3.7 Derivatives Of Logarithmic Functions

A Click here for answers.

S Click here for solutions.

1–11 Differentiate the function.

1. $f(x) = \ln(2 - x)$

2. $f(x) = \log_3(x^2 - 4)$

3. $f(x) = \log_{10}\left(\dfrac{x}{x-1}\right)$

4. $F(x) = \ln\sqrt{x}$

5. $G(x) = \sqrt[3]{\ln x}$

6. $F(x) = e^x \ln x$

7. $h(y) = \ln(y^3 \sin y)$

8. $y = \dfrac{\ln x}{1 + x}$

9. $y = (\ln \tan x)^2$

10. $y = \ln|x^3 - x^2|$

11. $y = \ln(x + \ln x)$

12–14 Find y' and y''.

12. $y = x \ln x$

13. $y = \ln(ax)$

14. $y = \ln(1 + x^2)$

15–17 Differentiate f and find the domain of f.

15. $f(x) = \ln(2x + 1)$

16. $f(x) = \cos(\ln x)$

17. $f(x) = \log_3(x^2 - 4)$

18. Find an equation of the tangent line to the curve $y = \ln(x^2 + 1)$ at the point $(1, \ln 2)$.

19–27 Use logarithmic differentiation to find the derivative of the function.

19. $y = (3x - 7)^4(8x^2 - 1)^3$

20. $y = x^{2/5}(x^2 + 8)^4 e^{x^2+x}$

21. $y = \dfrac{(x+1)^4(x-5)^3}{(x-3)^8}$

22. $y = \sqrt{\dfrac{x^2+1}{x+1}}$

23. $y = \dfrac{e^x\sqrt{x^5+2}}{(x+1)^4(x^2+3)^2}$

24. $y = \dfrac{(x^3+1)^4 \sin^2 x}{\sqrt[3]{x}}$

25. $y = x^{1/\ln x}$

26. $y = (\sin x)^{\cos x}$

27. $y = x^{x^x}$

Answers

E Click here for exercises.

S Click here for solutions.

1. $f'(x) = \dfrac{1}{x-2}$ **2.** $f'(x) = \dfrac{2x}{(x^2-4)\ln 3}$

3. $f'(x) = -\dfrac{1}{x(x-1)\ln 10}$ **4.** $F'(x) = \dfrac{1}{2x}$

5. $G'(x) = \dfrac{1}{3x(\ln x)^{2/3}}$ **6.** $F'(x) = e^x\left(\ln x + \dfrac{1}{x}\right)$

7. $h'(y) = \dfrac{3}{y} + \cot y$ **8.** $y' = \dfrac{1 + x - x\ln x}{x(1+x)^2}$

9. $y' = \dfrac{2(\ln\tan x)\sec^2 x}{\tan x}$ **10.** $y' = \dfrac{3x-2}{x(x-1)}$

11. $y' = \dfrac{x+1}{x(x+\ln x)}$ **12.** $y' = \ln x + 1,\ y'' = 1/x$

13. $y' = 1/x,\ y'' = -1/x^2$ **14.** $y' = \dfrac{2x}{x^2+1},\ y'' = \dfrac{2-2x^2}{(x^2+1)^2}$

15. $f'(x) = \dfrac{2}{2x+1};\ \left(-\frac{1}{2}, \infty\right)$

16. $f'(x) = -\dfrac{\sin(\ln x)}{x},\ (0, \infty)$

17. $f'(x) = \dfrac{2x}{(x^2-4)\ln 3};\ |x| > 2$ **18.** $y = x + \ln 2 - 1$

19. $y' = (3x-7)^4(8x^2-1)^3\left(\dfrac{12}{3x-7} + \dfrac{48x}{8x^2-1}\right)$

20. $y' = x^{2/5}(x^2+8)^4 e^{x^2+x}\left(\dfrac{2}{5x} + \dfrac{8x}{x^2+8} + 2x + 1\right)$

21. $y' = \dfrac{(x+1)^4(x-5)^3}{(x-3)^8}\left(\dfrac{4}{x+1} + \dfrac{3}{x-5} - \dfrac{8}{x-3}\right)$

22. $y' = \sqrt{\dfrac{x^2+1}{x+1}}\left[\dfrac{x}{x^2+1} - \dfrac{1}{2(x+1)}\right]$

23. $y' = \dfrac{e^x\sqrt{x^5+2}}{(x+1)^4(x^2+3)^2}\left[1 + \dfrac{5x^4}{2(x^5+2)} - \dfrac{4}{x+1} - \dfrac{4x}{x^2+3}\right]$

24. $y' = \dfrac{(x^3+1)^4\sin^2 x}{x^{1/3}}\left(\dfrac{12x^2}{x^3+1} + 2\cot x - \dfrac{1}{3x}\right)$

25. $y' = 0$

26. $y' = (\sin x)^{\cos x}(-\sin x \ln\sin x + \cos x\cot x)$

27. $y' = x^{x^x}[x^x(\ln x + 1)\ln x + x^{x-1}]$

Solutions

E Click here for exercises.

