3.5 The Chain Rule

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S Click here for solutions.

1–4 Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.] Then find the derivative dy/dx.

1. $y = (x^2 + 4x + 6)^5$

2. $y = \tan 3x$

3. $y = \cos(\tan x)$

4. $y = \sqrt[3]{1 + x^3}$

5–32 Find the derivative of the function.

5. $F(x) = (x^3 - 5x)^4$

6. $f(t) = (2t^2 + 6t + 1)^{-8}$

7. $g(x) = \sqrt{x^2 - 7x}$

8. $f(t) = \dfrac{1}{(t^2 - 2t - 5)^4}$

9. $h(t) = \left(t - \dfrac{1}{t}\right)^{3/2}$

10. $y = \sin \dfrac{1}{x}$

11. $G(x) = (3x - 2)^{10}(5x^2 - x + 1)^{12}$

12. $g(t) = (6t^2 + 5)^3(t^3 - 7)^4$

13. $F(y) = \left(\dfrac{y - 6}{y + 7}\right)^3$

14. $s(t) = \sqrt[4]{\dfrac{t^3 + 1}{t^3 - 1}}$

15. $f(z) = \dfrac{1}{\sqrt[5]{2z - 1}}$

16. $f(x) = \dfrac{x}{\sqrt{7 - 3x}}$

17. $y = 5^{-1/x}$

18. $y = \sqrt{1 + 2\tan x}$

19. $y = \sin^3 x + \cos^3 x$

20. $y = \sin^2(\cos kx)$

21. $y = \dfrac{e^{3x}}{1 + e^x}$

22. $y = e^{5\sin\theta}$

23. $y = \left(\sin\sqrt{x^2 + 1}\right)^{\sqrt{2}}$

24. $y = \cos^2(\cos x) + \sin^2(\cos x)$

25. $f(x) = [x^3 + (2x - 1)^3]^3$

26. $g(t) = \sqrt[4]{(1 - 3t)^4 + t^4}$

27. $y = \cos^2\left(\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}\right)$

28. $y = \sqrt{1 + \tan(x + (1/x))}$

29. $p(t) = \left[\left(1 + \dfrac{2}{t}\right)^{-1} + 3t\right]^{-2}$

30. $N(y) = \left(y + \sqrt[3]{y + \sqrt{2y - 9}}\right)^8$

31–37 Find an equation of the tangent line to the curve at the given point.

31. $y = \dfrac{8}{\sqrt{4 + 3x}}$, $(4, 2)$

32. $y = \sin x + \cos 2x$, $(\pi/6, 1)$

33. $y = 10^x$, $(1, 10)$

34. $y = (x^3 - x^2 + x - 1)^{10}$, $(1, 0)$

35. $y = \sqrt{x + (1/x)}$, $(1, \sqrt{2})$

36. $y = \dfrac{x}{(3 - x^2)^5}$, $(2, -2)$

37. $y = \cot^2 x$, $(\pi/4, 1)$

38–41 Find f' and state the domains of f and f'.

38. $f(x) = x^2 \sec^2 3x$

39. $f(x) = \sin\sqrt{2x + 1}$

40. $f(x) = \sqrt{\cos\sqrt{x}}$

41. $f(x) = \cos\sqrt{x} + \sqrt{\cos x}$

42–43 Find dy/dx.

42. $y = \sqrt{t} - t$, $y = t^3 - t$

43. $x = t \ln t$, $y = \sin^2 t$

44–47 Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

44. $x = t^2 + 1$, $y = t^2 - t$; $t = 0$

45. $x = t \sin t$, $y = t \cos t$; $t = \pi$

46. $y = t^2 + t$, $y = \sqrt{t}$; $t = 4$

47. $x = 2 \sin\theta$, $y = 3\cos\theta$; $\theta = \pi/4$

Answers

E Click here for exercises.

S Click here for solutions.

