, 3.0 and the midpoints are 1.25, 1.75, 2.25, 2.75.

$$\begin{aligned}\int_1^3 \frac{1}{2x-7} dx &\approx \sum_{i=1}^4 f(\bar{x}_i) \Delta x \\ &= 0.5 \left[ \frac{1}{2(1.25)-7} + \frac{1}{2(1.75)-7} \right. \\ &\quad \left. + \frac{1}{2(2.25)-7} + \frac{1}{2(2.75)-7} \right] \\ &\approx -0.7873\end{aligned}$$

3.  $\Delta x = (2 - 1) / 10 = 0.1$  so the partition points are 1.0, 1.1, ..., 2.0 and the midpoints are 1.05, 1.15, ..., 1.95.

$$\begin{aligned}\int_1^2 \sqrt{1+x^2} dx &\approx \sum_{i=1}^{10} f(\bar{x}_i) \Delta x \\ &= 0.1 \left[ \sqrt{1+(1.05)^2} + \sqrt{1+(1.15)^2} \right. \\ &\quad \left. + \cdots + \sqrt{1+(1.95)^2} \right] \\ &\approx 1.8100\end{aligned}$$

4.  $\Delta x = \frac{1}{4} (\frac{\pi}{4} - 0) = \frac{\pi}{16}$  so the partition points are 0,  $\frac{\pi}{16}$ ,  $\frac{2\pi}{16}$ ,  $\frac{3\pi}{16}$ ,  $\frac{4\pi}{16}$  and the midpoints are  $\frac{\pi}{32}$ ,  $\frac{3\pi}{32}$ ,  $\frac{5\pi}{32}$ ,  $\frac{7\pi}{32}$ . The Midpoint Rule gives

$$\begin{aligned}\int_0^{\pi/4} \tan x dx &\approx \left(\frac{\pi}{16}\right) \left( \tan \frac{\pi}{32} + \tan \frac{3\pi}{32} \right. \\ &\quad \left. + \tan \frac{5\pi}{32} + \tan \frac{7\pi}{32} \right) \\ &\approx 0.3450\end{aligned}$$

**A** Click here for answers.

5.  $\Delta x = (10 - 0) / 5 = 2$ , so the endpoints are 0, 2, 4, 6, 8, and 10, and the midpoints are 1, 3, 5, 7, and 9. The Midpoint Rule gives

$$\begin{aligned}\int_0^{10} \sin \sqrt{x} dx &\approx \sum_{i=1}^5 f(\bar{x}_i) \Delta x \\ &= 2 \left( \sin \sqrt{1} + \sin \sqrt{3} \right. \\ &\quad \left. + \sin \sqrt{5} + \sin \sqrt{7} + \sin \sqrt{9} \right) \\ &\approx 6.4643\end{aligned}$$

6.  $\Delta x = (\pi - 0) / 6 = \frac{\pi}{6}$ , so the endpoints are 0,  $\frac{\pi}{6}$ ,  $\frac{2\pi}{6}$ ,  $\frac{3\pi}{6}$ ,  $\frac{4\pi}{6}$ ,  $\frac{5\pi}{6}$ , and  $\frac{6\pi}{6}$ , and the midpoints are  $\frac{\pi}{12}$ ,  $\frac{3\pi}{12}$ ,  $\frac{5\pi}{12}$ ,  $\frac{7\pi}{12}$ ,  $\frac{9\pi}{12}$ , and  $\frac{11\pi}{12}$ . The Midpoint Rule gives

$$\begin{aligned}\int_0^{\pi} \sec(x/3) dx &\approx \sum_{i=1}^6 f(\bar{x}_i) \Delta x \\ &= \frac{\pi}{6} \left( \sec \frac{\pi}{36} + \sec \frac{3\pi}{36} + \sec \frac{5\pi}{36} \right. \\ &\quad \left. + \sec \frac{7\pi}{36} + \sec \frac{9\pi}{36} + \sec \frac{11\pi}{36} \right) \\ &\approx 3.9379\end{aligned}$$

7.  $\Delta x = (4 - 2) / 4 = 0.5$ , so the endpoints are 2, 2.5, 3, 3.5, and 4, and the midpoints are 2.25, 2.75, 3.25, and 3.75. The Midpoint Rule gives

$$\begin{aligned}\int_2^4 x \ln x dx &\approx \sum_{i=1}^4 f(\bar{x}_i) \Delta x \quad [f(x) = x \ln x] \\ &= 0.5 [f(2.25) + f(2.75) + f(3.25) + f(3.75)] \\ &\approx 6.6969\end{aligned}$$