A Click here for answers.

1. $f(x) = \ln(2-x) \quad \Rightarrow$

$$f'(x) = \frac{1}{2-x}\frac{d}{dx}(2-x) = \frac{-1}{2-x} = \frac{1}{x-2}$$

2. $f(x) = \log_3\left(x^2-4\right) \quad \Rightarrow$

$$f'(x) = \frac{1}{(x^2-4)\ln 3}(2x) = \frac{2x}{(x^2-4)\ln 3}$$

3. $f(x) = \log_{10}\left(\dfrac{x}{x-1}\right) = \log_{10} x - \log_{10}(x-1) \quad \Rightarrow$

$$f'(x) = \frac{1}{x\ln 10} - \frac{1}{(x-1)\ln 10} \text{ or } -\frac{1}{x(x-1)\ln 10}$$

4. $F(x) = \ln\sqrt{x} = \ln x^{1/2} = \frac{1}{2}\ln x \quad \Rightarrow$

$$F'(x) = \frac{1}{2}\left(\frac{1}{x}\right) = \frac{1}{2x}$$

5. $G(x) = \sqrt[3]{\ln x} = (\ln x)^{1/3} \quad \Rightarrow$

$$G'(x) = \tfrac{1}{3}(\ln x)^{-2/3}\cdot\frac{1}{x} = \frac{1}{3x(\ln x)^{2/3}}$$

6. $F(x) = e^x\ln x \quad \Rightarrow$

$$F'(x) = e^x\ln x + e^x\left(\frac{1}{x}\right) = e^x\left(\ln x + \frac{1}{x}\right)$$

7. $h(y) = \ln\left(y^3\sin y\right) = 3\ln y + \ln(\sin y) \quad \Rightarrow$

$$h'(y) = \frac{3}{y} + \frac{1}{\sin y}(\cos y) = \frac{3}{y} + \cot y$$

8. $y = \dfrac{\ln x}{1+x} \quad \Rightarrow$

$$y' = \frac{(1+x)(1/x) - (\ln x)(1)}{(1+x)^2} = \frac{\dfrac{1+x}{x} - \dfrac{x\ln x}{x}}{(1+x)^2} = \frac{1+x-x\ln x}{x(1+x)^2}$$

9. $y = (\ln\tan x)^2 \quad \Rightarrow$

$$y' = 2(\ln\tan x)\cdot\frac{1}{\tan x}\cdot\sec^2 x = \frac{2(\ln\tan x)\sec^2 x}{\tan x}$$

10. $y = \ln\left|x^3 - x^2\right| \quad \Rightarrow$

$$y' = \frac{1}{x^3-x^2}\left(3x^2-2x\right) = \frac{x(3x-2)}{x^2(x-1)} = \frac{3x-2}{x(x-1)}$$

11. $y = \ln(x+\ln x) \quad \Rightarrow$

$$y' = \frac{1}{x+\ln x}\left(1+\frac{1}{x}\right) = \frac{x+1}{x(x+\ln x)}$$

12. $y = x\ln x \quad \Rightarrow \quad y' = \ln x + x\left(\dfrac{1}{x}\right) = \ln x + 1 \quad \Rightarrow$

$$y'' = \frac{1}{x}$$

13. $y = \ln(ax) \quad \Rightarrow \quad y' = \dfrac{a}{ax} = \dfrac{1}{x} \quad \Rightarrow \quad y'' = -\dfrac{1}{x^2}$

14. $y = \ln\left(1+x^2\right) \quad \Rightarrow \quad y' = \dfrac{1}{1+x^2}\cdot 2x = \dfrac{2x}{x^2+1} \quad \Rightarrow$

$$y'' = \frac{\left(x^2+1\right)(2) - (2x)(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2}$$

15. $f(x) = \ln(2x+1) \quad \Rightarrow \quad f'(x) = \dfrac{1}{2x+1}\cdot 2 = \dfrac{2}{2x+1}$.

$\text{Dom}(f) = \{x \mid 2x+1 > 0\} = \left(-\frac{1}{2}, \infty\right)$

16. $f(x) = \cos(\ln x) \quad \Rightarrow \quad f'(x) = -\sin(\ln x)/x$.

$\text{Dom}(f) = (0, \infty)$

17. $f(x) = \log_3\left(x^2-4\right) \quad \Rightarrow \quad f'(x) = \dfrac{2x}{(x^2-4)\ln 3}$.

$$\text{Dom}(f) = \left\{x \mid x^2-4 > 0\right\} = \{x \mid |x| > 2\} = (-\infty, -2)\cup(2,\infty)$$

18. $y = f(x) = \ln\left(x^2+1\right) \quad \Rightarrow$

$f'(x) = \dfrac{1}{x^2+1}\cdot 2x = \dfrac{2x}{x^2+1} \quad \Rightarrow \quad f'(1) = 1$, so

an equation of the tangent line at $(1, \ln 2)$ is

$y - \ln 2 = 1(x-1)$, or $y = x + \ln 2 - 1$.

