3.1 Derivatives of Polynomials and Exponential Functions

A Click here for answers.

S Click here for solutions.

1–11 Differentiate the function.

1. $f(x) = x^2 - 10x + 100$

2. $g(x) = x^{100} + 50x + 1$

3. $s(t) = t^3 - 3t^2 + 12t$

4. $F(x) = (16x)^3$

5. $H(s) = (s/2)^5$

6. $y = \sqrt{5x}$

7. $y = x^{4/3} - x^{2/3}$

8. $y = 3x + 2e^x$

9. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2}$

10. $y = x + \sqrt[5]{x^2}$

11. $v = x\sqrt{x} + \dfrac{1}{x^2\sqrt{x}}$

12–14 Find $f'(x)$. Compare the graphs of f and f' and use them to explain why your answer is reasonable.

12. $f(x) = 2x^2 - x^4$

13. $f(x) = x - 3x^{1/3}$

14. $f(x) = x^2 + 2e^x$

15. (a) By zooming in on the graph of $f(x) = x^{2/5}$, estimate the value of $f'(2)$.
(b) Use the Power Rule to find the exact value of $f'(2)$ and compare with your estimate in part (a).

16. (a) By zooming in on the graph of $f(x) = x^2 - 2e^x$, estimate the value of $f'(1)$.
(b) Find the exact value of $f'(1)$ and compare with your estimate in part (a).

17–20 Find an equation of the tangent line to the curve at the given point. Illustrate by graphing the curve and the tangent line on the same screen.

17. $y = x + \dfrac{4}{x}$, $(2, 4)$

18. $y = x^{5/2}$, $(4, 32)$

19. $y = x + \sqrt{x}$, $(1, 2)$

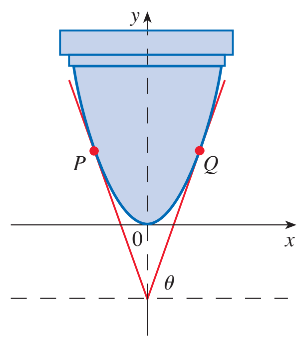
20. $y = x^2 + 2e^x$, $(0, 2)$

21. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent is horizontal.

22. For what values of x does the graph of $f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent?

23. At what point on the curve $y = x\sqrt{x}$ is the tangent line parallel to the line $3x - y + 6 = 0$?

24. A manufacturer of cartridges for stereo systems has designed a stylus with parabolic cross-section as shown in the figure. The equation of a parabola is $y = 16x^2$, where x and y are measured in millimeters. If the stylus sits in a record groove whose sides make an angle of θ with the horizontal direction, where $\tan\theta = 1.75$, find the points of contact P and Q of the stylus with the groove.



25. The **normal line** to a curve C at a point P is, by definition, the line that passes through P and is perpendicular to the tangent line to C at P. Find an equation of the normal line to the curve $y = \sqrt[3]{x}$ at the point $(-8, -2)$. Sketch the curve and its normal line.

26. At what point on the curve $y = x^4$ does the normal line have slope 16?

Answers

E Click here for exercises.

S Click here for solutions.

1. $f'(x) = 2x - 10$ **2.** $g'(x) = 100x^{99} + 50$

3. $s'(t) = 3t^2 - 6t + 12$ **4.** $F'(x) = 12{,}288x^2$

5. $H'(s) = \frac{5}{32}s^4$ **6.** $y' = \dfrac{\sqrt{5}}{2\sqrt{x}}$

7. $y' = \frac{4}{3}x^{1/3} - \frac{2}{3}x^{-1/3}$ **8.** $y' = 3 + 2e^x$

9. $y' = -\dfrac{B}{x^2} - 2\dfrac{C}{x^3}$ **10.** $y' = 1 + \dfrac{2}{5\sqrt[5]{x^3}}$

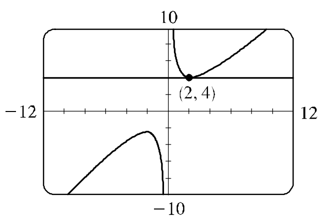
11. $v' = \frac{3}{2}\sqrt{x} - \dfrac{5}{2x^3\sqrt{x}}$ **12.** $4x - 4x^3$

