4.4 Graphing with Calculus and Calculators

A Click here for answers.

S Click here for solutions.

1–4 Produce graphs of f that reveal all the important aspects of the curve. In particular, you should use graphs of f' and f'' to estimate the intervals of increase and decrease, extreme values, intervals of concavity, and inflection points.

1. $f(x) = 4x^4 - 7x^2 + 4x + 6$

2. $f(x) = 8x^5 + 45x^4 + 80x^3 + 90x^2 + 200x$

3. $f(x) = x^2 \sin x, \quad -7 \leqslant x \leqslant 7$

4. $f(x) = \sin x + \frac{1}{3}\sin 3x$

5–6 Produce graphs of f that show all the important aspects of the curve. Estimate the local maximum and minimum values and then use calculus to find these values exactly. Use a graph of f'' to estimate the inflection points.

5. $f(x) = e^{x^3 - x}$

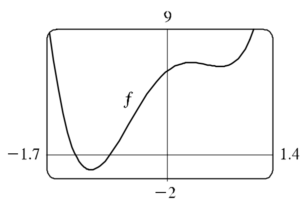
6. $f(x) = e^{\cos x}$

Answers

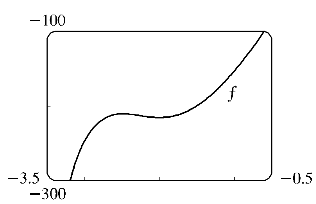
E Click here for exercises.

S Click here for solutions.

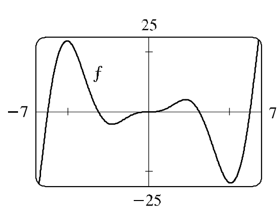
1. Inc. on $(-1.1, 0.3)$, $(0.7, \infty)$; dec. on $(-\infty, -1.1)$, $(0.3, 0.7)$; loc. max. $f(0.3) \approx 6.6$; loc. min. $f(-1.1) \approx -1.0$, $f(0.7) \approx 6.3$; CU on $(-\infty, -0.5)$, $(0.5, \infty)$; CD on $(-0.5, 0.5)$; IP $(-0.5, 2.1)$, $(0.5, 6.5)$



2. Inc. on $(-\infty, -2.5)$, $(-2.0, \infty)$; dec. on $(-2.5, -2.0)$; loc. max. $f(-2.5) \approx -211$, loc. min. $f(-2) \approx -216$; CU on $(-2.3, \infty)$, CD on $(-\infty, -2.3)$; IP $(-2.3, -213)$

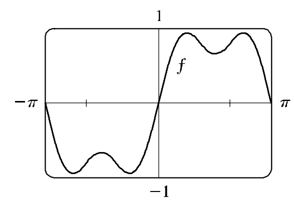


3. Inc. on $(-7, -5.1)$, $(-2.3, 2.3)$, $(5.1, 7)$; dec. on $(-5.1, -2.3)$, $(2.3, 5.1)$; loc. max. $f(-5.1) \approx 24.1$, $f(2.3) \approx 3.9$; loc. min. $f(-2.3) \approx -3.9$, $f(5.1) \approx -24.1$; CU on $(-7, -6.8)$, $(-4.0, -1.5)$, $(0, 1.5)$, $(4.0, 6.8)$; CD on $(-6.8, -4.0)$, $(-1.5, 0)$, $(1.5, 4.0)$, $(6.8, 7)$; IP $(-6.8, -24.4)$, $(-4.0, 12.0)$, $(-1.5, -2.3)$, $(0, 0)$, $(1.5, 2.3)$, $(4.0, -12.0)$, $(6.8, 24.4)$

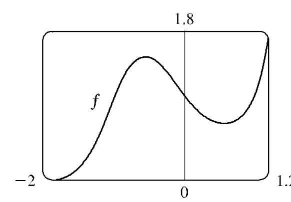


4. *Note:* Due to periodicity, we consider the function only on $[-\pi, \pi]$.

