3.3 Rates of Change in the Natural and Social Sciences

A Click here for answers.

S Click here for solutions.

1–5 A particle moves according to a law of motion $s = f(t)$, $t \geqslant 0$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t.
(b) What is the velocity after 2 s?
(c) When is the particle at rest?
(d) When is the particle moving in the positive direction?
(e) Find the total distance traveled during the first 4 s.
(f) Draw a diagram like Figure 2 to illustrate the motion of the particle.

1. $f(t) = t^2 - 6t + 9$

2. $f(t) = 4t^3 - 9t^2 + 6t + 2$

3. $f(t) = 2t^3 - 9t^2 + 12t + 1$

4. $f(t) = t^4 - 4t + 1$

5. $s = \sqrt{t}(5 - 5t + 2t^2)$

6. The interest rate on U.S. treasury bills is a function of time. The following table gives midyear values of this function $I(t)$ over a nine-year period (as a percent per year).

t	$I(t)$	t	$I(t)$
1983	8.62	1988	6.67
1984	9.57	1989	8.11
1985	7.49	1990	7.51
1986	5.97	1991	5.41
1987	5.83	1992	3.46

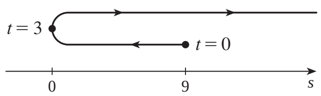
(a) Use a graphing calculator or computer to model these data by a fourth-degree polynomial.
(b) Use part (a) to find a model for $I'(t)$.
(c) Estimate the rate of change of interest rates in 1988 and 1991.
(d) Graph the data points and the models for I and I'.

Answers

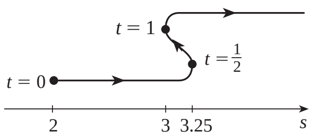
E Click here for exercises.

S Click here for solutions.

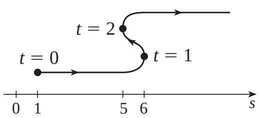
1. (a) $2t - 6$ (b) -2 ft/s (c) $t = 3$
(d) $t > 3$ (e) 10 ft
(f)



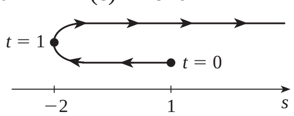
2. (a) $12t^2 - 18t + 6$ (b) 18 ft/s (c) $t = \frac{1}{2}, 1$
(d) $0 \leqslant t < \frac{1}{2}, t > 1$ (e) 136.5 ft
(f)



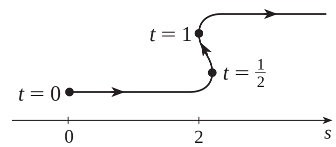
3. (a) $6t^2 - 18t + 12$ (b) 0 ft/s (c) $t = 1, 2$
(d) $0 \leqslant t < 1, t > 2$ (e) 34 ft
(f)



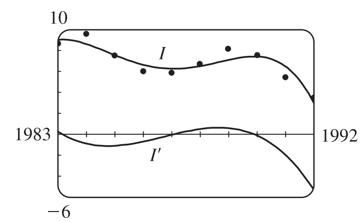
4. (a) $4t^3 - 4$ (b) 28 ft/s (c) $t = 1$
(d) $t > 1$ (e) 246 ft
(f)



5. (a) $\frac{5}{2}t^{-1/2}(1 - 3t + 2t^2)$ (b) $15\sqrt{2}/4$ ft/s (c) $t = \frac{1}{2}, 1$
(d) $0 \leqslant t < \frac{1}{2}, t > 1$ (e) $32 + 3\sqrt{2} - 2 \approx 34.24$ ft
(f)



6. (a) $I(t) = -0.0145512821t^4 + 115.636927t^3 - 344{,}605.8704t^2 + 456{,}421{,}256t - 2.266939 \times 10^{11}$
(b) $I'(t) = -0.0582051284t^3 + 346.910781t^2 - 689{,}211.7408t + 456{,}421{,}256$
(c) 0.5%/year, −2%/year
(d)



Solutions

E Click here for exercises.

A Click here for answers.

1. (a) $v(t) = f'(t) = 2t - 6$

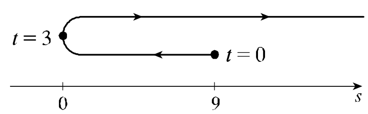
(b) $v(2) = 2(2) - 6 = -2$ ft/s

(c) It is at rest when $v(t) = 2t - 6 = 0 \quad \Leftrightarrow \quad t = 3$.

(d) It moves in the positive direction when $2t - 6 > 0 \quad \Leftrightarrow$ $t > 3$.

(e) Distance in positive direction $= |f(4) - f(3)| = |1 - 0| = 1$ ft
Distance in negative direction $= |f(3) - f(0)| = |0 - 9| = 9$ ft
Total distance traveled $= 1 + 9 = 10$ ft

(f)



2. (a) $v(t) = f'(t) = 12t^2 - 18t + 6$

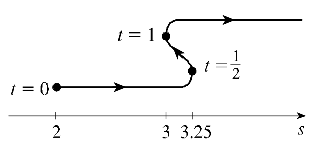
(b) $v(2) = 12(2)^2 - 18(2) + 6 = 18$ ft/s

(c) It is at rest when $v(t) = 12t^2 - 18t + 6 = 6(2t - 1)(t - 1) = 0 \quad \Rightarrow$ $t = \frac{1}{2}$ or 1.

