4.3 How Derivatives Affect the Shape of a Graph

A Click here for answers.

S Click here for solutions.

1–16
(a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.

1. $f(x) = 20 - x - x^2$

2. $f(x) = x^3 - x + 1$

3. $f(x) = x^3 + x + 1$

4. $f(x) = x^3 - 2x^2 + x$

5. $f(x) = 2x^2 - x^4$

6. $f(x) = x^2(1 - x)^2$

7. $f(x) = x^3(x - 4)^4$

8. $f(x) = 3x^5 - 25x^3 + 60x$

9. $f(x) = x\sqrt{6 - x}$

10. $f(x) = x\sqrt{1 - x^2}$

11. $f(x) = x^{1/5}(x + 1)$

12. $f(x) = x^{2/3}(x - 2)^2$

13. $f(x) = x\sqrt{x - x^2}$

14. $f(x) = \sqrt[3]{x} - \sqrt[3]{x^2}$

15. $f(x) = \sin^4 x + \cos^4 x, \quad 0 \leqslant x \leqslant 2\pi$

16. $f(x) = x \sin x + \cos x, \quad -\pi \leqslant x \leqslant \pi$

17–19 Find the intervals on which the function is increasing or decreasing.

17. $f(x) = x^3 + 2x^2 - x + 1$

18. $f(x) = x^5 + 4x^3 - 6$

19. $f(x) = 2 \tan x - \tan^2 x$

20–23 Find the intervals on which the curve is concave upward.

20. $y = 6x^2 - 2x^3 - x^4$

21. $y = \dfrac{x^2}{\sqrt{1 + x}}$

22. $y = \dfrac{x}{(1 + x)^2}$

23. $y = \dfrac{x^3}{x^2 - 3}$

24–26
(a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.
(c) Find the intervals of concavity and the inflection points.

24. $f(x) = x^6 + 192x + 17$

25. $f(x) = \dfrac{x}{(1 + x)^2}$

26. $f(x) = 2 \sin x + \sin^2 x, \quad 0 \leqslant x \leqslant 2\pi$

27–30 Sketch the graph of a function that satisfies all of the given conditions.

27. $f'(-1) = f'(1) = 0, \quad f'(x) < 0$ if $|x| < 1$,
$f'(x) > 0$ if $|x| > 1, \quad f(-1) = 4, \quad f(1) = 0$,
$f''(x) < 0$ if $x < 0, \quad f''(x) > 0$ if $x > 0$

28. $f'(-1) = 0, \quad f'(1)$ does not exist,
$f'(x) < 0$ if $|x| < 1, \quad f'(x) > 0$ if $|x| > 1$,
$f(-1) = 4, \quad f(1) = 0, \quad f''(x) < 0$ if $x \neq 1$

29. $f'(2) = 0, \quad f(2) = -1, \quad f(0) = 0$,
$f'(x) < 0$ if $0 < x < 2, \quad f'(x) > 0$ if $x > 2$,
$f''(x) < 0$ if $0 \leqslant x < 1$ or if $x > 4$,
$f''(x) > 0$ if $1 < x < 4, \quad \lim_{x\to\infty} f(x) = 1$,
$f(-x) = f(x)$ for all x

30. $\lim_{x\to 3} f(x) = -\infty, \quad f''(x) < 0$ if $x \neq 3, \quad f'(0) = 0$,
$f'(x) > 0$ if $x < 0$ or $x > 3, \quad f'(x) < 0$ if $0 < x < 3$

31–35
(a) Find the intervals of increase or decrease.
(b) Find the local maximum and minimum values.
(c) Find the intervals of concavity and the inflection points.
(d) Use the information from parts (a), (b), and (c) to sketch the graph. Check your work with a graphing device if you have one.

31. $P(x) = x\sqrt{x^2 + 1}$

32. $Q(x) = x - 3x^{1/3}$

33. $Q(x) = x^{1/3}(x + 3)^{2/3}$

34. $f(x) = \ln(1 + x^2)$

35. $f(\theta) = \sin^2\theta, \quad 0 \leqslant \theta \leqslant 2\pi$

36–37
(a) Use a graph of f to give a rough estimate of the intervals of concavity and the coordinates of the points of inflection.
(b) Use a graph of f'' to give better estimates.

36. $f(x) = 3x^5 - 40x^3 + 30x^2$

37. $f(x) = 2 \cos x + \sin 2x, \quad 0 \leqslant x \leqslant 2\pi$

38. Suppose f is continuous on $[2, 5]$ and $1 \leqslant f'(x) \leqslant 4$ for all x in $(2, 5)$. Show that $3 \leqslant f(5) - f(2) \leqslant 12$.

Answers

E Click here for exercises.

S Click here for solutions.

1. (a) Inc. on $\left(-\infty,-\frac{1}{2}\right)$, dec. on $\left(-\frac{1}{2},\infty\right)$

(b) Loc. max. $f\left(-\frac{1}{2}\right)=20.25$

2. (a) Inc. on $\left(-\infty,-\frac{1}{\sqrt{3}}\right)$, $\left(\frac{1}{\sqrt{3}},\infty\right)$, dec. on $\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

(b) Loc. max. $f\left(-\frac{1}{\sqrt{3}}\right)=1+\frac{2}{3\sqrt{3}}$, loc. min. $f\left(\frac{1}{\sqrt{3}}\right)=1-\frac{2}{3\sqrt{3}}$

3. (a) Inc. on $\mathbb{R}$

(b) None

4. (a) Inc. on $\left(-\infty,\frac{1}{3}\right)$, $(1,\infty)$, dec. on $\left(\frac{1}{3},1\right)$

(b) Loc. max. $f\left(\frac{1}{3}\right)=\frac{4}{27}$, loc. min. $f(1)=0$

5. (a) Inc. on $(-\infty,-1)$, $(0,1)$; dec. on $(-1,0)$, $(1,\infty)$

(b) Loc. max. $f(-1)=1$, $f(1)=1$; loc. min. $f(0)=0$

6. (a) Inc. on $\left(0,\frac{1}{2}\right)$, $(1,\infty)$; dec. on $(-\infty,0)$, $\left(\frac{1}{2},1\right)$

(b) Loc. max. $f\left(\frac{1}{2}\right)=\frac{1}{16}$; loc. min. $f(0)=0$, $f(1)=0$

7. (a) Inc. on $\left(-\infty,\frac{12}{7}\right)$, $(4,\infty)$; dec. on $\left(\frac{12}{7},4\right)$

(b) Loc. max. $f\left(\frac{12}{7}\right)=12^3\cdot\frac{16^4}{7^7}\approx 137.5$, loc. min. $f(4)=0$