19. $y = (3x-7)^4\left(8x^2-1\right)^3 \quad \Rightarrow$

$\ln|y| = 4\ln|3x-7| + 3\ln\left|8x^2-1\right|$

$$\Rightarrow \quad \frac{y'}{y} = \frac{12}{3x-7} + \frac{48x}{8x^2-1} \quad \Rightarrow$$

$$y' = (3x-7)^4\left(8x^2-1\right)^3\left(\frac{12}{3x-7} + \frac{48x}{8x^2-1}\right)$$

20. $y = x^{2/5}\left(x^2+8\right)^4 e^{x^2+x} \quad \Rightarrow$

$\ln|y| = \frac{2}{5}\ln|x| + 4\ln\left(x^2+8\right) + x^2 + x$

$$\Rightarrow \quad \frac{y'}{y} = \frac{2}{5}\cdot\frac{1}{x} + 4\frac{2x}{x^2+8} + 2x + 1 \quad \Rightarrow$$

$$y' = x^{2/5}\left(x^2+8\right)^4 e^{x^2+x}\left[\frac{2}{5x} + \frac{8x}{x^2+8} + 2x + 1\right]$$

21. $y = \dfrac{(x+1)^4(x-5)^3}{(x-3)^8} \quad \Rightarrow$

$\ln|y| = 4\ln|x+1| + 3\ln|x-5| - 8\ln|x-3|$

$$\Rightarrow \quad \frac{y'}{y} = \frac{4}{x+1} + \frac{3}{x-5} - \frac{8}{x-3} \quad \Rightarrow$$

$$y' = \frac{(x+1)^4(x-5)^3}{(x-3)^8}\left(\frac{4}{x+1} + \frac{3}{x-5} - \frac{8}{x-3}\right)$$

22. $y = \sqrt{\dfrac{x^2+1}{x+1}} \Rightarrow \ln y = \frac{1}{2}\left[\ln\left(x^2+1\right) - \ln(x+1)\right]$

$\Rightarrow \dfrac{y'}{y} = \dfrac{1}{2}\left(\dfrac{2x}{x^2+1} - \dfrac{1}{x+1}\right) \Rightarrow$

$y' = \sqrt{\dfrac{x^2+1}{x+1}}\left[\dfrac{x}{x^2+1} - \dfrac{1}{2(x+1)}\right]$

23. $y = \dfrac{e^x\sqrt{x^5+2}}{(x+1)^4\left(x^2+3\right)^2} \Rightarrow$

$\ln y = x + \frac{1}{2}\ln\left(x^5+2\right) - 4\ln|x+1| - 2\ln\left(x^2+3\right)$

$\Rightarrow \dfrac{y'}{y} = 1 + \dfrac{5x^4}{2\left(x^5+2\right)} - \dfrac{4}{x+1} - \dfrac{4x}{x^2+3}$. So

$$y' = \frac{e^x\sqrt{x^5+2}}{(x+1)^4\left(x^2+3\right)^2}\left[1 + \frac{5x^4}{2\left(x^5+2\right)} - \frac{4}{x+1} - \frac{4x}{x^2+3}\right]$$

24. $y = \dfrac{\left(x^3+1\right)^4\sin^2 x}{x^{1/3}} \Rightarrow$

$\ln|y| = 4\ln\left|x^3+1\right| + 2\ln|\sin x| - \frac{1}{3}\ln|x|$. So

$\dfrac{y'}{y} = 4\dfrac{3x^2}{x^3+1} + 2\dfrac{\cos x}{\sin x} - \dfrac{1}{3x} \Rightarrow$

$y' = \dfrac{\left(x^3+1\right)^4\sin^2 x}{x^{1/3}}\left(\dfrac{12x^2}{x^3+1} + 2\cot x - \dfrac{1}{3x}\right)$.

25. $y = x^{1/\ln x} \Rightarrow \ln y = \left(\dfrac{1}{\ln x}\right)\ln x = 1 \Rightarrow y = e$

$\Rightarrow y' = 0$

26. $y = (\sin x)^{\cos x} \Rightarrow \ln y = \cos x \ln(\sin x) \Rightarrow$

$\dfrac{y'}{y} = -\sin x \ln \sin x + \cos x\left(\dfrac{\cos x}{\sin x}\right) \Rightarrow$

$y' = (\sin x)^{\cos x}(-\sin x \ln \sin x + \cos x \cot x)$

27. $y = x^{x^x} \Rightarrow \ln y = x^x \ln x \Rightarrow$

$\dfrac{y'}{y} = x^x(\ln x + 1)\ln x + x^x\left(\dfrac{1}{x}\right)$, because $z = x^x$

$\Rightarrow \ln z = x\ln x \Rightarrow \dfrac{z'}{z} = \ln x + x\left(\dfrac{1}{x}\right)$

$\Rightarrow z' = x^x(\ln x + 1)$. Therefore,

$y' = x^{x^x}\left[x^x(\ln x + 1)\ln x + x^{x-1}\right]$.