8. On  $[0, 1]$ ,

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n [2(x_i^*)^2 - 5x_i^*] \Delta x = \int_0^1 (2x^2 - 5x) dx.$$

9. On  $[1, 4]$ ,  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{x_i^*} \Delta x = \int_1^4 \sqrt{x} dx$ .

$$\begin{aligned}10. \int_a^b c dx &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n c = \lim_{n \rightarrow \infty} \frac{b-a}{n} nc \\ &= \lim_{n \rightarrow \infty} (b-a)c = (b-a)c\end{aligned}$$

$$\begin{aligned}
11. \int_{-2}^7 (6-2x) dx &= \lim_{n \rightarrow \infty} \frac{9}{n} \sum_{i=1}^n \left[ 6 - 2 \left( -2 + \frac{9i}{n} \right) \right] \\
&= \lim_{n \rightarrow \infty} \frac{9}{n} \sum_{i=1}^n \left[ 10 - \frac{18i}{n} \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{90}{n} \sum_{i=1}^n 1 - \frac{162}{n^2} \sum_{i=1}^n i \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{90}{n} - \frac{162}{n^2} \frac{n(n+1)}{2} \right] \\
&= \lim_{n \rightarrow \infty} \left[ 90 - 81 \cdot 1 \left( 1 + \frac{1}{n} \right) \right] \\
&= 90 - 81 = 9
\end{aligned}$$

$$\begin{aligned}
12. \int_1^4 (x^2 - 2) dx &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ \left( 1 + \frac{3i}{n} \right)^2 - 2 \right] \\
&= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ \frac{9i^2}{n^2} + \frac{6i}{n} - 1 \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{27}{n^3} \sum_{i=1}^n i^2 + \frac{18}{n^2} \sum_{i=1}^n i - \frac{3}{n} \sum_{i=1}^n 1 \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{27}{n^3} \frac{n(n+1)(2n+1)}{6} + \frac{18}{n^2} \frac{n(n+1)}{2} - \frac{3}{n} n \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{9}{2} \cdot 1 \left( 1 + \frac{1}{n} \right) \left( 2 + \frac{1}{n} \right) + 9 \cdot 1 \left( 1 + \frac{1}{n} \right) - 3 \right] \\
&= \left( \frac{9}{2} \cdot 2 \right) + 9 - 3 = 15
\end{aligned}$$

$$\begin{aligned}
13. \int_1^5 (2+3x-x^2) dx &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left[ 2 + 3 \left( 1 + \frac{4i}{n} \right) - \left( 1 + \frac{4i}{n} \right)^2 \right] \\
&= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left[ -\frac{16i^2}{n^2} + \frac{4i}{n} + 4 \right] \\
&= \lim_{n \rightarrow \infty} \left[ -\frac{64}{n^3} \sum_{i=1}^n i^2 + \frac{16}{n^2} \sum_{i=1}^n i + \frac{16}{n} \sum_{i=1}^n 1 \right] \\
&= \lim_{n \rightarrow \infty} \left[ -\frac{64}{n^3} \frac{n(n+1)(2n+1)}{6} + \frac{16}{n^2} \frac{n(n+1)}{2} + \frac{16}{n} n \right] \\
&= \lim_{n \rightarrow \infty} \left[ -\frac{32}{3} \cdot 1 \left( 1 + \frac{1}{n} \right) \left( 2 + \frac{1}{n} \right) + 8 \cdot 1 \left( 1 + \frac{1}{n} \right) + 16 \right] \\
&= -\frac{64}{3} + 8 + 16 = \frac{8}{3}
\end{aligned}$$

$$\begin{aligned}
14. \int_0^1 (ax+b) dx &= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \left[ a \left( \frac{i}{n} \right) + b \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{a}{n^2} \sum_{i=1}^n i + \frac{b}{n} \sum_{i=1}^n 1 \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{a}{n^2} \frac{n(n+1)}{2} + \frac{b}{n} n \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{a}{2} \cdot 1 \left( 1 + \frac{1}{n} \right) + b \right] = \frac{a}{2} + b
\end{aligned}$$