13. $1 - x^{-2/3}$ **14.** $2x + 2e^x$

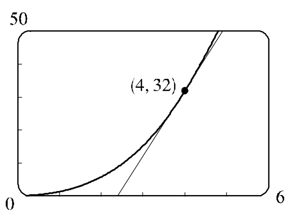
15. (a) 0.264 (b) $2^{2/5}/5 \approx 0.263902$

16. (a) -3.4455 (b) $2 - 2e \approx -3.436564$

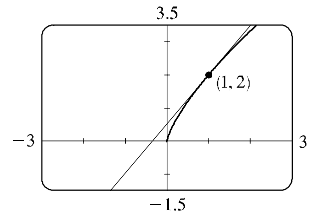
17. $y = 4$



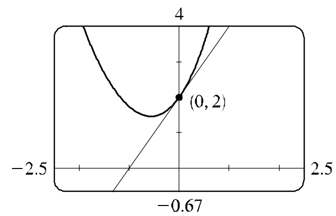
18. $y = 20x - 48$



19. $y = \frac{3}{2}x + \frac{1}{2}$



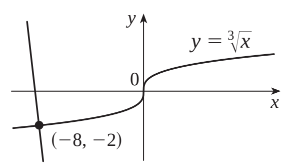
20. $y = 2x + 2$



21. $(1, 0), \left(-\frac{1}{3}, \frac{32}{27}\right)$ **22.** $x = \frac{1}{2}\left(1 \pm \sqrt{5}\right)$

23. $(4, 8)$ **24.** $\left(\pm\frac{7}{128}, \frac{49}{1024}\right)$

25. $12x + y + 98 = 0$



26. $\left(-\frac{1}{4}, \frac{1}{256}\right)$

Solutions

E Click here for exercises.

A Click here for answers.

1. $f(x) = x^2 - 10x + 100 \quad \Rightarrow \quad f'(x) = 2x - 10$

2. $g(x) = x^{100} + 50x + 1 \quad \Rightarrow \quad g'(x) = 100x^{99} + 50$

3. $s(t) = t^3 - 3t^2 + 12t \quad \Rightarrow$
$s'(t) = 3t^{3-1} - 3\left(2t^{2-1}\right) + 12 = 3t^2 - 6t + 12$

4. $F(x) = (16x)^3 = 4096x^3 \quad \Rightarrow$
$F'(x) = 4096\left(3x^2\right) = 12{,}288x^2$

5. $H(s) = (s/2)^5 = s^5/2^5 = \frac{1}{32}s^5 \quad \Rightarrow$
$H'(s) = \frac{1}{32}\left(5s^{5-1}\right) = \frac{5}{32}s^4$

6. $y = \sqrt{5x} = \sqrt{5}x^{1/2} \quad \Rightarrow \quad y' = \sqrt{5}\left(\frac{1}{2}\right)x^{-1/2} = \dfrac{\sqrt{5}}{2\sqrt{x}}$

7. $y = x^{4/3} - x^{2/3} \quad \Rightarrow \quad y' = \frac{4}{3}x^{1/3} - \frac{2}{3}x^{-1/3}$

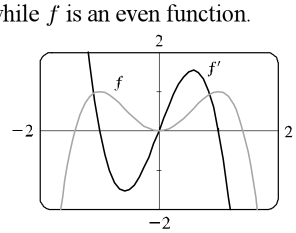
8. $y = 3x + 2e^x \quad \Rightarrow \quad y' = 3 + 2e^x$

9. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2} = A + Bx^{-1} + Cx^{-2} \quad \Rightarrow$
$y' = -Bx^{-2} - 2Cx^{-3} = -\dfrac{B}{x^2} - 2\dfrac{C}{x^3}$