1. $10(x^2+4x+6)^4(x+2)$ **2.** $3\sec^2 3x$

3. $-\sin(\tan x)\sec^2 x$ **4.** $\dfrac{x^2}{(1+x^3)^{2/3}}$

5. $F'(x)=4(x^3-5x)^3(3x^2-5)$

6. $f'(t)=-16(2t^2-6t+1)^{-9}(2t-3)$

7. $g'(x)=\dfrac{2x-7}{2\sqrt{x^2-7x}}$ **8.** $f'(t)=\dfrac{8(1-t)}{(t^2-2t-5)^5}$

9. $h'(t)=\frac{3}{2}(t-1/t)^{1/2}(1+1/t^2)$

10. $y'=-\dfrac{1}{x^2}\cos\dfrac{1}{x}$

11. $G'(x)=6(3x-2)^9(5x^2-x+1)^{11}(85x^2-51x+9)$

12. $g'(t)=12t(6t^2+5)^2(t^3-7)^3(9t^3+5t-21)$

13. $F'(y)=\dfrac{39(y-6)^2}{(y+7)^4}$

14. $s'(t)=\dfrac{1}{2}\left(\dfrac{t^3+1}{t^3+1}\right)^{-3/4}\dfrac{-3t^2}{(t^3-1)^2}$

15. $f'(z)=-\frac{2}{5}(2z-1)^{-6/5}$ **16.** $f'(x)=\dfrac{14-3x}{2(7-3x)^{3/2}}$

17. $y'=5^{-1/x}(\ln 5)/x^2$ **18.** $y'=\dfrac{\sec^2 x}{\sqrt{1+2\tan x}}$

19. $y'=3\sin x\cos x(\sin x-\cos x)$

20. $y'=-k\sin kx\sin(2\cos kx)$

21. $y'=\dfrac{3e^{3x}+2e^{4x}}{(1+e^x)^2}$ **22.** $y'=5\cos(5\theta)e^{\sin 5\theta}$

23. $y'=\sqrt{2}x\left(\sin\sqrt{x^2+1}\right)^{\sqrt{2}-1}\dfrac{\cos\sqrt{x^2+1}}{\sqrt{x^2+1}}$ **24.** $y'=0$

25. $f'(x)=9[x^3+(2x-1)^3]^2(9x^2-8x+2)$

26. $g'(t)=[(1-3t)^4+t^4]^{-3/4}[t^3-3(1-3t^3)]$

27. $y'=\dfrac{2}{\sqrt{x}(1+\sqrt{x})^2}\sin\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cos\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)$

28. $y'=\dfrac{(x^2-1)\sec^2\left(x+\dfrac{1}{x}\right)}{2x^2\sqrt{1+\tan\left(x+\dfrac{1}{x}\right)}}$

29. $p'(t)=-2[(1+2/t)^{-1}+3t]^{-3}[2(t+2)^{-2}+3]$

30. $N'(y)=8\left(y+\sqrt[3]{y+\sqrt{2y-9}}\right)^7\left[1+\frac{1}{3}\left(y+\sqrt{2y-9}\right)^{-2/3}\left(1+\dfrac{1}{\sqrt{2y-9}}\right)\right]$

31. $y=-\frac{3}{16}x+\frac{11}{4}$ **32.** $y=-\frac{\sqrt{3}}{2}x+1+\frac{\sqrt{3}\pi}{12}$

33. $y=10[(x-1)\ln 10+1]$ **34.** $y=0$

35. $y=\sqrt{2}$ **36.** $y=39x-80$ **37.** $4x+y=\pi+1$

38. $f'(x)=2x\sec^2 3x\,(1+3x\tan 3x)$,
$\left\{x \mid x\neq(2n-2)\frac{\pi}{6},\ n \text{ an integer}\right\}$ (both f and f')

39. $f'(x)=\dfrac{\cos\sqrt{2x+1}}{\sqrt{2x+1}}$,
$\text{dom}(f)=\left[-\frac{1}{2},\infty\right)$, $\text{dom}(f')=\left(-\frac{1}{2},\infty\right)$

