3.2 The Product and Quotient Rules

A Click here for answers.

S Click here for solutions.

1–15 Differentiate.

1. $h(x) = \dfrac{x + 2}{x - 1}$

2. $f(u) = \dfrac{1 - u^2}{1 + u^2}$

3. $G(s) = (s^2 + s + 1)(s^2 + 2)$

4. $g(x) = \left(1 + \sqrt{x}\right)(x - x^3)$

5. $H(x) = (x^3 - x + 1)(x^{-2} + 2x^{-3})$

6. $H(t) = e^t(1 + 3t^2 + 5t^4)$

7. $y = \dfrac{3t - 7}{t^2 + 5t - 4}$

8. $y = \dfrac{4t + 5}{2 - 3t}$

9. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

10. $y = \dfrac{u^2 - u - 2}{u + 1}$

11. $y = \dfrac{e^x}{x + e^x}$

12. $f(x) = \dfrac{x^5}{x^3 - 2}$

13. $s = \sqrt{t}\left(t^3 - \sqrt{t} + 1\right)$

14–18 Find an equation of the tangent line to the curve at the given point.

14. $y = x\sqrt{x}, \quad (1, 1)$

15. $y = \dfrac{x}{x - 3}, \quad (6, 2)$

16. $y = x + \dfrac{4}{x}, \quad (2, 4)$

17. $y = x^{5/2}, \quad (4, 32)$

18. $y = x + \sqrt{x}, \quad (1, 2)$

Answers

E Click here for exercises.

S Click here for solutions.

1. $h'(x) = -\dfrac{3}{(x-1)^2}$ **2.** $f'(u) = -\dfrac{4u}{(1+u^2)^2}$

3. $G'(s) = (2s+1)(s^2+2) + (s^2+s+1)(2s)$
$[= 4s^3 + 3s^2 + 6s + 2]$

4. $g'(x) = 1 - 3x^2 + \frac{3}{2}x^{1/2} - \frac{7}{2}x^{5/2}$

5. $H'(x) = 1 + x^{-2} + 2x^{-3} - 6x^{-4}$

6. $H'(t) = e^t(5t^4 + 20t^3 + 3t^2 + 6t + 1)$

7. $y' = \dfrac{-3t^2 + 14t + 23}{(t^2 + 5t - 4)^2}$ **8.** $y' = \dfrac{23}{(2-3t)^2}$

9. $y' = \frac{3}{2}\sqrt{x} + \dfrac{2}{\sqrt{x}} - \dfrac{3}{2x\sqrt{x}}$ **10.** $y' = 1$

11. $y' = \dfrac{e^x(x-1)}{(x+e^x)^2}$ **12.** $f'(x) = \dfrac{2x^4(x^3-5)}{(x^3-2)^2}$

13. $s' = \frac{7}{2}t^{5/2} - 1 + \dfrac{1}{2\sqrt{t}}$ **14.** $y = \frac{3}{2}x - \frac{1}{2}$

15. $x + 3y = 12$ **16.** $y = 4$ **17.** $y = 20x - 48$

18. $y = \frac{3}{2}x + \frac{1}{2}$

Solutions

E Click here for exercises.

A Click here for answers.

1. $h(x)=\dfrac{x+2}{x-1} \Rightarrow$

$$h'(x)=\frac{(x-1)(1)-(x+2)(1)}{(x-1)^2}=\frac{x-1-x-2}{(x-1)^2}=-\frac{3}{(x-1)^2}$$

2. $f(u)=\dfrac{1-u^2}{1+u^2} \Rightarrow$

$$f'(u)=\frac{(1+u^2)(-2u)-(1-u^2)(2u)}{(1+u^2)^2}=\frac{-2u-2u^3-2u+2u^3}{(1+u^2)^2}=-\frac{4u}{(1+u^2)^2}$$

3. $G(s)=(s^2+s+1)(s^2+2) \Rightarrow$

$$G'(s)=(s^2+s+1)(2s)+(s^2+2)(2s+1)=2s^3+2s^2+2s+2s^3+s^2+4s+2=4s^3+3s^2+6s+2$$

4. $g(x)=(1+\sqrt{x})(x-x^3)=x-x^3+x^{3/2}-x^{7/2} \Rightarrow$

$g'(x)=1-3x^2+\frac{3}{2}x^{1/2}-\frac{7}{2}x^{5/2}$

Another Method: Use the Product Rule.

