4.6 Optimization Problems

A Click here for answers.

S Click here for solutions.

1. Find the points on the hyperbola $y^2 - x^2 = 4$ that are closest to the point $(2, 0)$.

2. Find the point on the parabola $x + y^2 = 0$ that is closest to the point $(0, -3)$.

Answers

E Click here for exercises.

S Click here for solutions.

1. $(1, \pm\sqrt{5})$
2. $(-1, -1)$

Solutions

E Click here for exercises.

A Click here for answers.

1. By symmetry, the points are (x, y) and $(x, -y)$, where $y > 0$. The square of the distance is

$$\begin{aligned} D(x) &= (x-2)^2 + y^2 \\ &= (x-2)^2 + \left(4 + x^2\right) \\ &= 2x^2 - 4x + 8 \end{aligned}$$

So $D'(x) = 4x - 4 = 0 \quad \Rightarrow \quad x = 1$ and $y = \pm\sqrt{4+1} = \pm\sqrt{5}$. The points are $\left(1, \pm\sqrt{5}\right)$.

2. The square of the distance from a point (x, y) on the parabola $x = -y^2$ is $x^2 + (y+3)^2 = y^4 + y^2 + 6y + 9 = D(y)$. Now

$$\begin{aligned} D'(y) &= 4y^3 + 2y + 6 \\ &= 2(y+1)\left(2y^2 - 2y + 3\right) \end{aligned}$$

Since $2y^2 - 2y + 3 = 0$ has no real roots, $y = -1$ is the only critical number. Then $x = -(-1)^2 = -1$, so the point is $(-1, -1)$.