# 4.9 Antiderivatives

A Click here for answers.

S Click here for solutions.

**1–19** Find the most general antiderivative of the function. (Check your answer by differentiation.)

**1.** $f(x) = 12x^2 + 6x - 5$

**2.** $f(x) = x^3 - 4x^2 + 17$

**3.** $f(x) = 6x^9 - 4x^7 + 3x^2 + 1$

**4.** $f(x) = x^{99} - 2x^{49} - 1$

**5.** $f(x) = 6/x^5$

**6.** $f(x) = x^{2/3} + 2x^{-1/3}$

**7.** $f(x) = \sqrt{x} + 1/\sqrt{x}$

**8.** $g(t) = \sqrt[5]{t^4} + t^{-6}$

**9.** $f(x) = \sqrt{x} + \sqrt[3]{x}$

**10.** $f(x) = \sqrt[3]{x^2} - \sqrt{x^3}$

**11.** $f(x) = \dfrac{3}{x^2} - \dfrac{5}{x^4}$

**12.** $g(t) = \dfrac{t^3 + 2t^2}{\sqrt{t}}$

**13.** $f(t) = 3\cos t - 4\sin t$

**14.** $f(\theta) = e^\theta + \sec\theta\tan\theta$

**15.** $h(x) = \sin x - 2\cos x$

**16.** $f(t) = \sin t - 2\sqrt{t}$

**17.** $f(t) = \sec^2 t + t^2$

**18.** $f'(x) = x^4 - 2x^2 + x - 1$

**19.** $f'(x) = \sin x - \sqrt[5]{x^2}$

**20–42** Find $f(x)$.

**20.** $f''(x) = x^2 + x^3$

**21.** $f''(x) = 60x^4 - 45x^2$

**22.** $f''(x) = 1$

**23.** $f''(x) = \sin x$

**24.** $f'''(x) = 24x$

**25.** $f'''(x) = \sqrt{x}$

**26.** $f'(x) = 4x + 3, \quad f(0) = -9$

**27.** $f'(x) = 12x^2 - 24x + 1, \quad f(1) = -2$

**28.** $f'(x) = 3\sqrt{x} - 1/\sqrt{x}, \quad f(1) = 2$

**29.** $f'(x) = 1 + 1/x^2, \quad x > 0, \quad f(1) = 1$

**30.** $f'(x) = 3\cos x + 5\sin x, \quad f(0) = 4$

**31.** $f'(x) = \sin x - 2\sqrt{x} \quad f(0) = 0$

**32.** $f'(x) = 2 + \sqrt[5]{x^3}, \quad f(1) = 3$

**33.** $f'(x) = 4 - 3(1 + x^2)^{-1}, \quad f(1) = 0$

**34.** $f''(x) = -8, \quad f(0) = 6, \quad f'(0) = 5$

**35.** $f''(x) = x, \; f(0) = -3, \; f'(0) = 2$

**36.** $f''(x) = 20x^3 - 10, \quad f(1) = 1, \quad f'(1) = -5$

**37.** $f''(x) = x^2 + 3\cos x, \quad f(0) = 2, \quad f'(0) = 3$

**38.** $f''(x) = x + \sqrt{x}, \quad f(1) = 1, \quad f'(1) = 2$

**39.** $f''(x) = 6x + 6, \quad f(0) = 4, \quad f(1) = 3$

**40.** $f''(x) = 12x^2 - 6x + 2, \quad f(0) = 1, \quad f(2) = 11$

**41.** $f''(x) = 1/x^3, \quad x > 0, \quad f(1) = 0, \quad f(2) = 0$

**42.** $f''(x) = 3e^x + 5\sin x, \quad f(0) = 1, \quad f'(0) = 2$

**43–46** A particle is moving with the given data. Find the position of the particle.

**43.** $v(t) = 3 - 2t, \quad s(0) = 4$

**44.** $v(t) = 3\sqrt{t}, \quad s(1) = 5$

**45.** $a(t) = 3t + 8, \quad s(0) = 1, \quad v(0) = -2$

**46.** $a(t) = t^2 - t, \quad s(0) = 0, \quad s(6) = 12$

# Answers

E Click here for exercises.