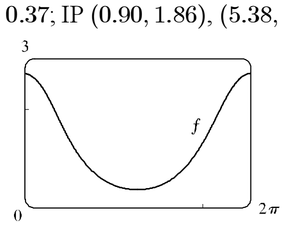
Inc. on $(-2.4, -1.6)$, $(-0.8, 0.8)$, $(1.6, 2.4)$; dec. on $(-\pi, -2.4)$, $(-1.6, -0.8)$, $(0.8, 1.6)$, $(2.4, \pi)$; loc. max. $f(-1.6) \approx -0.7$, $f(0.8) \approx 0.9$, $f(2.4) \approx 0.9$; loc. min. $f(-2.4) \approx -0.9$, $f(-0.8) \approx -0.9$, $f(1.6) \approx 0.7$; CU on $(-\pi, -2.0)$, $(-1.2, 0)$, $(1.2, 2)$; CD on $(-2.0, -1.2)$, $(0, 1.2)$, $(2.0, \pi)$; IP $(-\pi, 0)$, $(-2.0, -0.8)$, $(-1.2, -0.8)$, $(0, 0)$, $(1.2, 0.8)$, $(2.0, 0.8)$, $(\pi, 0)$



5. Loc. max. $f\left(-\frac{1}{\sqrt{3}}\right) = e^{2\sqrt{3}/9} \approx 1.5$, loc. min. $f\left(\frac{1}{\sqrt{3}}\right) = e^{-2\sqrt{3}/9} \approx 0.7$; IP $(-0.15, 1.15)$, $(-1.09, 0.82)$



6. Loc. max. $f(0) = f(2\pi) = e \approx 2.7$, loc. min. $f(\pi) = 1/e \approx 0.37$; IP $(0.90, 1.86)$, $(5.38, 1.86)$

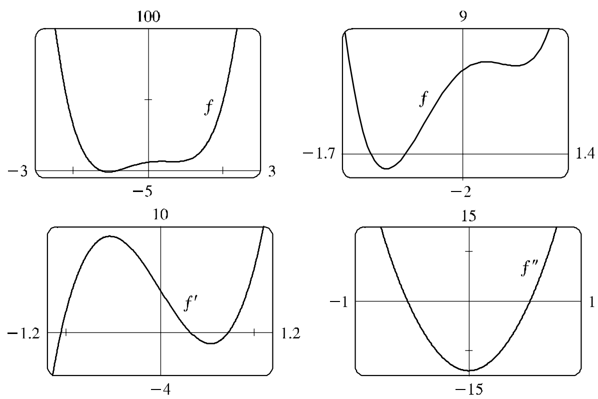


Solutions

E Click here for exercises.

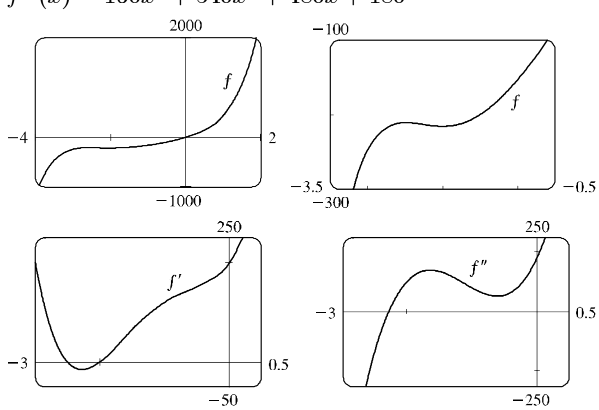
A Click here for answers.

1. $f(x) = 4x^4 - 7x^2 + 4x + 6 \quad \Rightarrow$
$f'(x) = 16x^3 - 14x + 4 \quad \Rightarrow \quad f''(x) = 48x^2 - 14$



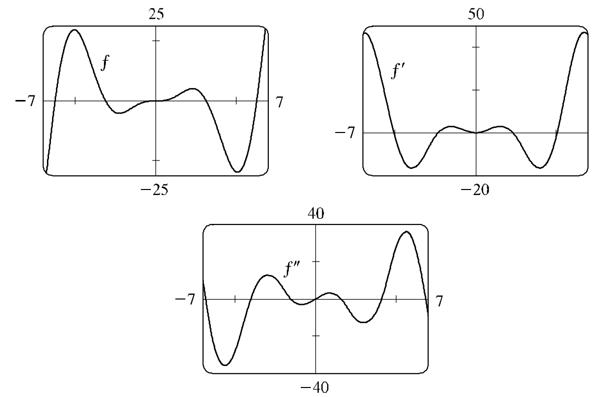
After finding suitable viewing rectangles (by ensuring that we have located all of the x-values where either $f' = 0$ or $f'' = 0$) we estimate from the graph of f' that f is increasing on $(-1.1, 0.3)$ and $(0.7, \infty)$ and decreasing on $(-\infty, -1.1)$ and $(0.3, 0.7)$, with a local maximum of $f(0.3) \approx 6.6$ and minima of $f(-1.1) \approx -1.0$ and $f(0.7) \approx 6.3$. We estimate from the graph of f'' that f is CU on $(-\infty, -0.5)$ and $(0.5, \infty)$ and CD on $(-0.5, 0.5)$, and that f has inflection points at about $(-0.5, 2.1)$ and $(0.5, 6.5)$.