40. $f'(x)=-\dfrac{\sin\sqrt{x}}{4\sqrt{x}\sqrt{\cos\sqrt{x}}}$,
$\text{dom}(f)=\{x \mid 0\leqslant x\leqslant \pi^2/4$ or
$[(4n-1)\pi/2]^2\leqslant x\leqslant[(4n+1)\pi/2]^2$
for some $n\in\{1,2,3,\dots\}\}$,
$\text{dom}(f')=\{x \mid 0<x<\pi^2/4$ or
$[(4n-1)\pi/2]^2<x<[(4n+1)\pi/2]^2$
for some $n\in\{1,2,3,\dots\}\}$

41. $f'(x)=-\dfrac{\sin\sqrt{x}}{2\sqrt{x}}-\dfrac{\sin x}{2\sqrt{\cos x}}$,
$\text{dom}(f)=\{x \mid 0\leqslant x\leqslant\pi/2$ or
$(4n-1)\pi/2\leqslant x\leqslant(4n+1)\pi/2$
for some $n=1,2,3,\dots\}$,
$\text{dom}(f')=\{x \mid 0<x<\pi/2$ or
$(4n-1)\pi/2<x<(4n+1)\pi/2$
for some $n=1,2,3,\dots\}$

42. $\dfrac{(3t^2-1)(2\sqrt{t})}{1-2\sqrt{t}}$ **43.** $\dfrac{2\sin t\cos t}{1+\ln t}$ **44.** $y=-x$

45. $y=\frac{1}{\pi}x-\pi$ **46.** $y=\frac{1}{36}x+\frac{13}{9}$ **47.** $y=-\frac{3}{2}x+3\sqrt{2}$

Solutions

E Click here for exercises.

A Click here for answers.

1. Let $u = g(x) = x^2 + 4x + 6$ and $y = f(u) = u^5$. Then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \left(5u^4\right)(2x+4) = 5\left(x^2+4x+6\right)^4(2x+4) = 10\left(x^2+4x+6\right)^4(x+2)$$

2. Let $u = g(x) = 3x$ and $y = f(u) = \tan u$. Then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \left(\sec^2 u\right)(3) = 3\sec^2 3x$.

3. Let $u = g(x) = \tan x$ and $y = f(u) = \cos u$. Then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (-\sin u)\left(\sec^2 x\right) = -\sin(\tan x)\sec^2 x$.

4. Let $u = g(x) = 1 + x^3$ and $y = f(u) = u^{1/3}$. Then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \tfrac{1}{3}u^{-2/3}\left(3x^2\right) = \left(1+x^3\right)^{-2/3}x^2 = \frac{x^2}{(1+x^3)^{2/3}}$$

5. $F(x) = \left(x^3-5x\right)^4 \Rightarrow$

$$F'(x) = 4\left(x^3-5x\right)^3\frac{d}{dx}\left(x^3-5x\right) = 4\left(x^3-5x\right)^3\left(3x^2-5\right)$$

6. $f(t) = \left(2t^2-6t+1\right)^{-8} \Rightarrow$

$$f'(t) = -8\left(2t^2-6t+1\right)^{-9}(4t-6) = -16\left(2t^2-6t+1\right)^{-9}(2t-3)$$

7. $g(x) = \sqrt{x^2-7x} = \left(x^2-7x\right)^{1/2} \Rightarrow$

$g'(x) = \frac{1}{2}\left(x^2-7x\right)^{-1/2}(2x-7) = \dfrac{2x-7}{2\sqrt{x^2-7x}}$