$$\begin{aligned}
15. \int_{-3}^0 (2x^2 - 3x - 4) dx &= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ 2 \left( -3 + \frac{3i}{n} \right)^2 - 3 \left( -3 + \frac{3i}{n} \right) - 4 \right] \\
&= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ \frac{18i^2}{n^2} - \frac{45i}{n} + 23 \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{54}{n^3} \sum_{i=1}^n i^2 - \frac{135}{n^2} \sum_{i=1}^n i + \frac{69}{n} \sum_{i=1}^n 1 \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{54}{n^3} \frac{n(n+1)(2n+1)}{6} - \frac{135}{n^2} \frac{n(n+1)}{2} + \frac{69}{n} n \right] \\
&= \lim_{n \rightarrow \infty} \left[ 9 \cdot 1 \left( 1 + \frac{1}{n} \right) \left( 2 + \frac{1}{n} \right) - \frac{135}{2} \cdot 1 \left( 1 + \frac{1}{n} \right) + 69 \right] \\
&= 9 \cdot 2 - \frac{135}{2} + 69 = 19.5
\end{aligned}$$

$$\begin{aligned}
16. \int_{-1}^1 (t^3 - t^2 + 1) dt &= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \left[ \left( -1 + \frac{2i}{n} \right)^3 - \left( -1 + \frac{2i}{n} \right)^2 + 1 \right] \\
&= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \left[ \left( \frac{8i^3}{n^3} - \frac{12i^2}{n^2} + \frac{6i}{n} - 1 \right) - \left( \frac{4i^2}{n^2} - \frac{4i}{n} + 1 \right) + 1 \right] \\
&= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \left[ \frac{8i^3}{n^3} - \frac{16i^2}{n^2} + \frac{10i}{n} - 1 \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{16}{n^4} \sum_{i=1}^n i^3 - \frac{32}{n^3} \sum_{i=1}^n i^2 + \frac{20}{n^2} \sum_{i=1}^n i - \frac{2}{n} \sum_{i=1}^n 1 \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{16}{n^4} \frac{n^2(n+1)^2}{4} - \frac{32}{n^3} \frac{n(n+1)(2n+1)}{6} + \frac{20}{n^2} \frac{n(n+1)}{2} - \frac{2}{n} n \right] \\
&= \lim_{n \rightarrow \infty} \left[ 4 \cdot 1^2 \left( 1 + \frac{1}{n} \right)^2 - \frac{16}{3} \cdot 1 \left( 1 + \frac{1}{n} \right) \left( 2 + \frac{1}{n} \right) + 10 \cdot 1 \left( 1 + \frac{1}{n} \right) - 2 \right] \\
&= 4 - \frac{32}{3} + 10 - 2 = \frac{4}{3}
\end{aligned}$$

$$\begin{aligned}
17. \int_a^b (Px^2 + Qx + R) dx &= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n \left[ P \left( a + \frac{b-a}{n} i \right)^2 + Q \left( a + \frac{b-a}{n} i \right) + R \right] \\
&= \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n \left[ P \frac{(b-a)^2}{n^2} i^2 + (2Pa + Q) \frac{b-a}{n} i + (Pa^2 + Qa + R) \right] \\
&= \lim_{n \rightarrow \infty} \left[ P \frac{(b-a)^3}{n^3} \sum_{i=1}^n i^2 + (2Pa + Q) \frac{(b-a)^2}{n^2} \sum_{i=1}^n i + (Pa^2 + Qa + R) \frac{b-a}{n} \sum_{i=1}^n 1 \right]
\end{aligned}$$

$$\begin{aligned}
&= \lim_{n \rightarrow \infty} \left[ P \frac{(b-a)^3 n(n+1)(2n+1)}{n^3} \right. \\
&\quad \left. + (2Pa + Q) \frac{(b-a)^2 n(n+1)}{n^2} \right. \\
&\quad \left. + (Pa^2 + Qa + R) \frac{b-a}{n} n \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{P(b-a)^3}{6} 1 \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \right. \\
&\quad \left. + \left(Pa + \frac{Q}{2}\right) (b-a)^2 1 \left(1 + \frac{1}{n}\right) \right. \\
&\quad \left. + (Pa^2 + Qa + R) (b-a) \right] \\
&= P \frac{(b-a)^3}{3} + \left(Pa + \frac{Q}{2}\right) (b-a)^2 \\
&\quad + (Pa^2 + Qa + R) (b-a) \\
&= P \left( \frac{b^3}{3} - b^2a + ba^2 - \frac{a^3}{3} + ab^2 - 2a^2b + a^3 + a^2b - a^3 \right) \\
&\quad + Q \left( \frac{b^2}{2} - ab + \frac{a^2}{2} + ab - a^2 \right) + R(b-a) \\
&= P \left( \frac{b^3}{3} - \frac{a^3}{3} \right) + Q \left( \frac{b^2}{2} - \frac{a^2}{2} \right) + R(b-a)
\end{aligned}$$

