4.7 Applications to Business and Economics

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S Click here for solutions.

1–6 For each cost function (given in dollars), find (a) the cost, average cost, and marginal cost at a production level of 1000 units; (b) the production level that will minimize the average cost; and (c) the minimum average cost.

1. $C(x) = 10{,}000 + 25x + x^2$
2. $C(x) = 1600 + 8x + 0.01x^2$
3. $C(x) = 45 + \dfrac{x}{2} + \dfrac{x^2}{560}$
4. $C(x) = 2000 + 10x + 0.001x^3$
5. $C(x) = 2\sqrt{x} + x^2/8000$
6. $C(x) = 1000 + 96x + 2x^{3/2}$

Answers

E Click here for exercises.

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1. (a) $1,035,000, $1035/unit, $2025/unit.
   (b) 100
   (c) $225/unit
2. (a) $19,600, $19.60/unit, $28/unit
   (b) 400
   (c) $16/unit
3. (a) $2330.71, $2.33/unit, $4.07/unit
   (b) 159
   (c) $1.07/unit
4. (a) $1,012,000, $1012/unit, $3010/unit
   (b) 100
   (c) $40/unit
5. (a) $188.25, $0.19/unit, $0.28/unit
   (b) 400
   (c) $0.15/unit
6. (a) $160,245.55, $160.25/unit, $190.87/unit
   (b) 100
   (c) $126/unit

Solutions

E Click here for exercises.

A Click here for answers.

1. (a) $C(x) = 10{,}000 + 25x + x^2$, $C(1000) = \$1{,}035{,}000$,
$c(x) = \dfrac{C(x)}{x} = \frac{10{,}000}{x} + 25 + x$, $c(1000) = \$1035$.
$C'(x) = 25 + 2x$, $C'(1000) = \$2025$/unit.

(b) We must have $c(x) = C'(x) \Rightarrow$
$10{,}000/x + 25 + x = 25 + 2x \Rightarrow 10{,}000/x = x$
$\Rightarrow x^2 = 10{,}000 \Rightarrow x = 100$. This is a minimum since $c''(x) = 20{,}000/x^3 > 0$.

(c) The minimum average cost is $c(100) = \$225$/unit.

2. (a) $C(x) = 1600 + 8x + 0.01x^2$, $C(1000) = \$19{,}600$.
$c(x) = \dfrac{1600}{x} + 8 + 0.01x$, $c(1000) = \$19.60$.
$C'(x) = 8 + 0.02x$, $C'(1000) = \$28$.

(b) We must have $C'(x) = c(x) \Leftrightarrow$
$8 + 0.02x = \dfrac{1600}{x} + 8 + 0.01x \Leftrightarrow 0.01x = \dfrac{1600}{x}$
$\Leftrightarrow x^2 = \dfrac{1600}{0.01} = 160{,}000 \Leftrightarrow x = 400$. This is a minimum since $c''(x) = 3200/x^3 > 0$ for $x > 0$.

(c) The minimum average cost is $c(400) = \$16$/unit.

3. (a) $C(x) = 45 + \dfrac{x}{2} + \dfrac{x^2}{560}$, $C(1000) = \$2330.71$.
$c(x) = \dfrac{45}{x} + \dfrac{1}{2} + \dfrac{x}{560}$, $c(1000) = \$2.33$/unit.
$C'(x) = \dfrac{1}{2} + \dfrac{x}{280}$, $C'(1000) = \$4.07$/unit.

(b) We must have $C'(x) = c(x) \Rightarrow$
$\dfrac{1}{2} + \dfrac{x}{280} = \dfrac{45}{x} + \dfrac{1}{2} + \dfrac{x}{560} \Rightarrow \dfrac{45}{x} = \dfrac{x}{560} \Rightarrow$
$x^2 = (45)(560) \Rightarrow x = \sqrt{25{,}200} \approx 159$. This is a minimum since $c''(x) = 90/x^2 > 0$.

(c) The minimum average cost is $c(159) = \$1.07$/unit.

4. (a) $C(x) = 2000 + 10x + 0.001x^3$,
$C(1000) = \$1{,}012{,}000$.
$c(x) = \dfrac{2000}{x} + 10 + 0.001x^2$, $c(1000) = \$1012$/unit.
$C'(x) = 10 + 0.003x^2$, $C'(1000) = \$3010$/unit.

(b) We must have $C'(x) = c(x) \Leftrightarrow$
$10 + 0.003x^2 = \dfrac{2000}{x} + 10 + 0.001x^2 \Leftrightarrow$
$\dfrac{2000}{x} = 0.002x^2 \Leftrightarrow x^3 = 2000/0.002 = 1{,}000{,}000$
$\Leftrightarrow x = 100$. This is a minimum since
$c''(x) = \dfrac{4000}{x^3} + 0.002 > 0$ for $x > 0$.

(c) The minimum average cost is $c(100) = \$40$/unit.

5. (a) $C(x) = 2\sqrt{x} + \dfrac{x^2}{8000}$, $C(1000) = \$188.25$.
$c(x) = \dfrac{2}{\sqrt{x}} + \dfrac{x}{8000}$, $c(1000) = \$0.19$/unit.
$C'(x) = \dfrac{1}{\sqrt{x}} + \dfrac{x}{4000}$, $C'(1000) = \$0.28$/unit.

(b) We must have $C'(x) = c(x) \Rightarrow$
$\dfrac{1}{\sqrt{x}} + \dfrac{x}{4000} = \dfrac{2}{\sqrt{x}} + \dfrac{x}{8000} \Rightarrow \dfrac{x}{8000} = \dfrac{1}{\sqrt{x}} \Rightarrow$
$x^{3/2} = 8000 \Rightarrow x = (8000)^{2/3} = 400$. This is a minimum since $c''(x) = \frac{3}{2}x^{-5/2} > 0$.

(c) The minimum average cost is $c(400) = \$0.15$/unit.

6. (a) $C(x) = 1000 + 96x + 2x^{3/2}$, $C(1000) = \$160{,}245.55$.
$c(x) = \dfrac{1000}{x} + 96 + 2\sqrt{x}$, $c(1000) = \$160.25$/unit.
$C'(x) = 96 + 3\sqrt{x}$, $C'(1000) = \$190.87$/unit.

(b) We must have $C'(x) = c(x) \Leftrightarrow$
$96 + 3\sqrt{x} = 1000/x + 96 + 2\sqrt{x} \Leftrightarrow \sqrt{x} = 1000/x$
$\Leftrightarrow x^{3/2} = 1000 \Leftrightarrow x = (1000)^{2/3} = 100$. Since $c'(x) = \left(x^{3/2} - 1000\right)/x^2 < 0$ for $0 < x < 100$ and $c'(x) > 0$ for $x > 100$, there is an absolute minimum at $x = 100$.

(c) The minimum average cost is $c(100) = \$126$/unit.