8. $f(t) = \dfrac{1}{(t^2-2t-5)^4} = \left(t^2-2t-5\right)^{-4} \Rightarrow$

$f'(t) = -4\left(t^2-2t-5\right)^{-5}(2t-2) = \dfrac{8(1-t)}{(t^2-2t-5)^5}$

9. $h(t) = (t-1/t)^{3/2} \Rightarrow$

$h'(t) = \frac{3}{2}(t-1/t)^{1/2}\left(1+1/t^2\right)$

10. $y = \sin\dfrac{1}{x} \Rightarrow y' = \cos\dfrac{1}{x}\left(-\dfrac{1}{x^2}\right) = -\dfrac{1}{x^2}\cos\dfrac{1}{x}$

11. $G(x) = (3x-2)^{10}\left(5x^2-x+1\right)^{12} \Rightarrow$

$$\begin{aligned} G'(x) &= (3x-2)^{10}(12)\left(5x^2-x+1\right)^{11}(10x-1) \\ &\quad + 10(3x-2)^9(3)\left(5x^2-x+1\right)^{12} \\ &= 6(3x-2)^9\left(5x^2-x+1\right)^{11} \\ &\quad \left[2(3x-2)(10x-1)+5\left(5x^2-x+1\right)\right] \\ &= 6(3x-2)^9\left(5x^2-x+1\right)^{11}\left(85x^2-51x+9\right) \end{aligned}$$

12. $g(t) = \left(6t^2+5\right)^3\left(t^3-7\right)^4 \Rightarrow$

$$\begin{aligned} g'(t) &= \left(6t^2+5\right)^3(4)\left(t^3-7\right)^3\left(3t^2\right) \\ &\quad + 3\left(6t^2+5\right)^2(12t)\left(t^3-7\right)^4 \\ &= 12t\left(6t^2+5\right)^2\left(t^3-7\right)^3\left[t\left(6t^2+5\right)+3\left(t^3-7\right)\right] \\ &= 12t\left(6t^2+5\right)^2\left(t^3-7\right)^3\left(9t^3+5t-21\right) \end{aligned}$$

13. $F(y) = \left(\dfrac{y-6}{y+7}\right)^3 \Rightarrow$

$$\begin{aligned} F'(y) &= 3\left(\frac{y-6}{y+7}\right)^2\frac{(y+7)(1)-(y-6)(1)}{(y+7)^2} \\ &= 3\left(\frac{y-6}{y+7}\right)^2\frac{13}{(y+7)^2} = \frac{39(y-6)^2}{(y+7)^4} \end{aligned}$$

14. $s(t) = \left(\dfrac{t^3+1}{t^3-1}\right)^{1/4} \Rightarrow$

$$\begin{aligned} s'(t) &= \frac{1}{4}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4}\frac{3t^2\left(t^3-1\right)-\left(t^3+1\right)\left(3t^2\right)}{(t^3-1)^2} \\ &= \frac{1}{2}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4}\frac{-3t^2}{(t^3-1)^2} \end{aligned}$$

15. $f(z) = (2z-1)^{-1/5} \Rightarrow$

$f'(z) = -\frac{1}{5}(2z-1)^{-6/5}(2) = -\frac{2}{5}(2z-1)^{-6/5}$

16. $f(x) = \dfrac{x}{\sqrt{7-3x}} \Rightarrow$

$$\begin{aligned} f'(x) &= \frac{\sqrt{7-3x} - x\left(\frac{1}{2}\right)(7-3x)^{-1/2}(-3)}{7-3x} \\ &= \frac{1}{\sqrt{7-3x}} + \frac{3x}{2(7-3x)^{3/2}} \text{ or } \frac{14-3x}{2(7-3x)^{3/2}} \end{aligned}$$

17. Using Formula 5 and the Chain Rule, $y = 5^{-1/x} \Rightarrow$

$y' = 5^{-1/x}(\ln 5)\left[-1\cdot\left(-x^{-2}\right)\right] = 5^{-1/x}(\ln 5)/x^2$

18. $y = \sqrt{1+2\tan x} \Rightarrow$

$y' = \frac{1}{2}(1+2\tan x)^{-1/2}\,2\sec^2 x = \dfrac{\sec^2 x}{\sqrt{1+2\tan x}}$