2. $f(x) = 8x^5 + 45x^4 + 80x^3 + 90x^2 + 200x \quad \Rightarrow$
$f'(x) = 40x^4 + 180x^3 + 240x^2 + 180x + 200 \quad \Rightarrow$
$f''(x) = 160x^3 + 540x^2 + 480x + 180$



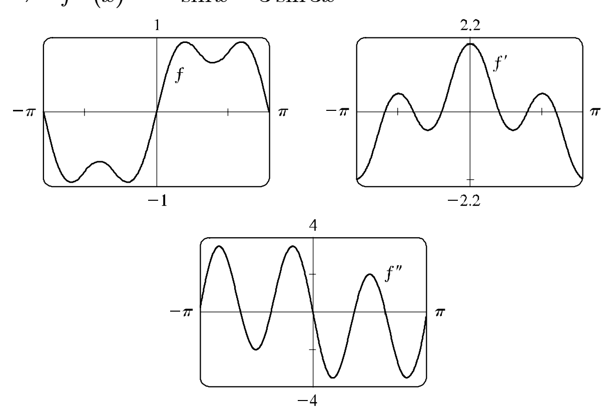
After finding suitable viewing rectangles, we estimate from the graph of f' that f is increasing on $(-\infty, -2.5)$ and $(-2.0, \infty)$ and decreasing on $(-2.5, -2.0)$. Maximum: $f(-2.5) \approx -211$. Minimum: $f(-2) \approx -216$. We estimate from the graph of f'' that f is CU on $(-2.3, \infty)$ and CD on $(-\infty, -2.3)$, and has an IP at $(-2.3, -213)$.

3. $f(x) = x^2 \sin x \quad \Rightarrow \quad f'(x) = 2x \sin x + x^2 \cos x \quad \Rightarrow$
$f''(x) = 2 \sin x + 4x \cos x - x^2 \sin x$



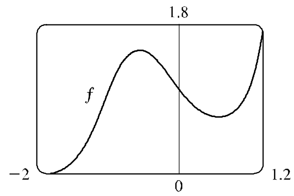
We estimate from the graph of f' that f is increasing on $(-7, -5.1)$, $(-2.3, 2.3)$, and $(5.1, 7)$ and decreasing on $(-5.1, -2.3)$, and $(2.3, 5.1)$. Local maxima: $f(-5.1) \approx 24.1$, $f(2.3) \approx 3.9$. Local minima: $f(-2.3) \approx -3.9$, $f(5.1) \approx -24.1$. From the graph of f'', we estimate that f is CU on $(-7, -6.8)$, $(-4.0, -1.5)$, $(0, 1.5)$, and $(4.0, 6.8)$, and CD on $(-6.8, -4.0)$, $(-1.5, 0)$, $(1.5, 4.0)$, and $(6.8, 7)$. f has IP at $(-6.8, -24.4)$, $(-4.0, 12.0)$, $(-1.5, -2.3)$, $(0, 0)$, $(1.5, 2.3)$, $(4.0, -12.0)$ and $(6.8, 24.4)$.

4. $f(x) = \sin x + \frac{1}{3} \sin 3x \quad \Rightarrow \quad f'(x) = \cos x + \cos 3x$
$\Rightarrow \quad f''(x) = -\sin x - 3 \sin 3x$