S Click here for solutions.

1. $4x^3+3x^2-5x+C$
2. $\frac{1}{4}x^4-\frac{4}{3}x^3+17x+C$
3. $\frac{3}{5}x^{10}-\frac{1}{2}x^8+x^3+x+C$
4. $\frac{1}{100}x^{100}-\frac{1}{25}x^{50}-x+C$
5. $-\dfrac{3}{2x^4}+C_1$ if $x<0$, $-\dfrac{3}{2x^4}+C_2$ if $x>0$
6. $\frac{3}{5}x^{5/3}+3x^{2/3}+C_1$ if $x>0$, $\frac{3}{5}x^{5/3}+3x^{2/3}+C_2$ if $x<0$
7. $\frac{2}{3}x^{3/2}+2x^{1/2}+C$
8. $\frac{5}{9}t^{9/5}-\dfrac{1}{5t^5}+C_1$ if $t>0$, $\frac{5}{9}t^{9/5}-\dfrac{1}{5t^5}+C_2$ if $t<0$
9. $\frac{2}{3}x^{3/2}+\frac{3}{4}x^{4/3}+C$
10. $\frac{3}{5}x^{5/3}-\frac{2}{5}x^{5/2}+C$
11. $-\dfrac{3}{x}+\dfrac{5}{3x^3}+C_1$ if $x<0$, $-\dfrac{3}{x}+\dfrac{5}{3x^3}+C_2$ if $x>0$
12. $\frac{2}{7}t^{7/2}+\frac{4}{5}t^{5/2}+C$
13. $3\sin t+4\cos t+C$
14. $e^\theta+\sec\theta+C_n$ on the interval $\left(n\pi-\frac{\pi}{2},n\pi+\frac{\pi}{2}\right)$
15. $-\cos x-2\sin x+C$
16. $-\cos t-\frac{4}{3}t^{3/2}+C$
17. $\tan t+\frac{1}{3}t^3+C_n$ on the interval $\left(n\pi-\frac{\pi}{2},n\pi+\frac{\pi}{2}\right)$
18. $\frac{1}{5}x^5-\frac{2}{3}x^3+\frac{1}{2}x^2-x+C$
19. $-\cos x-\frac{5}{7}x^{7/5}+C$
20. $\frac{1}{12}x^4+\frac{1}{20}x^5+Cx+D$
21. $2x^6-\frac{15}{4}x^4+Cx+D$
22. $\frac{1}{2}x^2+Cx+D$
23. $-\sin x+Cx+D$
24. $x^4+\frac{1}{2}Cx^2+Dx+E$
25. $\frac{8}{105}x^{7/2}+\frac{1}{2}Cx^2+Dx+E$
26. $2x^2+3x-9$
27. $4x^3-12x^2+x+5$
28. $2x^{3/2}-2x^{1/2}+2$
29. $1+x-1/x$
30. $3\sin x-5\cos x+9$
31. $-\cos x-\frac{4}{3}x^{3/2}+1$
32. $2x+\frac{5}{8}x^{8/5}+\frac{3}{8}$
33. $4x-3\arctan x+\frac{3\pi}{4}-4$
34. $-4x^2+5x+6$
35. $\frac{1}{6}x^3+2x-3$
36. $x^5-5x^2+5$
37. $\frac{1}{12}x^4-3\cos x+3x+5$
38. $\frac{1}{6}x^3+\frac{4}{15}x^{5/2}+\frac{5}{6}x-\frac{4}{15}$
39. $x^3+3x^2-5x+4$
40. $x^4-x^3+x^2-x+1$
41. $\dfrac{1}{2x}+\frac{1}{4}x-\frac{3}{4}$
42. $3e^x-5\sin x+4x-2$
43. $s(t)=3t-t^2+4$
44. $s(t)=2t^{3/2}+3$
45. $s(t)=\frac{1}{2}t^3+4t^2-2t+1$
46. $s(t)=\frac{1}{12}t^4-\frac{1}{6}t^3-10t$

# Solutions

E Click here for exercises.