8. (a) Inc. on $(-\infty,-2)$, $(-1,1)$, $(2,\infty)$; dec. on $(-2,1)$, $(1,2)$

(b) Loc. max. $f(-2)=-16$, $f(1)=38$; loc. min. $f(-1)=-38$, $f(2)=16$

9. (a) Inc. on $(-\infty,4)$, dec. on $(4,6)$

(b) Loc. max. $f(4)=4\sqrt{2}$

10. (a) Inc. on $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$; dec. on $\left(-1,-\frac{1}{\sqrt{2}}\right)$, $\left(\frac{1}{\sqrt{2}},1\right)$

(b) Loc. max. $f\left(\frac{1}{\sqrt{2}}\right)=\frac{1}{2}$, loc. min. $f\left(-\frac{1}{\sqrt{2}}\right)=-\frac{1}{2}$

11. (a) Inc. on $\left(-\frac{1}{6},\infty\right)$, dec. on $\left(-\infty,-\frac{1}{6}\right)$

(b) Loc. min. $f\left(-\frac{1}{6}\right)=-\frac{5}{6^{6/5}}\approx -0.58$

12. (a) Inc. on $\left(0,\frac{1}{2}\right)$, $(2,\infty)$; dec. on $(-\infty,0)$, $\left(\frac{1}{2},2\right)$

(b) Loc. max. $f\left(\frac{1}{2}\right)=\left(\frac{9}{4}\right)^{4/3}\approx 1.42$; loc. min. $f(0)=0$, $f(2)=0$

13. (a) Inc. on $\left(0,\frac{3}{4}\right)$, dec. on $\left(\frac{3}{4},1\right)$

(b) Loc. max. $f\left(\frac{3}{4}\right)=\frac{3\sqrt{3}}{16}$

14. (a) Inc. on $\left(-\infty,\frac{1}{8}\right)$, dec. on $\left(\frac{1}{8},\infty\right)$

(b) Loc. max. $f\left(\frac{1}{8}\right)=\frac{1}{4}$

15. (a) Inc. on $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$, $\left(\frac{3\pi}{4},\pi\right)$, $\left(\frac{5\pi}{4},\frac{3\pi}{2}\right)$, $\left(\frac{7\pi}{4},2\pi\right)$; dec. on $\left(0,\frac{\pi}{4}\right)$, $\left(\frac{\pi}{2},\frac{3\pi}{4}\right)$, $\left(\pi,\frac{5\pi}{4}\right)$, $\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

(b) Loc. max. $f\left(\frac{\pi}{2}\right)=f(\pi)=f\left(\frac{3\pi}{2}\right)=1$, loc. min. $f\left(\frac{\pi}{4}\right)=f\left(\frac{3\pi}{4}\right)=f\left(\frac{5\pi}{4}\right)=f\left(\frac{7\pi}{4}\right)=\frac{1}{2}$

16. (a) Inc. on $\left(-\pi,-\frac{\pi}{2}\right)$, $\left(0,\frac{\pi}{2}\right)$; dec. on $\left(-\frac{\pi}{2},0\right)$, $\left(\frac{\pi}{2},\pi\right)$

(b) Loc. max. $f\left(-\frac{\pi}{2}\right)=f\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$, loc. min. $f(0)=1$

17. Inc. on $\left(-\infty,\frac{-2-\sqrt{7}}{3}\right)$, $\left(\frac{-2+\sqrt{7}}{3},\infty\right)$; dec. on $\left(\frac{-2-\sqrt{7}}{3},\frac{-2+\sqrt{7}}{3}\right)$

18. Inc. on $\mathbb{R}$

19. Inc. on $\left(n\pi-\frac{\pi}{2},n\pi+\frac{\pi}{4}\right)$, dec. on $\left(n\pi+\frac{\pi}{4},n\pi+\frac{\pi}{2}\right)$, n an integer

20. $\left(\frac{-1-\sqrt{5}}{2},\frac{-1+\sqrt{5}}{2}\right)$

21. $(-1,\infty)$

22. $(2,\infty)$

23. $(-\sqrt{3},0)$, $(\sqrt{3},\infty)$

24. (a) Inc. on $(-2,\infty)$, dec. on $(-\infty,-2)$

(b) Loc. min. $f(-2)=-303$

(c) CU on $\mathbb{R}$

25. (a) Inc. on $(-1,1)$; dec. on $(-\infty,-1)$, $(1,\infty)$

(b) Loc. max. $f(1)=\frac{1}{4}$

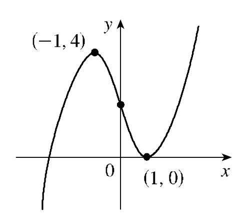
(c) CU on $(2,\infty)$; CD on $(-\infty,-1)$, $(-1,2)$; IP $\left(2,\frac{2}{9}\right)$

26. (a) Inc. on $\left(0,\frac{\pi}{2}\right)$, $\left(\frac{3\pi}{2},2\pi\right)$; dec. on $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$

(b) Loc. max. $f\left(\frac{\pi}{2}\right)=3$; loc. min. $f\left(\frac{3\pi}{2}\right)=-1$

(c) CU on $\left(0,\frac{\pi}{6}\right)$, $\left(\frac{5\pi}{6},2\pi\right)$; CD on $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$; IP $\left(\frac{\pi}{6},\frac{5}{4}\right)$, $\left(\frac{5\pi}{6},\frac{5}{4}\right)$

27.



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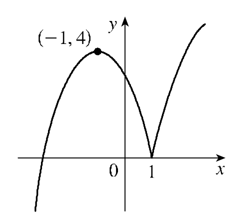

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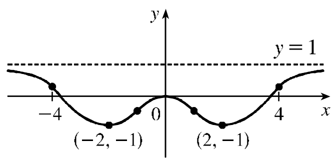

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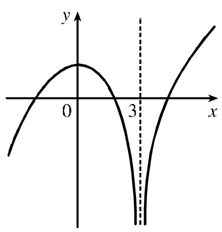
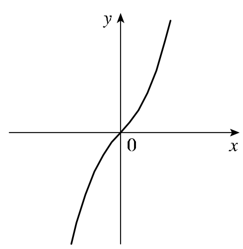


31. (a) Inc. on $\mathbb{R}$

(b) None

(c) CU on $(0, \infty)$, CD on $(-\infty, 0)$, IP $(0, 0)$

(d)