(d) It moves in the positive direction when $v(t) = 6(2t - 1)(t - 1) > 0 \quad \Leftrightarrow \quad 0 \le t < \frac{1}{2}$ or $t > 1$.

(e) Distance in positive direction $= \left|f\left(\frac{1}{2}\right) - f(0)\right| + |f(4) - f(1)| = |3.25 - 2| + |138 - 3| = 136.25$ ft
Distance in negative direction $= \left|f(1) - f\left(\frac{1}{2}\right)\right| = |3 - 3.25| = 0.25$ ft
Total distance traveled $= 136.25 + 0.25 = 136.5$ ft

(f)



3. (a) $v(t) = f'(t) = 6t^2 - 18t + 12$

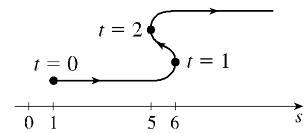
(b) $v(2) = 6(2)^2 - 18(2) + 12 = 0$ ft/s

(c) It is at rest when $v(t) = 6t^2 - 18t + 12 = 6(t - 1)(t - 2) = 0 \quad \Leftrightarrow$ $t = 1$ or 2.

(d) It moves in the positive direction when $6(t - 1)(t - 2) > 0 \quad \Leftrightarrow \quad 0 \le t < 1$ or $t > 2$.

(e) Distance in positive direction $= |f(4) - f(2)| + |f(1) - f(0)| = |33 - 5| + |6 - 1| = 33$ ft
Distance in negative direction $= |f(2) - f(1)| = |5 - 6| = 1$ ft
Total distance traveled $= 33 + 1 = 34$ ft

(f)



4. (a) $v(t) = f'(t) = 4t^3 - 4$

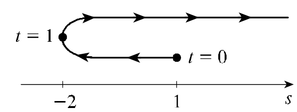
(b) $v(2) = 4(2)^3 - 4 = 28$ ft/s

(c) It is at rest when $v(t) = 4\left(t^3 - 1\right) = 4(t - 1)\left(t^2 + t + 1\right) = 0 \quad \Leftrightarrow$ $t = 1$.

(d) It moves in the positive direction when $4\left(t^3 - 1\right) > 0$ $\Leftrightarrow \quad t > 1$.

(e) Distance in positive direction $= |f(4) - f(1)| = |241 - (-2)| = 243$ ft
Distance in negative direction $= |f(1) - f(0)| = |-2 - 1| = 3$ ft
Total distance traveled $= 243 + 3 = 246$ ft

(f)



5. $s = f(t) = \sqrt{t}\left(5 - 5t + 2t^2\right) = 5t^{1/2} - 5t^{3/2} + 2t^{5/2}$

(a) $v(t) = f'(t) = \frac{5}{2}t^{-1/2} - \frac{15}{2}t^{1/2} + 5t^{3/2} = \frac{5}{2}t^{-1/2}\left(1 - 3t + 2t^2\right)$

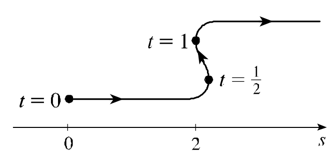
(b) $v(2) = \frac{5}{2}(2)^{-1/2}\left[1 - 3(2) + 2(2)^2\right] = 15/\left(2\sqrt{2}\right) = 15\sqrt{2}/4$ ft/s

(c) It is at rest when $v = 0 \quad \Leftrightarrow$ $1 - 3t + 2t^2 = (2t - 1)(t - 1) = 0 \quad \Leftrightarrow \quad t = \frac{1}{2}$ or 1.

(d) It moves in the positive direction when $v > 0 \quad \Leftrightarrow$ $(2t - 1)(t - 1) > 0 \quad \Leftrightarrow \quad 0 \le t < \frac{1}{2}$ or $t > 1$.

(e) Distance in positive direction $= \left|f\left(\frac{1}{2}\right) - f(0)\right| + |f(4) - f(1)| = \left|\frac{3}{\sqrt{2}}\right| + |34 - 2| = \left(32 + \frac{3}{2}\sqrt{2}\right)$ ft
Distance in negative direction $= \left|f(1) - f\left(\frac{1}{2}\right)\right| = \left|2 - \frac{3}{2}\sqrt{2}\right| = \left(\frac{3}{2}\sqrt{2} - 2\right)$ ft
Total distance traveled $= 32 + 3\sqrt{2} - 2 \approx 34.24$ft

(f)



6. (a) $I(t) = at^4 + bt^3 + ct^2 + dt + e$ for $1983 \leq t \leq 1992$, where $a = -0.0145512821$, $b = 115.636927$, $c = -344{,}605.8704$, $d = 456{,}421{,}256$, and $e = -2.266939 \times 10^{11}$.

(b) Using the values in part (a), $I'(t) = 4at^3 + 3bt^2 + 2ct + d$.

(c) $I'(1988) \approx 0.49559$ and $I'(1991) \approx -1.95946$, so the interest rate was increasing at about $\frac{1}{2}$ percent per year in 1988 and decreasing at about 2 percent per year in 1991.

(d)