19. $y = \sin^3 x + \cos^3 x \;\Rightarrow$

$$\begin{aligned} y' &= 3\sin^2 x \cos x + 3\cos^2 x\,(-\sin x) \\ &= 3\sin x \cos x\,(\sin x - \cos x) \end{aligned}$$

20. $y = \sin^2(\cos kx) \;\Rightarrow$

$$\begin{aligned} y' &= 2\sin(\cos kx)\cos(\cos kx)\,(-\sin kx)\,(k) \\ &= -k\sin kx \sin(2\cos kx) \end{aligned}$$

21. $y = \dfrac{e^{3x}}{1+e^x} \;\Rightarrow$

$$\begin{aligned} y' &= \frac{3e^{3x}(1+e^x) - e^{3x}(e^x)}{(1+e^x)^2} \\ &= \frac{3e^{3x} + 3e^{4x} - e^{4x}}{(1+e^x)^2} = \frac{3e^{3x} + 2e^{4x}}{(1+e^x)^2} \end{aligned}$$

22. $y = e^{\sin 5\theta} \;\Rightarrow\; y' = 5\cos(5\theta)\,e^{\sin 5\theta}$

23. $y = \left(\sin\sqrt{x^2+1}\right)^{\sqrt{2}} \;\Rightarrow$

$$\begin{aligned} y' &= \sqrt{2}\left(\sin\sqrt{x^2+1}\right)^{\sqrt{2}-1}\left(\cos\sqrt{x^2+1}\right) \\ &\qquad \left(\frac{1}{2}\right)\left(x^2+1\right)^{-1/2}(2x) \\ &= \sqrt{2}x\left(\sin\sqrt{x^2+1}\right)^{\sqrt{2}-1}\frac{\cos\sqrt{x^2+1}}{\sqrt{x^2+1}} \end{aligned}$$

24. $y = \cos^2(\cos x) + \sin^2(\cos x) = 1 \;\Rightarrow\; y' = 0$

25. $f(x) = \left[x^3 + (2x-1)^3\right]^3 \;\Rightarrow$

$$\begin{aligned} f'(x) &= 3\left[x^3 + (2x-1)^3\right]^2\left[3x^2 + 3(2x-1)^2(2)\right] \\ &= 9\left[x^3 + (2x-1)^3\right]^2\left[9x^2 - 8x + 2\right] \end{aligned}$$

26. $g(t) = \sqrt[4]{(1-3t)^4 + t^4} \;\Rightarrow$

$$\begin{aligned} g'(t) &= \frac{1}{4}\left[(1-3t)^4 + t^4\right]^{-3/4}\left[4(1-3t)^3(-3) + 4t^3\right] \\ &= \left[(1-3t)^4 + t^4\right]^{-3/4}\left[t^3 - 3\left(1-3t^3\right)\right] \end{aligned}$$

27. $y = \cos^2\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right) \;\Rightarrow$

$$\begin{aligned} y' &= 2\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)(-1)\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) \\ &\qquad \frac{(1+\sqrt{x})\left(-\frac{1}{2\sqrt{x}}\right) - (1-\sqrt{x})\frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2} \\ &= \frac{2}{\sqrt{x}(1+\sqrt{x})^2}\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) \end{aligned}$$

28. $y = \sqrt{1 + \tan\left(x + \dfrac{1}{x}\right)} \;\Rightarrow$

$$\begin{aligned} y' &= \frac{1}{2\sqrt{1+\tan\left(x+\frac{1}{x}\right)}}\left[\sec^2\left(x+\frac{1}{x}\right)\right]\left(1 - \frac{1}{x^2}\right) \\ &= \frac{\left(x^2-1\right)\sec^2\left(x+\frac{1}{x}\right)}{2x^2\sqrt{1+\tan\left(x+\frac{1}{x}\right)}} \end{aligned}$$