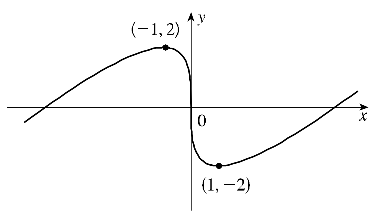


32. (a) Inc. on $(-\infty, -1)$, $(1, \infty)$; dec. on $(-1, 1)$

(b) Loc. max. $Q(-1) = 2$, loc. min. $Q(1) = -2$

(c) CU on $(0, \infty)$, CD on $(-\infty, 0)$, IP $(0, 0)$

(d)

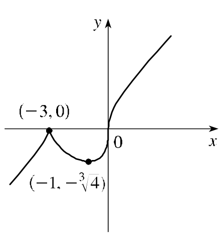


33. (a) Inc. on $(-\infty, -3)$, $(-1, \infty)$; dec. on $(-3, -1)$

(b) Loc. max. $Q(-3) = 0$, loc. min. $Q(-1) = -4^{1/3} \approx -1.6$

(c) CU on $(-\infty, -3)$, $(-3, 0)$; CD on $(0, \infty)$; IP $(0, 0)$

(d)

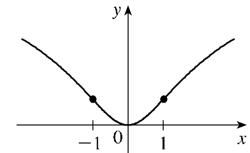


34. (a) Inc. on $(0, \infty)$, dec. on $(-\infty, 0)$

(b) Loc. min. $f(0) = 0$

(c) CU on $(-1, 1)$; CD on $(-\infty, -1)$, $(1, \infty)$; IP $(\pm 1, \ln 2)$

(d)

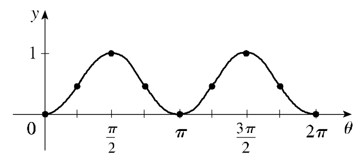


35. (a) Inc. on $\left(0, \frac{\pi}{2}\right)$, $\left(\pi, \frac{3\pi}{2}\right)$; dec. on $\left(\frac{\pi}{2}, \pi\right)$, $\left(\frac{3\pi}{2}, 2\pi\right)$

(b) Loc. max. $f\left(\frac{\pi}{2}\right) = f\left(\frac{3\pi}{2}\right) = 1$, loc. min. $f(\pi) = 0$

(c) CU on $\left(0, \frac{\pi}{4}\right)$, $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, $\left(\frac{7\pi}{4}, 2\pi\right)$; CD on $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$, $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$; IP $\left(\frac{\pi}{4}, \frac{1}{2}\right)$, $\left(\frac{3\pi}{4}, \frac{1}{2}\right)$, $\left(\frac{5\pi}{4}, \frac{1}{2}\right)$, $\left(\frac{7\pi}{4}, \frac{1}{2}\right)$

(d)



36. (b) CU on $(-2.1, 0.25)$, $(1.9, \infty)$; CD on $(-\infty, -2.1)$, $(0.25, 1.9)$, IP $(-2.1, 386)$, $(0.25, 1.3)$, $(1.9, -87)$

37. (b) CU on $(1.57, 3.39)$, $(4.71, 6.03)$; CD on $(0, 1.57)$, $(3.39, 4.71)$, $(6.03, 2\pi)$; IP $(1.57, 0)$, $(3.39, -1.45)$, $(4.71, 0)$, $(6.03, 1.45)$

Solutions

E Click here for exercises.

A Click here for answers.

1. $f(x) = 20 - x - x^2$, $f'(x) = -1 - 2x = 0 \quad \Rightarrow$ $x = -\frac{1}{2}$ (the only critical number)

(a) $f'(x) > 0 \quad \Leftrightarrow \quad -1 - 2x > 0 \quad \Leftrightarrow \quad x < -\frac{1}{2}$, $f'(x) < 0 \quad \Leftrightarrow \quad x > -\frac{1}{2}$, so f is increasing on $\left(-\infty, -\frac{1}{2}\right)$ and decreasing on $\left(-\frac{1}{2}, \infty\right)$.

(b) By the First Derivative Test, $f\left(-\frac{1}{2}\right) = 20.25$ is a local maximum.

2. $f(x) = x^3 - x + 1$. $f'(x) = 3x^2 - 1 = 0 \quad \Rightarrow \quad x = \pm\frac{1}{\sqrt{3}}$ (the only critical numbers)

(a) $f'(x) > 0 \quad \Leftrightarrow \quad 3x^2 > 1 \quad \Leftrightarrow \quad |x| > \frac{1}{3} \quad \Leftrightarrow$ $x < -\frac{1}{\sqrt{3}}$ or $x > \frac{1}{\sqrt{3}}$ and $f'(x) < 0 \quad \Leftrightarrow$ $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$. So f is increasing on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$ and decreasing on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

(b) By the First Derivative Test, $f\left(-\frac{1}{\sqrt{3}}\right) = 1 + \frac{2}{3\sqrt{3}}$ is a local maximum and $f\left(\frac{1}{\sqrt{3}}\right) = 1 - \frac{2}{3\sqrt{3}}$ is a local minimum.

3. $f(x) = x^3 + x + 1 \quad \Rightarrow \quad f'(x) = 3x^2 + 1 > 0$ for all $x \in \mathbb{R}$.

(a) f is increasing on $\mathbb{R}$.

(b) f has no local maximum or minimum.

4. $f(x) = x^3 - 2x^2 + x$.
$f'(x) = 3x^2 - 4x + 1 = (3x - 1)(x - 1)$. So the critical numbers are $x = \frac{1}{3}, 1$.

(a) $f'(x) > 0 \quad \Leftrightarrow \quad (3x - 1)(x - 1) > 0 \quad \Leftrightarrow \quad x < \frac{1}{3}$ or $x > 1$ and $f'(x) < 0 \quad \Leftrightarrow \quad \frac{1}{3} < x < 1$. So f is increasing on $\left(-\infty, \frac{1}{3}\right)$ and $(1, \infty)$ and f is decreasing on $\left(\frac{1}{3}, 1\right)$.

(b) The local maximum is $f\left(\frac{1}{3}\right) = \frac{4}{27}$ and the local minimum is $f(1) = 0$.

5. $f(x) = 2x^2 - x^4$.
$f'(x) = 4x - 4x^3 = 4x\left(1 - x^2\right) = 4x(1 + x)(1 - x)$, so the critical numbers are $x = 0, \pm 1$.

(a)

Interval	$4x$	$1+x$	$1-x$	$f'(x)$
$x < -1$	$-$	$-$	$+$	$+$
$-1 < x < 0$	$-$	$+$	$+$	$-$
$0 < x < 1$	$+$	$+$	$+$	$+$
$x > 1$	$+$	$+$	$-$	$-$

So f is increasing on $(-\infty, -1)$, decreasing on $(-1, 0)$, increasing on $(0, 1)$, and decreasing on $(1, \infty)$.