A Click here for answers.

**1.** $f(x) = 12x^2 + 6x - 5 \Rightarrow$
$F(x) = 12\left(\frac{1}{3}x^3\right) + 6\left(\frac{1}{2}x^2\right) - 5x + C = 4x^3 + 3x^2 - 5x + C$

**2.** $f(x) = x^3 - 4x^2 + 17 \Rightarrow$
$F(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 17x + C$

**3.** $f(x) = 6x^9 - 4x^7 + 3x^2 + 1 \Rightarrow$
$F(x) = 6\left(\frac{1}{10}x^{10}\right) - 4\left(\frac{1}{8}x^8\right) + 3\left(\frac{1}{3}x^3\right) + x + C$
$= \frac{3}{5}x^{10} - \frac{1}{2}x^8 + x^3 + x + C$

**4.** $f(x) = x^{99} - 2x^{49} - 1 \Rightarrow$
$F(x) = \left(\frac{1}{100}x^{100}\right) - 2\left(\frac{1}{50}x^{50}\right) - x + C$
$= \frac{1}{100}x^{100} - \frac{1}{25}x^{50} - x + C$

**5.** $f(x) = \dfrac{6}{x^5} = 6x^{-5} \Rightarrow$
$F(x) = 6\dfrac{x^{-4}}{-4} + C_1 = -\dfrac{3}{2x^4} + C_1$ if $x < 0$ and
$F(x) = -\dfrac{3}{2x^4} + C_2$ if $x > 0$.

**6.** $f(x) = x^{2/3} + 2x^{-1/3}$ has domain $(-\infty, 0) \cup (0, \infty)$, so
$F(x) = \dfrac{x^{5/3}}{5/3} + \dfrac{2x^{2/3}}{2/3} + C_1 = \frac{3}{5}x^{5/3} + 3x^{2/3} + C_1$ if $x > 0$ and $F(x) = \frac{3}{5}x^{5/3} + 3x^{2/3} + C_2$ if $x < 0$.

**7.** $f(x) = \sqrt{x} + 1/\sqrt{x} = x^{1/2} + x^{-1/2} \Rightarrow$
$F(x) = \dfrac{x^{3/2}}{3/2} + \dfrac{x^{1/2}}{1/2} + C = \frac{2}{3}x^{3/2} + 2x^{1/2} + C$

**8.** $g(t) = t^{4/5} + t^{-6}$ has domain $(-\infty, 0) \cup (0, \infty)$, so
$G(t) = \dfrac{t^{9/5}}{9/5} + \dfrac{t^{-5}}{-5} + C_1 = \frac{5}{9}t^{9/5} - \dfrac{1}{5t^5} + C_1$ if $t > 0$ and
$G(t) = \frac{5}{9}t^{9/5} - \dfrac{1}{5t^5} + C_2$ if $t < 0$.

**9.** $f(x) = \sqrt{x} + \sqrt[3]{x} = x^{1/2} + x^{1/3} \Rightarrow$
$F(x) = \frac{1}{3/2}x^{3/2} + \frac{1}{4/3}x^{4/3} + C = \frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$

**10.** $f(x) = \sqrt[3]{x^2} - \sqrt{x^3} = x^{2/3} - x^{3/2} \Rightarrow$
$F(x) = \frac{1}{5/3}x^{5/3} - \frac{1}{5/2}x^{5/2} + C = \frac{3}{5}x^{5/3} - \frac{2}{5}x^{5/2} + C$

**11.** $f(x) = 3x^{-2} - 5x^{-4}$ has domain $(-\infty, 0) \cup (0, \infty)$, so
$F(x) = \dfrac{3x^{-1}}{-1} - \dfrac{5x^{-3}}{-3} + C_1 = -\dfrac{3}{x} + \dfrac{5}{3x^3} + C_1$ if $x < 0$
and $F(x) = -\dfrac{3}{x} + \dfrac{5}{3x^3} + C_2$ if $x > 0$.