18.  $\int_0^b (x^3 + 4x) dx$

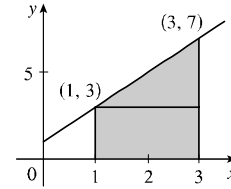
$$\begin{aligned}
&= \lim_{n \rightarrow \infty} \frac{b}{n} \sum_{i=1}^n \left[ \left(\frac{bi}{n}\right)^3 + 4 \left(\frac{bi}{n}\right) \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{b^4}{n^4} \sum_{i=1}^n i^3 + 4 \frac{b^2}{n^2} \sum_{i=1}^n i \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{b^4}{n^4} \frac{n^2(n+1)^2}{4} + \frac{4b^2}{n^2} \frac{n(n+1)}{2} \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{b^4}{4} \cdot 1^2 \left(1 + \frac{1}{n}\right)^2 + 2b^2 \cdot 1 \left(1 + \frac{1}{n}\right) \right] \\
&= \frac{b^4}{4} + 2b^2
\end{aligned}$$

19.  $\int_2^5 (t^3 - 2t + 3) dt$

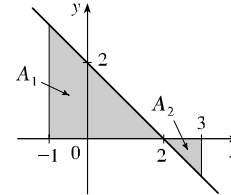
$$\begin{aligned}
&= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ \left(2 + \frac{3i}{n}\right)^3 - 2 \left(2 + \frac{3i}{n}\right) + 3 \right] \\
&= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ \frac{27i^3}{n^3} + \frac{54i^2}{n^2} + \frac{36i}{n} + 8 - 4 - \frac{6i}{n} + 3 \right] \\
&= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[ \frac{27i^3}{n^3} + \frac{54i^2}{n^2} + \frac{30i}{n} + 7 \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{81}{n^4} \sum_{i=1}^n i^3 + \frac{162}{n^3} \sum_{i=1}^n i^2 + \frac{90}{n^2} \sum_{i=1}^n i + \frac{21}{n} \sum_{i=1}^n 1 \right] \\
&= \lim_{n \rightarrow \infty} \left[ \frac{81}{n^4} \frac{n^2(n+1)^2}{4} + \frac{162}{n^3} \frac{n(n+1)(2n+1)}{6} \right. \\
&\quad \left. + \frac{90}{n^2} \frac{n(n+1)}{2} + \frac{21}{n} n \right]
\end{aligned}$$

$$\begin{aligned}
&= \lim_{n \rightarrow \infty} \left[ \frac{81}{4} \cdot 1^2 \left(1 + \frac{1}{n}\right)^2 + 27 \cdot 1 \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \right. \\
&\quad \left. + 45 \cdot 1 \left(1 + \frac{1}{n}\right) + 21 \right] \\
&= \frac{81}{4} + 54 + 45 + 21 = 140.25
\end{aligned}$$

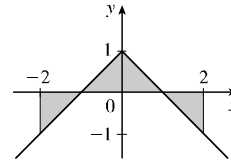
20.  $\int_1^3 (1 + 2x) dx$  can be interpreted as the area under the graph of  $f(x) = 1 + 2x$  between  $x = 1$  and  $x = 3$ . This is equal to the area of the rectangle plus the area of the triangle, so  $\int_1^3 (1 + 2x) dx = A = 2 \cdot 3 + \frac{1}{2} \cdot 2 \cdot 4 = 10$ .  
Or: Use the formula for the area of a trapezoid:  
 $A = \frac{1}{2} (2) (3 + 7) = 10$ .



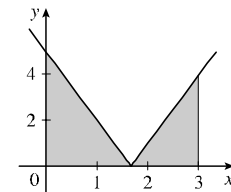
21.  $\int_{-1}^3 (2 - x) dx$  can be interpreted as  $A_1 - A_2$ , where  $A_1$  and  $A_2$  are the areas of the triangles shown. Thus,  $\int_{-1}^3 (2 - x) dx = \frac{1}{2} \cdot 3 \cdot 3 - \frac{1}{2} \cdot 1 \cdot 1 = 4$ .