(b) Local maximum $f(-1) = 1$, local minimum $f(0) = 0$, local maximum $f(1) = 1$.

6. $f(x) = x^2(1 - x)^2$.

$$\begin{aligned} 0 &= f'(x) = 2x(1-x)^2 + x^2\left[2(1-x)(-1)\right] \\ &= 2x(1-x)(1-2x) \end{aligned}$$

So the critical numbers are $x = 0, \frac{1}{2}, 1$.

(a)

Interval	$2x$	$1-x$	$1-2x$	$f'(x)$
$x < 0$	$-$	$+$	$+$	$-$
$0 < x < \frac{1}{2}$	$+$	$+$	$+$	$+$
$\frac{1}{2} < x < 1$	$+$	$+$	$-$	$-$
$x > 1$	$+$	$-$	$-$	$+$

So f is decreasing on $(-\infty, 0)$, increasing on $\left(0, \frac{1}{2}\right)$, decreasing on $\left(\frac{1}{2}, 1\right)$, and increasing on $(1, \infty)$.

(b) Local minimum $f(0) = 0$, local maximum $f\left(\frac{1}{2}\right) = \frac{1}{16}$, local minimum $f(1) = 0$.

7. $f(x) = x^3(x - 4)^4$.

$$\begin{aligned} f'(x) &= 3x^2(x-4)^4 + x^3\left[4(x-4)^3\right] \\ &= x^2(x-4)^3(7x-12) \end{aligned}$$

The critical numbers are $x = 0, 4, \frac{12}{7}$.

(a) $x^2(x-4)^2 \geq 0$ so $f'(x) \geq 0 \quad \Leftrightarrow$ $(x - 4)(7x - 12) \geq 0 \quad \Leftrightarrow \quad x \leq \frac{12}{7}$ or $x \geq 4$. $f'(x) \leq 0 \quad \Leftrightarrow \quad \frac{12}{7} \leq x \leq 4$. So f is increasing on $\left(-\infty, \frac{12}{7}\right)$ and $(4, \infty)$ and decreasing on $\left(\frac{12}{7}, 4\right)$.

(b) Local maximum $f\left(\frac{12}{7}\right) = 12^3 \cdot \frac{16^4}{7^7} \approx 137.5$, local minimum $f(4) = 0$.

8. $f(x) = 3x^5 - 25x^3 + 60x$.

$$\begin{aligned} f'(x) &= 15x^4 - 75x^2 + 60 \\ &= 15\left(x^4 - 5x^2 + 4\right) = 15\left(x^2 - 4\right)\left(x^2 - 1\right) \\ &= 15(x-2)(x+2)(x+1)(x-1) \end{aligned}$$

So the critical numbers are $x = \pm 2, \pm 1$.

(a)

Interval	$x+2$	$x-2$	$x+1$	$x-1$	$f'(x)$
$x < -2$	$-$	$-$	$-$	$-$	$+$
$-2 < x < -1$	$+$	$-$	$-$	$-$	$-$
$-1 < x < 1$	$+$	$-$	$+$	$-$	$+$
$1 < x < 2$	$+$	$-$	$+$	$+$	$-$
$x > 2$	$+$	$+$	$+$	$+$	$+$

So f is increasing on $(-\infty, -2)$, decreasing on $(-2, -1)$, increasing on $(-1, 2)$, decreasing on $(1, 2)$, and increasing on $(2, \infty)$.

(b) Local maximum $f(-2) = -16$, local minimum $f(-1) = -38$, local maximum $f(1) = 38$, local minimum $f(2) = 16$.

9. $f(x) = x\sqrt{6-x}$.

$f'(x) = \sqrt{6-x} + x\left(-\dfrac{1}{2\sqrt{6-x}}\right) = \dfrac{3(4-x)}{2\sqrt{6-x}}$. Critical numbers are $x = 4, 6$.

(a) $f'(x) > 0 \Leftrightarrow 4 - x > 0$ (and $x < 6$) $\Leftrightarrow x < 4$ and $f'(x) < 0 \Leftrightarrow 4 - x < 0$ (and $x < 6$) $\Leftrightarrow 4 < x < 6$. So f is increasing on $(-\infty, 4)$ and decreasing on $(4, 6)$.

(b) Local maximum $f(4) = 4\sqrt{2}$

10. $f(x) = x\sqrt{1-x^2}$.

$f'(x) = \sqrt{1-x^2} - \dfrac{x^2}{\sqrt{1-x^2}} = \dfrac{1-2x^2}{\sqrt{1-x^2}}$. Critical numbers are $\pm\frac{1}{\sqrt{2}}$ and ± 1.

(a) $f'(x) > 0 \Leftrightarrow 1 - 2x^2 > 0 \Leftrightarrow x^2 < \frac{1}{2} \Leftrightarrow |x| < \frac{1}{\sqrt{2}} \Leftrightarrow -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$. $f'(x) < 0 \Leftrightarrow -1 < x < -\frac{1}{\sqrt{2}}$ or $\frac{1}{\sqrt{2}} < x < 1$. So f is increasing on $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and decreasing on $\left(-1, -\frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{\sqrt{2}}, 1\right)$.

(b) Local minimum $f\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}$, local maximum $f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$.

11. $f(x) = x^{1/5}(x+1)$.

$f'(x) = \frac{1}{5}x^{-4/5}(x+1) + x^{1/5} = \frac{1}{5}x^{-4/5}(6x+1)$. The critical numbers are $x = 0, -\frac{1}{6}$.

(a) $f'(x) > 0 \Leftrightarrow 6x + 1 > 0\ (x \neq 0) \Leftrightarrow x > -\frac{1}{6}$ $(x \neq 0)$ and $f'(x) < 0 \Leftrightarrow x < -\frac{1}{6}$. So f is increasing on $\left(-\frac{1}{6}, \infty\right)$ and decreasing on $\left(-\infty, -\frac{1}{6}\right)$.

(b) Local minimum $f\left(-\frac{1}{6}\right) = -\frac{5}{6^{6/5}} \approx -0.58$

12. $f(x) = x^{2/3}(x-2)^2$. Domain is $\mathbb{R}$.

$$\begin{aligned} f'(x) &= \tfrac{2}{3}x^{-1/3}(x-2)^2 + x^{2/3}[2(x-2)] \\ &= \tfrac{2}{3}x^{-1/3}(x-2)(4x-2) \end{aligned}$$

Critical numbers are $x = 0, \frac{1}{2}, 2$.