**12.** $g(t) = \dfrac{t^3 + 2t^2}{\sqrt{t}} = t^{5/2} + 2t^{3/2} \Rightarrow$
$G(t) = \dfrac{t^{7/2}}{7/2} + \dfrac{2t^{5/2}}{5/2} + C = \frac{2}{7}t^{7/2} + \frac{4}{5}t^{5/2} + C$. Note that $g$ has domain $(0, \infty)$.

**13.** $f(t) = 3\cos t - 4\sin t \Rightarrow$
$F(t) = 3(\sin t) - 4(-\cos t) + C = 3\sin t + 4\cos t + C$

**14.** $f(\theta) = e^\theta + \sec\theta\tan\theta \Rightarrow F(\theta) = e^\theta + \sec\theta + C_n$ on the interval $\left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}\right)$.

**15.** $h(x) = \sin x - 2\cos x \Rightarrow$
$H(x) = -\cos x - 2\sin x + C$

**16.** $f(t) = \sin t - 2\sqrt{t} \Rightarrow$
$F(t) = -\cos t - 2\left(\frac{1}{3/2}\right)t^{3/2} + C = -\cos t - \frac{4}{3}t^{3/2} + C$

**17.** $f(t) = \sec^2 t + t^2 \Rightarrow F(t) = \tan t + \frac{1}{3}t^3 + C_n$ on the interval $\left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}\right)$.

**18.** $f'(x) = x^4 - 2x^2 + x - 1 \Rightarrow$
$f(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$

**19.** $f'(x) = \sin x - x^{2/5} \Rightarrow f(x) = -\cos x - \frac{5}{7}x^{7/5} + C$

**20.** $f''(x) = x^2 + x^3 \Rightarrow f'(x) = \frac{1}{3}x^3 + \frac{1}{4}x^4 + C \Rightarrow$
$f(x) = \frac{1}{12}x^4 + \frac{1}{20}x^5 + Cx + D$

**21.** $f''(x) = 60x^4 - 45x^2 \Rightarrow$
$f'(x) = 60\left(\frac{1}{5}x^5\right) - 45\left(\frac{1}{3}x^3\right) + C = 12x^5 - 15x^3 + C$
$\Rightarrow$
$f(x) = 12\left(\frac{1}{6}x^6\right) - 15\left(\frac{1}{4}x^4\right) + Cx + D$
$= 2x^6 - \frac{15}{4}x^4 + Cx + D$

**22.** $f''(x) = 1 \Rightarrow f'(x) = x + C \Rightarrow$
$f(x) = \frac{1}{2}x^2 + Cx + D$

**23.** $f''(x) = \sin x \Rightarrow f'(x) = -\cos x + C \Rightarrow$
$f(x) = -\sin x + Cx + D$

**24.** $f'''(x) = 24x \Rightarrow f''(x) = 12x^2 + C$
$\Rightarrow f'(x) = 4x^3 + Cx + D \Rightarrow$
$f(x) = x^4 + \frac{1}{2}Cx^2 + Dx + E$

**25.** $f'''(x) = x^{1/2} \Rightarrow f''(x) = \frac{2}{3}x^{3/2} + C \Rightarrow$
$f'(x) = \frac{2}{3} \cdot \frac{2}{5}x^{5/2} + Cx + D = \frac{4}{15}x^{5/2} + Cx + D \Rightarrow$
$f(x) = \frac{4}{15} \cdot \frac{2}{7}x^{7/2} + C\left(\frac{1}{2}x^2\right) + Dx + E$
$= \frac{8}{105}x^{7/2} + \frac{1}{2}Cx^2 + Dx + E$

**26.** $f'(x) = 4x + 3 \Rightarrow f(x) = 2x^2 + 3x + C \Rightarrow$
$-9 = f(0) = C \Rightarrow f(x) = 2x^2 + 3x - 9$

**27.** $f'(x) = 12x^2 - 24x + 1 \Rightarrow$
$f(x) = 4x^3 - 12x^2 + x + C \Rightarrow$
$f(1) = 4 - 12 + 1 + C = -2 \Rightarrow C = 5$, so
$f(x) = 4x^3 - 12x^2 + x + 5$