29. $p(t) = \left[\left(1 + \dfrac{2}{t}\right)^{-1} + 3t\right]^{-2} \;\Rightarrow$

$$\begin{aligned} p'(t) &= -2\left[\left(1+\frac{2}{t}\right)^{-1} + 3t\right]^{-3}\left[-\left(1+\frac{2}{t}\right)^{-2}\left(-\frac{2}{t^2}\right) + 3\right] \\ &= -2\left[\left(1+\frac{2}{t}\right)^{-1} + 3t\right]^{-3}\left[2(t+2)^{-2} + 3\right] \end{aligned}$$

30. $N(y) = \left(y + \sqrt[3]{y + \sqrt{2y-9}}\right)^8 =$

$\left\{y + \left[y + (2y-9)^{1/2}\right]^{1/3}\right\}^8 \;\Rightarrow$

$$\begin{aligned} N'(y) &= 8\left(y + \sqrt[3]{y+\sqrt{2y-9}}\right)^7 \\ &\qquad \left[1 + \tfrac{1}{3}\left(y + \sqrt{2y-9}\right)^{-2/3}\right. \\ &\qquad \left.\left[1 + \tfrac{1}{2}(2y-9)^{-1/2}(2)\right]\right] \\ &= 8\left(y + \sqrt[3]{y+\sqrt{2y-9}}\right)^7 \\ &\qquad \left[1 + \tfrac{1}{3}\left(y+\sqrt{2y-9}\right)^{-2/3}\left(1 + \frac{1}{\sqrt{2y-9}}\right)\right] \end{aligned}$$

31. $y = f(x) = \dfrac{8}{\sqrt{4+3x}} = 8(4+3x)^{-1/2} \;\Rightarrow$

$f'(x) = 8\left(-\tfrac{1}{2}\right)(4+3x)^{-3/2}(3) = -12(4+3x)^{-3/2}$.

The slope of the tangent at $(4,2)$ is $f'(4) = -\tfrac{12}{64} = -\tfrac{3}{16}$

and its equation is $y - 2 = -\tfrac{3}{16}(x-4)$ or $y = -\tfrac{3}{16}x + \tfrac{11}{4}$.

32. $y = f(x) = \sin x + \cos 2x \;\Rightarrow$

$f'(x) = \cos x - 2\sin 2x$. The slope of the tangent at $\left(\tfrac{\pi}{6}, 1\right)$ is $f'\left(\tfrac{\pi}{6}\right) = \tfrac{\sqrt{3}}{2} - 2\left(\tfrac{\sqrt{3}}{2}\right) = -\tfrac{\sqrt{3}}{2}$ and its equation is

$y - 1 = -\tfrac{\sqrt{3}}{2}\left(x - \tfrac{\pi}{6}\right)$ or $\sqrt{3}x + 2y = 2 + \tfrac{\sqrt{3}}{6}\pi$.

33. $f(x) = 10^x \;\Rightarrow\; f'(x) = 10^x \ln 10$, so the slope of the tangent at $(1, 10)$ is $f'(1) = 10\ln 10$ and an equation is $y - 10 = 10\ln 10\,(x-1)$ or $y = 10\left[(x-1)\ln 10 + 1\right]$.

34. $y = f(x) = (x^3 - x^2 + x - 1)^{10} \Rightarrow$
$f'(x) = 10(x^3 - x^2 + x - 1)^9(3x^2 - 2x + 1)$. The slope of the tangent at $(1, 0)$ is $f'(1) = 0$ and its equation is $y - 0 = 0(x - 1)$ or $y = 0$.

35. $y = f(x) = \sqrt{x + 1/x} \Rightarrow$
$f'(x) = \frac{1}{2}\left(x + \frac{1}{x}\right)^{-1/2}\left(1 - \frac{1}{x^2}\right)$. The slope of the tangent at $(1, \sqrt{2})$ is $f'(1) = 0$ and its equation is $y - \sqrt{2} = 0(x - 1)$ or $y = \sqrt{2}$.