(a)

Interval	$x^{-1/3}$	$x-2$	$2x-1$	$f'(x)$
$x < 0$	$-$	$-$	$-$	$-$
$0 < x < \frac{1}{2}$	$+$	$-$	$-$	$+$
$\frac{1}{2} < x < 2$	$+$	$-$	$+$	$-$
$x > 2$	$+$	$+$	$+$	$+$

So f is decreasing on $(-\infty, 0)$, increasing on $\left(0, \frac{1}{2}\right)$, decreasing on $\left(\frac{1}{2}, 2\right)$, and increasing on $(2, \infty)$.

(b) Local minimum $f(0) = 0$, local maximum $f\left(\frac{1}{2}\right) = \left(\frac{9}{4}\right)^{4/3} \approx 1.42$, local minimum $f(2) = 0$.

13. $f(x) = x\sqrt{x - x^2}$. The domain of f is $\{x \mid x(1-x) \geq 0\} = [0, 1]$.

$f'(x) = \sqrt{x - x^2} + x\dfrac{1-2x}{2\sqrt{x-x^2}} = \dfrac{x(3-4x)}{2\sqrt{x-x^2}}$. So the critical numbers are $x = 0, \frac{3}{4}, 1$.

(a) $f'(x) > 0 \Leftrightarrow 3 - 4x > 0 \Leftrightarrow 0 < x < \frac{3}{4}$. $f'(x) < 0 \Leftrightarrow \frac{3}{4} < x < 1$. So f is increasing on $\left(0, \frac{3}{4}\right)$ and decreasing on $\left(\frac{3}{4}, 1\right)$

(b) Local maximum $f\left(\frac{3}{4}\right) = \frac{3\sqrt{3}}{16}$

14. $f(x) = \sqrt[3]{x} - \sqrt[3]{x^2} = x^{1/3} - x^{2/3}$.

$f'(x) = \frac{1}{3}x^{-2/3} - \frac{2}{3}x^{-1/3} = \frac{1}{3}x^{-2/3}\left(1 - 2x^{1/3}\right)$. So the critical numbers are $x = 0, \frac{1}{8}$.

(a) $f'(x) > 0 \Leftrightarrow 1 - 2x^{1/3} > 0 \Leftrightarrow \frac{1}{2} > x^{1/3} \Leftrightarrow x < \frac{1}{8}\ (x \neq 0)$. $f'(x) < 0 \Leftrightarrow x > \frac{1}{8}$. So f is increasing on $\left(-\infty, \frac{1}{8}\right)$ and decreasing on $\left(\frac{1}{8}, \infty\right)$.

(b) Local maximum $f\left(\frac{1}{8}\right) = \frac{1}{4}$

15. $f(x) = \sin^4 x + \cos^4 x$, $0 \le x \le 2\pi$.

$$\begin{aligned} f'(x) &= 4\sin^3 x \cos x - 4\cos^3 x \sin x \\ &= -4\sin x \cos x \left(\cos^2 x - \sin^2 x\right) \\ &= -2\sin 2x \cos 2x = -\sin 4x \end{aligned}$$

$f'(x) = 0 \Leftrightarrow \sin 4x = 0 \Leftrightarrow 4x = n\pi \Leftrightarrow x = n\frac{\pi}{4}$. So the critical numbers are $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi$.

(a) $f'(x) > 0 \Leftrightarrow \sin 4x < 0 \Leftrightarrow \frac{\pi}{4} < x < \frac{\pi}{2}$ or $\frac{3\pi}{4} < x < \pi$ or $\frac{5\pi}{4} < x < \frac{3\pi}{2}$ or $\frac{7\pi}{4} < x < 2\pi$. f is increasing on these intervals. f is decreasing on $\left(0, \frac{\pi}{4}\right)$, $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$, $\left(\pi, \frac{5\pi}{4}\right)$, $\left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$.

(b) Local maxima $f\left(\frac{\pi}{2}\right) = f(\pi) = f\left(\frac{3\pi}{2}\right) = 1$, local minima $f\left(\frac{\pi}{4}\right) = f\left(\frac{3\pi}{4}\right) = f\left(\frac{5\pi}{4}\right) = f\left(\frac{7\pi}{4}\right) = \frac{1}{2}$.

16. $f(x) = x\sin x + \cos x$, $-\pi \le x \le \pi$.
$f'(x) = \sin x + x\cos x - \sin x = x\cos x$, $f'(x) = 0 \Leftrightarrow x = -\frac{\pi}{2}, 0, \frac{\pi}{2}$.

(a) $f'(x) > 0 \Leftrightarrow x\cos x > 0 \Leftrightarrow -\pi \le x < -\frac{\pi}{2}$ or $0 < x < \frac{\pi}{2}$. So f is increasing on $\left(-\pi, -\frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(-\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, \pi\right)$.

(b) Local maxima $f\left(-\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, local minimum $f(0) = 1$.

17. $f(x) = x^3 + 2x^2 - x + 1$. $f'(x) = 3x^2 + 4x - 1 = 0 \Rightarrow x = \frac{-4\pm\sqrt{28}}{6} = \frac{-2\pm\sqrt{7}}{3}$. Now $f'(x) > 0$ for $x < \frac{-2-\sqrt{7}}{3}$ or $x > \frac{-2+\sqrt{7}}{3}$ and $f'(x) < 0$ for $\frac{-2-\sqrt{7}}{3} < x < \frac{-2+\sqrt{7}}{3}$. f is increasing on $\left(-\infty, \frac{-2-\sqrt{7}}{3}\right)$ and $\left(\frac{-2+\sqrt{7}}{3}, \infty\right)$ and decreasing on $\left(\frac{-2-\sqrt{7}}{3}, \frac{-2+\sqrt{7}}{3}\right)$.

18. $f(x) = x^5 + 4x^3 - 6$. $f'(x) = 5x^4 + 12x^2 > 0$ for all $x \ne 0$. So f is increasing on $\mathbb{R}$.

19. $f(x) = 2\tan x - \tan^2 x$.
$f'(x) = 2\sec^2 x - 2\tan x \sec^2 x = 2\sec^2 x(1 - \tan x)$.
So $f'(x) > 0 \Leftrightarrow 1 - \tan x > 0 \Leftrightarrow \tan x < 1 \Leftrightarrow x \in \left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{4}\right)$, n an integer. So f is increasing on $\left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{4}\right)$, n an integer, and decreasing on $\left(n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{2}\right)$, n an integer.