22.  $\int_{-2}^2 (1 - |x|) dx$  can be interpreted as the area of the middle triangle minus the areas of the outside ones, so  $\int_{-2}^2 (1 - |x|) dx = \frac{1}{2} \cdot 2 \cdot 1 - 2 \cdot \frac{1}{2} \cdot 1 \cdot 1 = 0$ .



23.  $\int_0^3 |3x - 5| dx$  can be interpreted as the area under the graph of the function  $f(x) = |3x - 5|$  between  $x = 0$  and  $x = 3$ . This is equal to the sum of the areas of the two triangles, so  $\int_0^3 |3x - 5| dx = \frac{1}{2} \cdot \frac{5}{3} \cdot 5 + \frac{1}{2} \left(3 - \frac{5}{3}\right) 4 = \frac{41}{6}$ .



24.  $\int_1^3 f(x) dx + \int_3^6 f(x) dx + \int_6^{12} f(x) dx$   
 $= \int_1^6 f(x) dx + \int_6^{12} f(x) dx = \int_1^{12} f(x) dx$

$$\begin{aligned} 25. \int_5^8 f(x) dx + \int_0^5 f(x) dx \\ = \int_0^5 f(x) dx + \int_5^8 f(x) dx = \int_0^8 f(x) dx \end{aligned}$$

$$\begin{aligned} 26. \int_2^{10} f(x) dx - \int_2^7 f(x) dx \\ = \int_2^7 f(x) dx + \int_7^{10} f(x) dx - \int_2^7 f(x) dx = \int_7^{10} f(x) dx \end{aligned}$$

$$\begin{aligned} 27. \int_{-3}^5 f(x) dx - \int_{-3}^0 f(x) dx + \int_5^6 f(x) dx \\ = \int_{-3}^0 f(x) dx + \int_0^5 f(x) dx - \int_{-3}^0 f(x) dx + \int_5^6 f(x) dx \\ = \int_0^6 f(x) dx \end{aligned}$$

$$\begin{aligned} 28. \int_2^5 f(x) dx + \int_5^8 f(x) dx = \int_2^8 f(x) dx \Rightarrow \\ \int_2^5 f(x) dx + 2.5 = 1.7 \Rightarrow \int_2^5 f(x) dx = -0.8 \end{aligned}$$

$$\begin{aligned} 29. \int_0^1 f(t) dt + \int_1^3 f(t) dt + \int_3^4 f(t) dt = \int_0^4 f(t) dt \\ \Rightarrow 2 + \int_1^3 f(t) dt + 1 = -6 \Rightarrow \\ \int_1^3 f(t) dt = -6 - 2 - 1 = -9 \end{aligned}$$

30. If  $f(x) = x^2 + 2x$ ,  $-3 \leq x \leq 0$ , then  $f'(x) = 2x + 2 = 0$  when  $x = -1$ , and  $f(-1) = -1$ . At the endpoints,  $f(-3) = 3$ ,  $f(0) = 0$ . Thus the absolute minimum is  $m = -1$  and the absolute maximum is  $M = 3$ . Thus  $-1[0 - (-3)] \leq \int_{-3}^0 (x^2 + 2x) dx \leq 3[0 - (-3)]$  or  $-3 \leq \int_{-3}^0 (x^2 + 2x) dx \leq 9$ .

31. If  $\frac{\pi}{4} \leq x \leq \frac{\pi}{3}$ , then  $\frac{1}{2} \leq \cos x \leq \frac{\sqrt{2}}{2}$ , so

$$\begin{aligned} \frac{1}{2} \left( \frac{\pi}{3} - \frac{\pi}{4} \right) \leq \int_{\pi/4}^{\pi/3} \cos x dx \leq \frac{\sqrt{2}}{2} \left( \frac{\pi}{3} - \frac{\pi}{4} \right) \text{ or} \\ \frac{\pi}{24} \leq \int_{\pi/4}^{\pi/3} \cos x dx \leq \frac{\sqrt{2}\pi}{24}. \end{aligned}$$

32. For  $-1 \leq x \leq 1$ ,  $0 \leq x^4 \leq 1$  and  $1 \leq \sqrt{1+x^4} \leq \sqrt{2}$ , so

$$\begin{aligned} 1[1 - (-1)] \leq \int_{-1}^1 \sqrt{1+x^4} dx \leq \sqrt{2}[1 - (-1)] \text{ or} \\ 2 \leq \int_{-1}^1 \sqrt{1+x^4} dx \leq 2\sqrt{2}. \end{aligned}$$