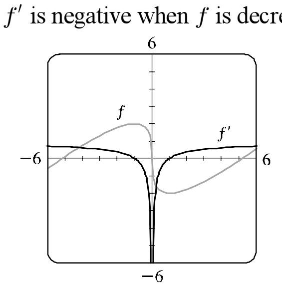
10. $y = x + \sqrt[5]{x^2} = x + x^{2/5} \quad \Rightarrow$
$y' = 1 + \frac{2}{5}x^{-3/5} = 1 + \dfrac{2}{5\sqrt[5]{x^3}}$

11. $v = x\sqrt{x} + \dfrac{1}{x^2\sqrt{x}} = x^{3/2} + x^{-5/2} \quad \Rightarrow$
$v' = \frac{3}{2}x^{1/2} - \frac{5}{2}x^{-7/2} = \frac{3}{2}\sqrt{x} - \dfrac{5}{2x^3\sqrt{x}}$

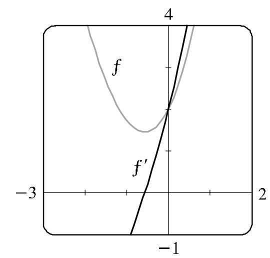
12. $f(x) = 2x^2 - x^4 \quad \Rightarrow \quad f'(x) = 4x - 4x^3$. Notice that $f'(x) = 0$ when f has a horizontal tangent and that f' is an odd function while f is an even function.



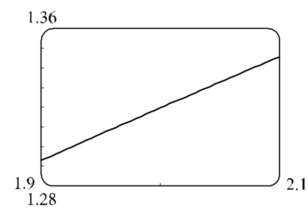
13. $f(x) = x - 3x^{1/3} \quad \Rightarrow$
$f'(x) = 1 - x^{-2/3} = 1 - 1/x^{2/3}$. Note that $f'(x) = 0$ when f has a horizontal tangent, f' is positive when f is increasing, and f' is negative when f is decreasing.



14. $f(x) = x^2 + 2e^x \quad \Rightarrow \quad f'(x) = 2x + 2e^x$. Note that $f'(x) = 0$ when f has a horizontal tangent, f' is positive when f is increasing, and f' is negative when f is decreasing.



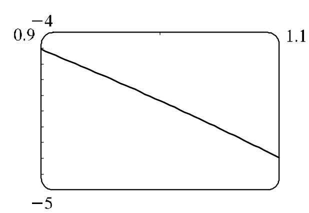
15. (a)



The endpoints of f in this graph are about $(1.9, 1.2927)$ and $(2.1, 1.3455)$. An estimate of $f'(2)$ is $\frac{1.3455 - 1.2927}{2.1 - 1.9} = \frac{0.0528}{0.2} = 0.264$.

(b) $f(x) = x^{2/5} \quad \Rightarrow \quad f'(x) = \frac{2}{5}x^{-3/5} = 2\Big/\left(5x^{3/5}\right)$.
$f'(2) = 2\Big/\left(5 \cdot 2^{3/5}\right) \approx 0.263902$.

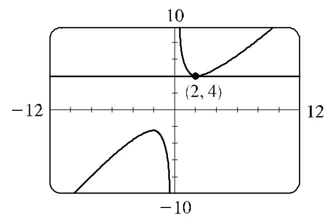
16. (a)



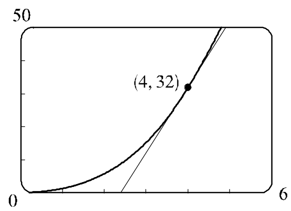
The endpoints of f in this graph are about $(0.9, -4.1092)$ and $(1.1, -4.7983)$. An estimate of $f'(1)$ is $\frac{-4.7983 - (-4.1092)}{1.1 - 0.9} = \frac{-0.6891}{0.2} = -3.4455$.

(b) $f(x) = x^2 - 2e^x \quad \Rightarrow f'(x) = 2x - 2e^x$.
$f'(1) = 2 - 2e \approx -3.436564$.

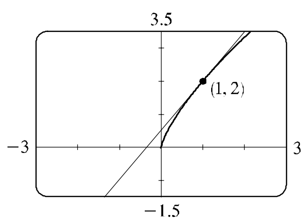
17. $y = f(x) = x + \dfrac{4}{x} \quad\Rightarrow\quad f'(x) = 1 - \dfrac{4}{x^2}$. So the slope of the tangent line at $(2, 4)$ is $f'(2) = 0$ and its equation is $y - 4 = 0$ or $y = 4$.