36. $y = f(x) = \dfrac{x}{(3 - x^2)^5} \Rightarrow$

$$f'(x) = \frac{(3 - x^2)^5(1) - x(5)(3 - x^2)^4(-2x)}{(3 - x^2)^{10}} = \frac{9x^2 + 3}{(3 - x^2)^6}$$

The slope of the tangent at $(2, -2)$ is $f'(2) = 39$ and its equation is $y + 2 = 39(x - 2)$ or $y = 39x - 80$.

37. $y = f(x) = \cot^2 x \Rightarrow$
$y' = 2\cot x(-\csc^2 x) = -2\cot x \csc^2 x$. The slope of the tangent at $\left(\frac{\pi}{4}, 1\right)$ is $f'\left(\frac{\pi}{4}\right) = -2(1)(\sqrt{2})^2 = -4$ and its equation is $y - 1 = -4\left(x - \frac{\pi}{4}\right)$ or $4x + y = \pi + 1$.

38. $f(x) = x^2 \sec^2 3x \Rightarrow$

$$\begin{aligned} f'(x) &= 2x\sec^2 3x + x^2(2\sec 3x)(\sec 3x \tan 3x)(3) \\ &= 2x\sec^2 3x(1 + 3x\tan 3x) \end{aligned}$$

Domain of f = domain of $f' = \{x \mid \cos 3x \neq 0\}$
$= \left\{x \mid x \neq (2n - 1)\frac{\pi}{6}, n \text{ an integer}\right\}$

39. $f(x) = \sin\sqrt{2x + 1} \Rightarrow$
$f'(x) = \cos\sqrt{2x + 1}\left(\dfrac{1}{2\sqrt{2x + 1}}\right)(2) = \dfrac{\cos\sqrt{2x + 1}}{\sqrt{2x + 1}}$.
$\text{Dom}(f) = \{x \mid 2x + 1 \geq 0\} = \left[-\frac{1}{2}, \infty\right)$.
$\text{Dom}(f') = \{x \mid 2x + 1 > 0\} = \left(-\frac{1}{2}, \infty\right)$.

40. $f(x) = \sqrt{\cos\sqrt{x}} \Rightarrow$

$$\begin{aligned} f'(x) &= \tfrac{1}{2}(\cos\sqrt{x})^{-1/2}(-\sin\sqrt{x})\left(\tfrac{1}{2}\right)x^{-1/2} \\ &= -\frac{\sin\sqrt{x}}{4\sqrt{x}\sqrt{\cos\sqrt{x}}} \end{aligned}$$

Domain of $f = \{x \mid x \geq 0 \text{ and } \cos\sqrt{x} \geq 0\}$
$= \left\{x \mid 0 \leq x \leq \frac{\pi^2}{4} \text{ or } \left[(4n - 1)\frac{\pi}{2}\right]^2 \leq x \leq \left[(4n + 1)\frac{\pi}{2}\right]^2 \text{ for some } n \in \{1, 2, 3, \ldots\}\right\}$
Domain of $f' = \{x \mid x > 0 \text{ and } \cos\sqrt{x} > 0\}$
$= \left\{x \mid 0 < x < \frac{\pi^2}{4} \text{ or } \left[(4n - 1)\frac{\pi}{2}\right]^2 < x < \left[(4n + 1)\frac{\pi}{2}\right]^2 \text{ for some } n \in \{1, 2, 3, \ldots\}\right\}$

41. $f(x) = \cos\sqrt{x} + \sqrt{\cos x} \Rightarrow$

$$\begin{aligned} f'(x) &= -\sin\sqrt{x}\tfrac{1}{2}x^{-1/2} + \tfrac{1}{2}(\cos x)^{-1/2}(-\sin x) \\ &= -\frac{\sin\sqrt{x}}{2\sqrt{x}} - \frac{\sin x}{2\sqrt{\cos x}} \end{aligned}$$