5. $H(x)=(x^3-x+1)(x^{-2}+2x^{-3})$

$=(x^3-x+1)(x^{-2})+(x^3-x+1)(2x^{-3})$

$=x-x^{-1}+x^{-2}+2-2x^{-2}+2x^{-3}$

$=2+x-x^{-1}-x^{-2}+2x^{-3}$

$\Rightarrow \quad H'(x)=1+x^{-2}+2x^{-3}-6x^{-4}$

Another Method: Use the Product Rule.

6. $H(t)=e^t(1+3t^2+5t^4) \Rightarrow$

$$H'(t)=e^t(6t+20t^3)+(1+3t^2+5t^4)e^t=e^t(5t^4+20t^3+3t^2+6t+1)$$

7. $y=\dfrac{3t-7}{t^2+5t-4} \Rightarrow$

$$y'=\frac{(t^2+5t-4)(3)-(3t-7)(2t+5)}{(t^2+5t-4)^2}=\frac{-3t^2+14t+23}{(t^2+5t-4)^2}$$

8. $y=\dfrac{4t+5}{2-3t} \Rightarrow$

$$y'=\frac{(2-3t)(4)-(4t+5)(-3)}{(2-3t)^2}=\frac{23}{(2-3t)^2}$$

9. $y=\dfrac{x^2+4x+3}{\sqrt{x}}=x^{3/2}+4x^{1/2}+3x^{-1/2} \Rightarrow$

$$y'=\frac{3}{2}x^{1/2}+4\left(\frac{1}{2}\right)x^{-1/2}+3\left(-\frac{1}{2}\right)x^{-3/2}=\frac{3}{2}\sqrt{x}+\frac{2}{\sqrt{x}}-\frac{3}{2x\sqrt{x}}$$

Another Method: Use the Quotient Rule.

10. $y=\dfrac{u^2-u-2}{u+1}=\dfrac{(u-2)(u+1)}{u+1}=u-2$ for $u\neq -1$.

$$y'=\frac{d}{du}(u-2)=1$$

11. $y=\dfrac{e^x}{x+e^x} \Rightarrow$

$$y'=\frac{(x+e^x)(e^x)-e^x(1+e^x)}{(x+e^x)^2}=\frac{e^x(x+e^x-1-e^x)}{(x+e^x)^2}=\frac{e^x(x-1)}{(x+e^x)^2}$$

12. $f(x)=\dfrac{x^5}{x^3-2} \Rightarrow$

$$f'(x)=\frac{(x^3-2)(5x^4)-x^5(3x^2)}{(x^3-2)^2}=\frac{2x^4(x^3-5)}{(x^3-2)^2}$$

13. $s=\sqrt{t}(t^3-\sqrt{t}+1)=t^{7/2}-t+t^{1/2} \Rightarrow$

$s'=\frac{7}{2}t^{5/2}-1+\dfrac{1}{2\sqrt{t}}$.

Another Method: Use the Product Rule.

14. $y=x\sqrt{x}=x^{3/2} \Rightarrow y'=\frac{3}{2}x^{1/2}$. At $(1,1)$, $y'=\frac{3}{2}$, and an equation of the tangent line is $y-1=\frac{3}{2}(x-1)$, or $y=\frac{3}{2}x-\frac{1}{2}$.

15. $y=f(x)=\dfrac{x}{x-3} \Rightarrow$

$f'(x)=\dfrac{(x-3)1-x(1)}{(x-3)^2}=\dfrac{-3}{(x-3)^2}$. So the slope of the tangent line at $(6,2)$ is $f'(6)=-\frac{1}{3}$ and its equation is $y-2=-\frac{1}{3}(x-6)$ or $x+3y=12$.

16. $y=f(x)=x+\dfrac{4}{x} \Rightarrow f'(x)=1-\dfrac{4}{x^2}$. So the slope of the tangent line at $(2,4)$ is $f'(2)=0$ and its equation is $y-4=0$ or $y=4$.

17. $y = f(x) = x^{5/2} \Rightarrow f'(x) = \frac{5}{2}x^{3/2}$. So the slope of the tangent line at $(4, 32)$ is $f'(4) = 20$ and its equation is $y - 32 = 20(x - 4)$ or $y = 20x - 48$.

18. $y = f(x) = x + \sqrt{x} \Rightarrow f'(x) = 1 + \frac{1}{2}x^{-1/2}$. So the slope of the tangent line at $(1, 2)$ is $f'(1) = 1 + \frac{1}{2}(1) = \frac{3}{2}$ and its equation is $y - 2 = \frac{3}{2}(x - 1)$ or $y = \frac{3}{2}x + \frac{1}{2}$ or $3x - 2y + 1 = 0$.