20. $f(x) = 6x^2 - 2x^3 - x^4 \Rightarrow f'(x) = 12x - 6x^2 - 4x^3 \Rightarrow f''(x) = 12 - 12x - 12x^2 = 0 \Leftrightarrow x^2 + x - 1 = 0 \Rightarrow x = \frac{-1\pm\sqrt{5}}{2}$. For $x < \frac{-1-\sqrt{5}}{2}$, $f''(x) < 0$. For $\frac{-1-\sqrt{5}}{2} < x < \frac{-1+\sqrt{5}}{2}$, $f''(x) > 0$, and if $x > \frac{-1+\sqrt{5}}{2}$ then $f''(x) < 0$. Therefore f is CU on $\left(\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$.

21. $y = \dfrac{x^2}{\sqrt{1+x}}$, $D = \{x \mid x > -1\} \Rightarrow$

$$y' = \frac{2x\sqrt{1+x} - \frac{1}{2}(1+x)^{-1/2}\cdot x^2}{1+x} = \frac{4x + 3x^2}{2(1+x)^{3/2}} \Rightarrow$$

$$y'' = \frac{(4+6x)\,2(1+x)^{3/2} - 3(1+x)^{1/2}\left(4x+3x^2\right)}{4(1+x)^3} = \frac{3x^2 + 8x + 8}{4(1+x)^{5/2}} > 0 \Leftrightarrow$$

$3x^2 + 8x + 8 > 0$, which is true for all x since the discriminant is negative, so the function is CU on its domain, which is $(-1, \infty)$.

22. $f(x) = x(1+x)^{-2} \Rightarrow$
$f'(x) = (1+x)^{-2} - 2x(1+x)^{-3} = (1+x)^{-3}(1-x) \Rightarrow$

$$\begin{aligned} f''(x) &= -3(1+x)^{-4}(1-x) - (1+x)^{-3} \\ &= (1+x)^{-4}(2x-4) > 0 \Leftrightarrow \end{aligned}$$

$(2x - 4) > 0 \Leftrightarrow x > 2$. Therefore f is CU on $(2, \infty)$.

23. $y = \dfrac{x^3}{x^2-3} \Rightarrow y' = \dfrac{x^4 - 9x^2}{\left(x^2-3\right)^2} \Rightarrow$

$$y'' = \frac{\left(4x^3 - 18x\right)\left(x^2-3\right)^2 - 4x\left(x^2-3\right)\left(x^4 - 9x^2\right)}{\left(x^2-3\right)^4} = \frac{6x\left(x^2+9\right)}{\left(x^2-3\right)^3}$$

Now since $x^2 + 9 > 0$, the quotient is positive $\Leftrightarrow \dfrac{x}{x^2-3} = \dfrac{x}{\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)} > 0$.

Interval	x	$x+\sqrt{3}$	$x-\sqrt{3}$	$\dfrac{x}{x^2-3}$
$x < -\sqrt{3}$	$-$	$-$	$-$	$-$
$-\sqrt{3} < x < 0$	$-$	$-$	$-$	$-$
$0 < x < \sqrt{3}$	$+$	$+$	$-$	$-$
$x > \sqrt{3}$	$+$	$+$	$+$	$+$

So y is concave upward on $\left(-\sqrt{3}, 0\right)$ and $\left(\sqrt{3}, \infty\right)$.

24. (a) $f(x) = x^6 + 192x + 17 \Rightarrow$
$f'(x) = 6x^5 + 192 = 6\left(x^5 + 32\right)$. So $f'(x) > 0 \Leftrightarrow x^5 > -32 \Leftrightarrow x > -2$ and $f'(x) < 0 \Leftrightarrow x < -2$. So f is increasing on $(-2, \infty)$ and decreasing on $(-\infty, -2)$.

(b) f changes from decreasing to increasing at its only critical number, $x = -2$. Thus, $f(-2) = -303$ is a local minimum.

(c) $f''(x) = 30x^4 \ge 0$ for all x, so the concavity of f doesn't change and there is no inflection point. f is concave upward on $(-\infty, \infty)$.

25. (a) $f(x) = x/(1+x)^2 \Rightarrow$

$$f'(x) = \frac{(1+x)^2(1) - (x)2(1+x)}{\left[(1+x)^2\right]^2} = \frac{(1+x)\left[(1+x)-2x\right]}{(1+x)^4} = \frac{(1+x)(1-x)}{(1+x)^4} = \frac{1-x}{(1+x)^3}$$

So $f'(x) > 0 \Leftrightarrow -1 < x < 1$ and $f'(x) < 0 \Leftrightarrow x < -1$ or $x > 1$. So f is increasing on $(-1, 1)$ and f is decreasing on $(-\infty, -1)$ and $(1, \infty)$.

(b) f changes from increasing to decreasing at $x = 1$. $x = -1$ is not in the domain of f. Thus, $f(1) = \frac{1}{4}$ is a local maximum.

(c) $$f''(x) = \frac{(1+x)^3(-1) - (1-x)3(1+x)^2}{\left[(1+x)^3\right]^2} = \frac{(1+x)^2\left[-1(1+x) - 3(1-x)\right]}{(1+x)^6} = \frac{2x-4}{(1+x)^4}$$

$f''(x) > 0 \Leftrightarrow x > 2$ and $f''(x) < 0 \Leftrightarrow x < 2$ $(x \neq -1)$. Thus, f is concave upward on $(2, \infty)$ and f is concave downward on $(-\infty, -1)$ and $(-1, 2)$. There is an inflection point at $\left(2, \frac{2}{9}\right)$.

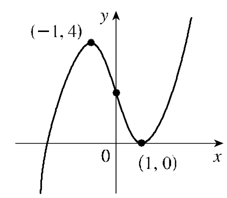
26. (a) $f(x) = 2\sin x + \sin^2 x$ on $[0, 2\pi] \Rightarrow$ $f'(x) = 2\cos x + 2\sin x\cos x = 2\cos x(1 + \sin x)$. $f'(x) > 0 \Leftrightarrow \cos x > 0$ (since $1 + \sin x \geq 0$ with equality when $x = \frac{3\pi}{2}$, a value where $\cos x = 0$) $\Leftrightarrow$ $0 \leq x < \frac{\pi}{2}$ or $\frac{3\pi}{2} < x \leq 2\pi$. So f is increasing on $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$, and f is decreasing on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

(b) Since f changes from increasing to decreasing at $x = \frac{\pi}{2}$, $f\left(\frac{\pi}{2}\right) = 3$ is a local maximum. Since f changes from decreasing to increasing at $x = \frac{3\pi}{2}$, $f\left(\frac{3\pi}{2}\right) = -1$ is a local minimum.