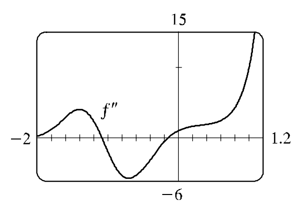
Note that f is periodic with period 2π, so we consider it on the interval $[-\pi, \pi]$. From the graph of f', we estimate that f is increasing on $(-2.4, -1.6)$, $(-0.8, 0.8)$, and $(1.6, 2.4)$ and decreasing on $(-\pi, -2.4)$, $(-1.6, -0.8)$, $(0.8, 1.6)$ and $(2.4, \pi)$. Maxima: $f(-1.6) \approx -0.7$, $f(0.8) \approx 0.9$, $f(2.4) \approx 0.9$. Minima: $f(-2.4) \approx -0.9$, $f(-0.8) \approx -0.9$, $f(1.6) \approx 0.7$. We estimate from the graph of f'' that f is CD on $(-2.0, -1.2)$, $(0, 1.2)$ and $(2.0, \pi)$ and CU on $(-\pi, -2.0)$, $(-1.2, 0)$ and $(1.2, 2)$. f has IP at $(-\pi, 0)$, $(-2.0, -0.8)$, $(-1.2, -0.8)$, $(0, 0)$, $(1.2, 0.8)$, $(2.0, 0.8)$, and $(\pi, 0)$.

5.



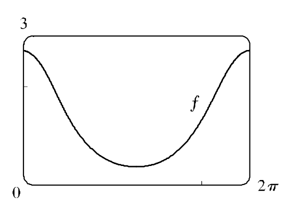
$f(x) = e^{x^3-x} \to 0$ as $x \to -\infty$, and $f(x) \to \infty$ as $x \to \infty$. From the graph, it appears that f has a local minimum of about $f(0.58) = 0.68$, and a local maximum of about $f(-0.58) = 1.47$. To find the exact values, we calculate $f'(x) = \left(3x^2 - 1\right) e^{x^3-x}$, which is 0 when $3x^2 - 1 = 0 \quad \Leftrightarrow \quad x = \pm\frac{1}{\sqrt{3}}$. The negative root corresponds to the local maximum $f\left(-\frac{1}{\sqrt{3}}\right) = e^{(-1/\sqrt{3})^3-(-1/\sqrt{3})} = e^{2\sqrt{3}/9}$, and the positive root corresponds to the local minimum $f\left(\frac{1}{\sqrt{3}}\right) = e^{(1/\sqrt{3})^3-(1/\sqrt{3})} = e^{-2\sqrt{3}/9}$. To estimate the inflection points, we calculate and graph

$$\begin{aligned} f''(x) &= \frac{d}{dx}\left[\left(3x^2-1\right)e^{x^3-x}\right] \\ &= \left(3x^2-1\right)e^{x^3-x}\left(3x^2-1\right) + e^{x^3-x}(6x) \\ &= e^{x^3-x}\left(9x^4 - 6x^2 + 6x + 1\right) \end{aligned}$$



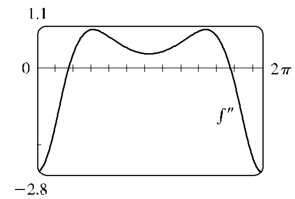
From the graph, it appears that $f''(x)$ changes sign (and thus f has inflection points) at $x \approx -0.15$ and $x \approx -1.09$. From the graph of f, we see that these x-values correspond to inflection points at about $(-0.15, 1.15)$ and $(-1.09, 0.82)$.

6.



The function $f(x) = e^{\cos x}$ is periodic with period 2π, so we consider it only on the interval $[0, 2\pi]$. We see that it has local maxima of about $f(0) \approx 2.72$ and $f(2\pi) \approx 2.72$, and a local minimum of about $f(3.14) \approx 0.37$. To find the exact values, we calculate $f'(x) = -\sin x\, e^{\cos x}$. This is 0 when $-\sin x = 0 \quad \Leftrightarrow \quad x = 0, \pi$ or 2π (since we are only considering $x \in [0, 2\pi]$). Also $f'(x) > 0 \quad \Leftrightarrow \quad \sin x < 0 \quad \Leftrightarrow \quad \pi < x < 2\pi$. So $f(0) = f(2\pi) = e$ (both maxima) and $f(\pi) = e^{\cos \pi} = 1/e$ (minimum). To find the inflection points, we calculate and graph

$$\begin{aligned} f''(x) &= \frac{d}{dx}\left(-\sin x\, e^{\cos x}\right) \\ &= -\cos x\, e^{\cos x} - \sin x\left(e^{\cos x}\right)(-\sin x) \\ &= e^{\cos x}\left(\sin^2 x - \cos x\right) \end{aligned}$$



From the graph of $f''(x)$, we see that f has inflection points at $x \approx 0.90$ and at $x \approx 5.38$. These x-coordinates correspond to inflection points $(0.90, 1.86)$ and $(5.38, 1.86)$.