**28.** $f'(x) = 3\sqrt{x} - 1/\sqrt{x} = 3x^{1/2} - x^{-1/2} \Rightarrow$
$f(x) = 3\left(\frac{1}{3/2}\right)x^{3/2} - \frac{1}{1/2}x^{1/2} + C \Rightarrow$
$2 = f(1) = 2 - 2 + C = C \Rightarrow$
$f(x) = 2x^{3/2} - 2x^{1/2} + 2$

**29.** $f'(x) = 1 + x^{-2}, x > 0 \Rightarrow f(x) = x - 1/x + C$. Now $f(1) = 1 - 1 + C = 1 \Rightarrow C = 1$, so $f(x) = 1 + x - 1/x$.

**30.** $f'(x) = 3\cos x + 5\sin x \Rightarrow$
$f(x) = 3\sin x - 5\cos x + C \Rightarrow 4 = f(0) = -5 + C$
$\Rightarrow C = 9 \Rightarrow f(x) = 3\sin x - 5\cos x + 9$

**31.** $f'(x) = \sin x - 2x^{1/2} \Rightarrow f(x) = -\cos x - \frac{4}{3}x^{3/2} + C$
$\Rightarrow f(0) = -1 - 0 + C = 0 \Rightarrow C = 1$, so
$f(x) = -\cos x - \frac{4}{3}x^{3/2} + 1$.

**32.** $f'(x) = 2 + x^{3/5} \Rightarrow f(x) = 2x + \frac{5}{8}x^{8/5} + C \Rightarrow$
$3 = f(1) = 2 + \frac{5}{8} + C \Rightarrow C = \frac{3}{8} \Rightarrow$
$f(x) = 2x + \frac{5}{8}x^{8/5} + \frac{3}{8}$

**33.** $f'(x) = 4 - \dfrac{3}{1+x^2} \Rightarrow f(x) = 4x - 3\arctan x + C$.
Now $f(1) = 4 \cdot 1 - 3\arctan 1 + C = 0 \Rightarrow$
$C = 3 \cdot \frac{\pi}{4} - 4$. Therefore $f(x) = 4x - 3\arctan x + \frac{3\pi}{4} - 4$.

**34.** $f''(x) = -8 \Rightarrow f'(x) = -8x + C \Rightarrow$
$5 = f'(0) = C \Rightarrow f'(x) = -8x + 5 \Rightarrow$
$f(x) = -4x^2 + 5x + D \Rightarrow 6 = f(0) = D \Rightarrow$
$f(x) = -4x^2 + 5x + 6$

**35.** $f''(x) = x \Rightarrow f'(x) = \frac{1}{2}x^2 + C \Rightarrow$
$2 = f'(0) = C \Rightarrow f'(x) = \frac{1}{2}x^2 + 2 \Rightarrow$
$f(x) = \frac{1}{6}x^3 + 2x + D \Rightarrow -3 = f(0) = D \Rightarrow$
$f(x) = \frac{1}{6}x^3 + 2x - 3$

**36.** $f''(x) = 20x^3 - 10 \Rightarrow f'(x) = 5x^4 - 10x + C \Rightarrow$
$-5 = f'(1) = 5 - 10 + C \Rightarrow C = 0 \Rightarrow$
$f'(x) = 5x^4 - 10x \Rightarrow f(x) = x^5 - 5x^2 + D \Rightarrow$
$1 = f(1) = 1 - 5 + D \Rightarrow D = 5 \Rightarrow$
$f(x) = x^5 - 5x^2 + 5$

**37.** $f''(x) = x^2 + 3\cos x \Rightarrow f'(x) = \frac{1}{3}x^3 + 3\sin x + C$
$\Rightarrow 3 = f'(0) = C \Rightarrow f'(x) = \frac{1}{3}x^3 + 3\sin x + 3$
$\Rightarrow f(x) = \frac{1}{12}x^4 - 3\cos x + 3x + D \Rightarrow$
$2 = f(0) = -3 + D \Rightarrow D = 5 \Rightarrow$
$f(x) = \frac{1}{12}x^4 - 3\cos x + 3x + 5$