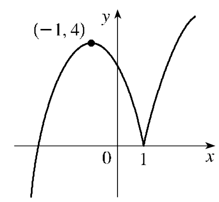
(c) $$\begin{aligned} f''(x) &= 2\cos x(\cos x) + (1+\sin x)(-2\sin x) \\ &= 2\cos^2 x - 2\sin x - 2\sin^2 x \\ &= 2\left(1 - \sin^2 x\right) - 2\sin x - 2\sin^2 x \\ &= 2 - 2\sin x - 4\sin^2 x \\ &= 2(1+\sin x)(1 - 2\sin x) \end{aligned}$$

$f''(x) > 0 \Leftrightarrow 1 - 2\sin x > 0 \Leftrightarrow \sin x < \frac{1}{2} \Leftrightarrow$ $0 \leq x < \frac{\pi}{6}$ or $\frac{5\pi}{6} < x \leq 2\pi$, so f is concave upward on $\left(0, \frac{\pi}{6}\right)$ and $\left(\frac{5\pi}{6}, 2\pi\right)$, and concave downward on $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$. There are inflection points at $\left(\frac{\pi}{6}, \frac{5}{4}\right)$ and $\left(\frac{5\pi}{6}, \frac{5}{4}\right)$.

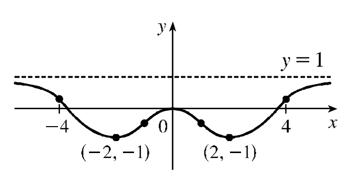
27. $f(-1) = 4$ and $f(1) = 0$ gives us two points to start with. $f'(-1) = f'(1) = 0 \Rightarrow$ horizontal tangents at $x = \pm 1$. $f'(x) < 0$ if $|x| < 1 \Rightarrow f$ is decreasing on $(-1, 1)$. $f'(x) > 0$ if $|x| > 1 \Rightarrow f$ is increasing on $(-\infty, -1)$ and $(1, \infty)$. $f''(x) < 0$ if $x < 0 \Rightarrow f$ is concave downward on $(-\infty, 0)$. $f''(x) > 0$ if $x > 0 \Rightarrow f$ is concave upward on $(0, \infty)$ and there is an inflection point at $x = 0$.



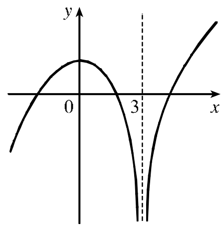
28. Since $f'(-1) = 0$ and $f'(1)$ does not exist, we have a horizontal tangent at $x = -1$ and a vertical tangent at $x = 1$. $f'(x) < 0$ if $|x| < 1 \Rightarrow f$ is decreasing on $(-1, 1)$, and $f'(x) > 0$ if $|x| > 1 \Rightarrow f$ is increasing on $(-\infty, -1)$ and $(1, \infty)$. $f''(x) < 0$ if $x \neq 1 \Rightarrow f$ is concave downward on $(-\infty, 1)$ and $(1, \infty)$.



29. First we plot the points which are known to be on the graph: $(2, -1)$ and $(0, 0)$. We can also draw a short line segment of slope 0 at $x = 2$, since we are given that $f'(2) = 0$. Now we know that $f'(x) < 0$ (that is, the function is decreasing) on $(0, 2)$, and that $f''(x) < 0$ on $(0, 1)$ and $f''(x) > 0$ on $(1, 2)$. So we must join the points $(0, 0)$ and $(2, -1)$ in such a way that the curve is concave down on $(0, 1)$ and concave up on $(1, 2)$. The curve must be concave up and increasing on $(2, 4)$ and concave down and increasing on $(4, \infty)$. Now we just need to reflect the curve in the y-axis, since we are given that f is an even function.



30. $\lim_{x \to 3} f(x) = -\infty \quad \Rightarrow$ there is a vertical asymptote at $x = 3$. $f'(0) = 0$ means that there is a horizontal tangent at $x = 0$. $f'(x) > 0$ if $x < 0$ or $x > 3$ and $f'(x) < 0$ if $0 < x < 3$ indicates that there is a local maximum at $x = 0$, since f is increasing on $(-\infty, 0)$ and decreasing on $(0, 3)$, and then increasing on $(3, \infty)$. $f''(x) < 0$ if $x \neq 3 \quad \Rightarrow \quad f$ is concave downward on $(-\infty, 3)$ and $(3, \infty)$.



31. (a) $P(x) = x\sqrt{x^2+1} \quad \Rightarrow$

$$P'(x) = \sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}} = \frac{2x^2+1}{\sqrt{x^2+1}} > 0, \text{ so } P$$

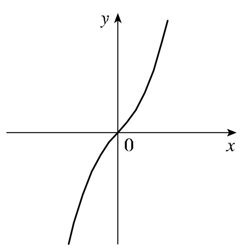
is increasing on $\mathbb{R}$.

(b) No maximum or minimum

(c) $$P''(x) = \frac{4x\sqrt{x^2+1} - (2x^2+1)\dfrac{x}{\sqrt{x^2+1}}}{x^2+1} = \frac{x(2x^2+3)}{(x^2+1)^{3/2}} > 0 \quad \Leftrightarrow$$

$x > 0$, so P is CU on $(0, \infty)$ and CD on $(-\infty, 0)$. IP at $(0, 0)$

(d)



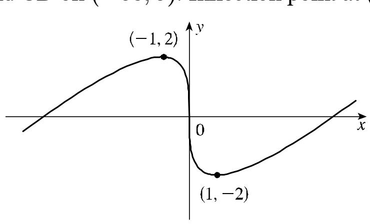
32. (a) $Q(x) = x - 3x^{1/3} \quad \Rightarrow \quad Q'(x) = 1 - \dfrac{1}{x^{2/3}} > 0 \quad \Leftrightarrow$

$x^{2/3} > 1 \quad \Leftrightarrow \quad x^2 > 1 \quad \Leftrightarrow \quad x < -1$ or $x > 1$, so Q is increasing on $(-\infty, -1)$, and $(1, \infty)$, and decreasing on $(-1, 1)$.

(b) $Q'(x) = 0 \quad \Leftrightarrow \quad x = \pm 1$; $Q(1) = -2$ is a local minimum, and $Q(-1) = 2$ is a local maximum.

