

## ||| 4.1 Related Rates

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**A** [Click here for answers.](#)

**S** [Click here for solutions.](#)

1. If  $xy = 1$  and  $dx/dt = 4$ , find  $dy/dt$  when  $x = 2$ .
2. If  $x^2 + 3xy + y^2 = 1$  and  $dy/dt = 2$ , find  $dx/dt$  when  $y = 1$ .
3. A spherical snowball is melting in such a way that its volume is decreasing at a rate of  $1 \text{ cm}^3/\text{min}$ . At what rate is the diameter decreasing when the diameter is  $10 \text{ cm}$ ?

## ||| Answers

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**E** [Click here for exercises.](#)

**S** [Click here for solutions.](#)

1.  $-1$
2.  $-\frac{4}{3}$  (if  $x = 0$ ),  $-\frac{14}{3}$  (if  $x = -3$ )
3.  $1/(50\pi)$  cm/min


**Solutions**

[E Click here for exercises.](#)

[A Click here for answers.](#)

1.  $xy = 1 \Rightarrow x \frac{dy}{dt} + y \frac{dx}{dt} = 0$ . If  $\frac{dx}{dt} = 4$  and  $x = 2$ , then

$$y = \frac{1}{2}, \text{ so } \frac{dy}{dt} = -\frac{y}{x} \frac{dx}{dt} = -\frac{1/2}{2} (4) = -1.$$

2.  $x^2 + 3xy + y^2 = 1 \Rightarrow$

$$2x \frac{dx}{dt} + 3y \frac{dx}{dt} + 3x \frac{dy}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow$$

$$\frac{dx}{dt} = -\frac{3x + 2y}{2x + 3y} \frac{dy}{dt}. \text{ When } y = 1, \text{ we have } x^2 + 3x = 0$$

$$\Rightarrow x = 0 \text{ or } -3. \text{ If } \frac{dy}{dt} = 2 \text{ and } x = 0 \text{ and } y = 1, \text{ then}$$

$$\frac{dx}{dt} = -\frac{3(0) + 2(1)}{2(0) + 3(1)} (2) = -\frac{4}{3}. \text{ If } x = -3, \text{ then}$$

$$\frac{dx}{dt} = -\frac{3(-3) + 2(1)}{2(-3) + 3(1)} (2) = -\frac{14}{3}.$$

3. If the radius is  $r$  and the diameter  $x$ , then  $V = \frac{4}{3}\pi r^3 = \frac{\pi}{6}x^3$

$$\Rightarrow -1 = \frac{dV}{dt} = \frac{\pi}{2}x^2 \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -\frac{2}{\pi x^2}. \text{ When}$$

$$x = 10, \frac{dx}{dt} = -\frac{2}{\pi(100)} = -\frac{1}{50\pi}. \text{ So the rate of decrease}$$

$$\text{is } \frac{1}{50\pi} \frac{\text{cm}}{\text{min}}.$$