**38.** $f''(x) = x + x^{1/2} \Rightarrow f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + C$
$\Rightarrow 2 = f'(1) = \frac{1}{2} + \frac{2}{3} + C \Rightarrow$
$C = \frac{5}{6} \Rightarrow f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + \frac{5}{6}$
$\Rightarrow f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x + D \Rightarrow$
$1 = f(1) = \frac{1}{6} + \frac{4}{15} + \frac{5}{6} + D \Rightarrow D = -\frac{4}{15} \Rightarrow$
$f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x - \frac{4}{15}$

**39.** $f''(x) = 6x + 6 \Rightarrow f'(x) = 3x^2 + 6x + C \Rightarrow$
$f(x) = x^3 + 3x^2 + Cx + D$. $4 = f(0) = D$ and
$3 = f(1) = 1 + 3 + C + D = 4 + C + 4 \Rightarrow C = -5$,
so $f(x) = x^3 + 3x^2 - 5x + 4$.

**40.** $f''(x) = 12x^2 - 6x + 2 \Rightarrow$
$f'(x) = 4x^3 - 3x^2 + 2x + C \Rightarrow$
$f(x) = x^4 - x^3 + x^2 + Cx + D$. $1 = f(0) = D$ and
$11 = f(2) = 16 - 8 + 4 + 2C + D = 13 + 2C \Rightarrow$
$C = -1$, so $f(x) = x^4 - x^3 + x^2 - x + 1$.

**41.** $f''(x) = x^{-3} \Rightarrow f'(x) = -\frac{1}{2}x^{-2} + C \Rightarrow$
$f(x) = \frac{1}{2}x^{-1} + Cx + D \Rightarrow 0 = f(1) = \frac{1}{2} + C + D$
and $0 = f(2) = \frac{1}{4} + 2C + D$. Solving these equations, we get $C = \frac{1}{4}$, $D = -\frac{3}{4}$, so $f(x) = 1/(2x) + \frac{1}{4}x - \frac{3}{4}$.

**42.** $f''(x) = 3e^x + 5\sin x \Rightarrow f'(x) = 3e^x - 5\cos x + C$
$\Rightarrow 2 = f'(0) = 3 - 5 + C \Rightarrow C = 4$,
so $f'(x) = 3e^x - 5\cos x + 4 \Rightarrow$
$f(x) = 3e^x - 5\sin x + 4x + D \Rightarrow 1 = f(0) = 3 + D$
$\Rightarrow D = -2$, so $f(x) = 3e^x - 5\sin x + 4x - 2$.

**43.** $v(t) = s'(t) = 3 - 2t \Rightarrow s(t) = 3t - t^2 + C \Rightarrow$
$4 = s(0) = C \Rightarrow s(t) = 3t - t^2 + 4$

**44.** $v(t) = s'(t) = 3\sqrt{t} \Rightarrow s(t) = 2t^{3/2} + C \Rightarrow$
$5 = s(1) = 2 + C \Rightarrow C = 3$, so $s(t) = 2t^{3/2} + 3$

**45.** $a(t) = v'(t) = 3t + 8 \Rightarrow v(t) = \frac{3}{2}t^2 + 8t + C \Rightarrow$
$-2 = v(0) = C \Rightarrow v(t) = \frac{3}{2}t^2 + 8t - 2 \Rightarrow$
$s(t) = \frac{1}{2}t^3 + 4t^2 - 2t + D \Rightarrow 1 = s(0) = D \Rightarrow$
$s(t) = \frac{1}{2}t^3 + 4t^2 - 2t + 1$

**46.** $a(t) = v'(t) = t^2 - t \Rightarrow v(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 + C \Rightarrow$
$s(t) = \frac{1}{12}t^4 - \frac{1}{6}t^3 + Ct + D \Rightarrow 0 = s(0) = D$ and
$12 = s(6) = 108 - 36 + 6C + 0 \Rightarrow C = -10 \Rightarrow$
$s(t) = \frac{1}{12}t^4 - \frac{1}{6}t^3 - 10t$