(c) $Q''(x) = \frac{2}{3}x^{-5/3} > 0 \quad \Leftrightarrow \quad x > 0$, so Q is CU on $(0, \infty)$ and CD on $(-\infty, 0)$. Inflection point at $(0, 0)$

(d)



33. (a) $Q(x) = x^{1/3}(x+3)^{2/3} \quad \Rightarrow$

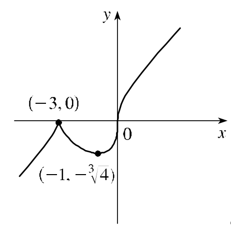
$$Q'(x) = \tfrac{1}{3}x^{-2/3}(x+3)^{2/3} + x^{1/3}\left(\tfrac{2}{3}\right)(x+3)^{-1/3} = \frac{x+1}{x^{2/3}(x+3)^{1/3}}$$

The critical numbers are -3, -1, and 0. Note that $x^{2/3} \geq 0$ for all x. So $Q'(x) > 0$ when $x < -3$ or $x > -1$ and $Q'(x) < 0$ when $-3 < x < -1 \quad \Rightarrow \quad Q$ is increasing on $(-\infty, -3)$ and $(-1, \infty)$ and decreasing on $(-3, -1)$.

(b) $Q(-3) = 0$ is a local maximum and $Q(-1) = -4^{1/3} \approx -1.6$ is a local minimum.

(c) $Q''(x) = -\dfrac{2}{x^{5/3}(x+3)^{4/3}} \quad \Rightarrow \quad Q''(x) > 0$ when $x < 0$, so Q is CU on $(-\infty, -3)$ and $(-3, 0)$ and CD on $(0, \infty)$. IP at $(0, 0)$

(d)



34. (a) $f(x) = \ln(1+x^2) \quad \Rightarrow \quad f'(x) = \dfrac{2x}{1+x^2} > 0 \quad \Leftrightarrow$

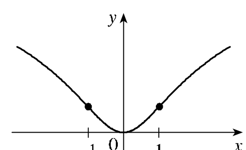
$x > 0$, so f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

(b) $f(0) = 0$ is a local minimum.

(c) $$f''(x) = \frac{2(1+x^2) - 2x(2x)}{(1+x^2)^2} = \frac{2(1-x^2)}{(1+x^2)^2} > 0$$

$\Leftrightarrow \quad |x| < 1$, so f is CU on $(-1, 1)$, CD on $(-\infty, -1)$ and $(1, \infty)$. There are IP at $(1, \ln 2)$ and $(-1, \ln 2)$.

(d)



35. (a) $f(\theta) = \sin^2\theta \quad \Rightarrow$

$f'(\theta) = 2\sin\theta\cos\theta = \sin 2\theta > 0 \quad \Leftrightarrow$

$2\theta \in (0, \pi) \cup (2\pi, 3\pi) \quad \Leftrightarrow \quad \theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$. So f is increasing on $\left(0, \frac{\pi}{2}\right)$ and $\left(\pi, \frac{3\pi}{2}\right)$, and decreasing on $\left(\frac{\pi}{2}, \pi\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$.

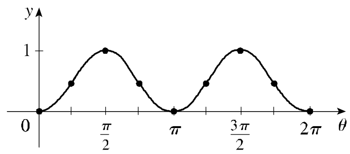
(b) Local minimum $f(\pi) = 0$, local maxima $f\left(\frac{\pi}{2}\right) = f\left(\frac{3\pi}{2}\right) = 1$

(c) $f''(\theta) = 2\cos 2\theta > 0 \quad \Leftrightarrow$

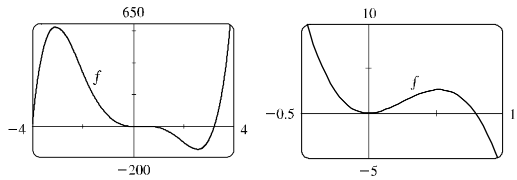
$2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right) \cup \left(\frac{7\pi}{2}, 4\pi\right) \quad \Leftrightarrow$

$\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$, so f is CU on these intervals and CD on $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ and $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$. IP at $\left(\frac{n\pi}{4}, \frac{1}{2}\right)$, $n = 1, 3, 5, 7$

(d)

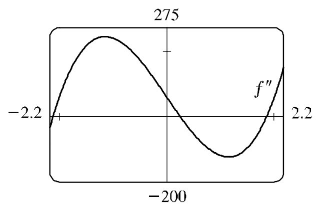


36. (a)



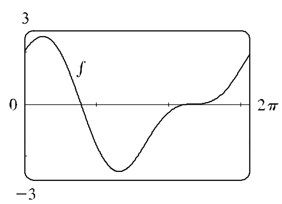
From the graphs of $f(x) = 3x^5 - 40x^3 + 30x^2$, it seems that f is concave upward on $(-2, 0.25)$ and $(2, \infty)$, and concave downward on $(-\infty, -2)$ and $(0.25, 2)$, with inflection points at about $(-2, 350)$, $(0.25, 1)$, and $(2, -100)$.

(b)



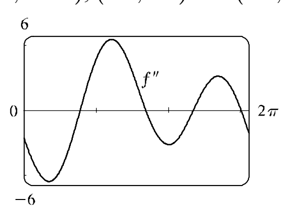
From the graph of $f''(x) = 60x^3 - 240x + 60$, it seems that f is CU on $(-2.1, 0.25)$ and $(1.9, \infty)$, and CD on $(-\infty, -2.1)$ and $(0.25, 1.9)$, with inflection points at about $(-2.1, 386)$, $(0.25, 1.3)$ and $(1.9, -87)$. (We have to check back on the graph of f to find the y-coordinates of the inflection points.)

37. (a)



From the graph of $f(x) = 2\cos x + \sin 2x$, it seems that f is CU on $(1.5, 3.5)$ and $(4.5, 6.0)$, and CD on $(0, 1.5)$, $(3.5, 4.5)$ and $(6.0, 2\pi)$, with inflection points at about $(1.5, 0.3)$, $(3.5, -1.3)$, $(4.5, 0.0)$ and $(6.0, 1.5)$.

(b)



From the graph of $f''(x) = -2\cos x - 4\sin 2x$, it seems that f is CU on $(1.57, 3.39)$ and $(4.71, 6.03)$ and CD on $(0, 1.57)$, $(3.39, 4.71)$ and $(6.03, 2\pi)$, with inflection points at about $(1.57, 0.00)$, $(3.39, -1.45)$, $(4.71, 0.00)$ and $(6.03, 1.45)$.

38. By the Mean Value Theorem, $\dfrac{f(5) - f(2)}{5 - 2} = f'(c)$ for some $c \in (2, 5)$. Since $1 \le f'(x) \le 4$, we have $1 \le \dfrac{f(5) - f(2)}{5 - 2} \le 4$ or $1 \le \dfrac{f(5) - f(2)}{3} \le 4$ or $3 \le f(5) - f(2) \le 12$.