Domain of $f = \{x \mid x \geq 0 \text{ and } \cos x \geq 0\}$
$= \left\{x \mid 0 \leq x \leq \frac{\pi}{2} \text{ or } (4n - 1)\frac{\pi}{2} \leq x \leq (4n + 1)\frac{\pi}{2} \text{ for some } n \in \{1, 2, 3, \ldots\}\right\}$
Domain of $f' = \{x \mid x > 0 \text{ and } \cos x > 0\}$
$= \left\{x \mid 0 < x < \frac{\pi}{2} \text{ or } (4n - 1)\frac{\pi}{2} < x < (4n + 1)\frac{\pi}{2} \text{ for some } n \in \{1, 2, 3, \ldots\}\right\}$

42. $x = \sqrt{t} - t,\ y = t^3 - t \Rightarrow \dfrac{dy}{dt} = 3t^2 - 1$,
$\dfrac{dx}{dt} = \dfrac{1}{2\sqrt{t}} - 1$, and

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 1}{1/(2\sqrt{t}) - 1} = \frac{(3t^2 - 1)(2\sqrt{t})}{1 - 2\sqrt{t}}$$

43. $x = t\ln t,\ y = \sin^2 t \Rightarrow \dfrac{dy}{dt} = 2\sin t\cos t$,
$\dfrac{dx}{dt} = t\left(\dfrac{1}{t}\right) + (\ln t)\cdot 1 = 1 + \ln t$, and
$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2\sin t\cos t}{1 + \ln t}$

44. $x = t^2 + t,\ y = t^2 - t;\ t = 0.\ \dfrac{dy}{dt} = 2t - 1,\ \dfrac{dx}{dt} = 2t + 1$,
so $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t - 1}{2t + 1}$. When $t = 0$, $x = y = 0$ and $\dfrac{dy}{dx} = -1$. An equation of the tangent is $y - 0 = (-1)(x - 0)$ or $y = -x$.

45. $x = t\sin t,\ y = t\cos t;\ t = \pi.\ \dfrac{dy}{dt} = \cos t - t\sin t$,
$\dfrac{dx}{dt} = \sin t + t\cos t$, and $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\cos t - t\sin t}{\sin t + t\cos t}$.
When $t = \pi$, $(x, y) = (0, -\pi)$ and $\dfrac{dy}{dx} = \dfrac{-1}{-\pi} = \dfrac{1}{\pi}$, so an equation of the tangent is $y + \pi = \frac{1}{\pi}(x - 1)$ or $y = \frac{1}{\pi}x - \pi$.

46. $x = t^2 + t,\ y = \sqrt{t};\ t = 4.\ \dfrac{dy}{dt} = \dfrac{1}{2\sqrt{t}},\ \dfrac{dx}{dt} = 2t + 1$, so
$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{1}{2\sqrt{t}(2t + 1)}$. When $t = 4$,
$(x, y) = (20, 2)$ and $\dfrac{dy}{dx} = \dfrac{1}{36}$, so an equation of the tangent is $y - 2 = \frac{1}{36}(x - 20)$ or $y = \frac{1}{36}x + \frac{13}{9}$.

47. $x = 2\sin\theta,\ y = 3\cos\theta;\ \theta = \frac{\pi}{4}.\ \dfrac{dx}{d\theta} = 2\cos\theta$,
$\dfrac{dy}{d\theta} = -3\sin\theta,\ \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = -\frac{3}{2}\tan\theta$. When $\theta = \frac{\pi}{4}$,
$(x, y) = \left(\sqrt{2}, \frac{3\sqrt{2}}{2}\right)$, and $dy/dx = -\frac{3}{2}$, so an equation of the tangent is $y - \frac{3\sqrt{2}}{2} = -\frac{3}{2}(x - \sqrt{2})$ or $y = -\frac{3}{2}x + 3\sqrt{2}$.