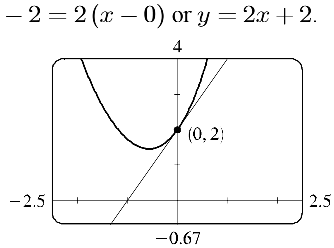
18. $y = f(x) = x^{5/2} \quad\Rightarrow\quad f'(x) = \frac{5}{2}x^{3/2}$. So the slope of the tangent line at $(4, 32)$ is $f'(4) = 20$ and its equation is $y - 32 = 20(x - 4)$ or $y = 20x - 48$.



19. $y = f(x) = x + \sqrt{x} \quad\Rightarrow\quad f'(x) = 1 + \frac{1}{2}x^{-1/2}$. So the slope of the tangent line at $(1, 2)$ is $f'(1) = 1 + \frac{1}{2}(1) = \frac{3}{2}$ and its equation is $y - 2 = \frac{3}{2}(x - 1)$ or $y = \frac{3}{2}x + \frac{1}{2}$.



20. $y = f(x) = x^2 + 2e^x \quad\Rightarrow\quad f'(x) = 2x + 2e^x$. So the slope of the tangent line at $(0, 2)$ is $f'(0) = 2e^0 = 2$ and its equation is $y - 2 = 2(x - 0)$ or $y = 2x + 2$.



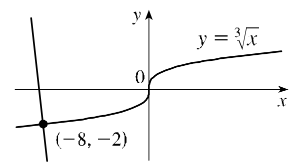
21. $y = x^3 - x^2 - x + 1$ has a horizontal tangent when $y' = 3x^2 - 2x - 1 = 0$. $(3x + 1)(x - 1) = 0 \quad\Leftrightarrow\quad x = 1$ or $-\frac{1}{3}$. Therefore, the points are $(1, 0)$ and $\left(-\frac{1}{3}, \frac{32}{27}\right)$.

22. $f(x) = 2x^3 - 3x^2 - 6x + 87$ has a horizontal tangent when $f'(x) = 6x^2 - 6x - 6 = 0 \quad\Leftrightarrow\quad x^2 - x - 1 = 0 \quad\Leftrightarrow\quad x = \frac{1 \pm \sqrt{5}}{2}$.

23. $y = x\sqrt{x} = x^{3/2} \quad\Rightarrow\quad y' = \frac{3}{2}\sqrt{x}$, so the tangent line is parallel to $3x - y + 6 = 0$ when $\frac{3}{2}\sqrt{x} = 3 \quad\Leftrightarrow\quad \sqrt{x} = 2 \quad\Leftrightarrow\quad x = 4$. So the point is $(4, 8)$.

24. The sides of the groove must be tangent to the parabola $y = 16x^2$. $y' = 32x = 1.75$ when $x = \frac{1.75}{32} = \frac{7}{128}$, which implies that $y = 16\left(\frac{7}{128}\right)^2 = \frac{49}{1024}$. Therefore the points of contact are $\left(\pm\frac{7}{128}, \frac{49}{1024}\right)$.

25. $y = f(x) = \sqrt[3]{x} = x^{1/3} \quad\Rightarrow\quad f'(x) = \frac{1}{3}x^{-2/3}$, so the tangent line at $(-8, -2)$ has slope $f'(-8) = \frac{1}{12}$. The normal line has slope $-1/\left(\frac{1}{12}\right) = -12$ and equation $y + 2 = -12(x + 8) \quad\Leftrightarrow\quad 12x + y + 98 = 0$.



26. If the normal line has slope 16, then the tangent has slope $-\frac{1}{16}$, so $y' = 4x^3 = -\frac{1}{16} \quad\Rightarrow\quad x^3 = -\frac{1}{64} \quad\Rightarrow\quad x = -\frac{1}{4}$. The point is $\left(-\frac{1}{4}, \frac{1}{256}\right)$.