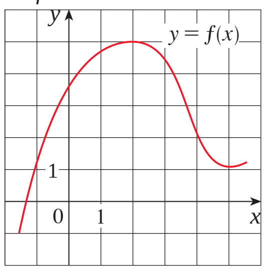
2.8 The Derivative as a Function

A Click here for answers.

S Click here for solutions.

1. Use the given graph to estimate the value of each derivative. Then sketch the graph of f'.

(a) $f'(0)$
(b) $f'(1)$
(c) $f'(2)$
(d) $f'(3)$
(e) $f'(4)$
(f) $f'(5)$



2–6 ▥ Trace or copy the graph of the given function f. (Assume that the axes have equal scales.) Then use the method of Example 1 to sketch the graph of f' below it.

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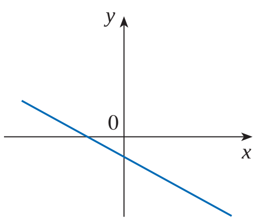

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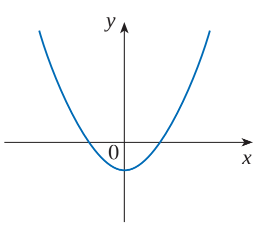

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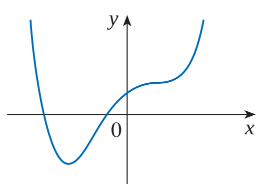

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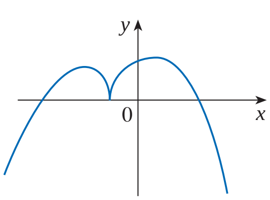

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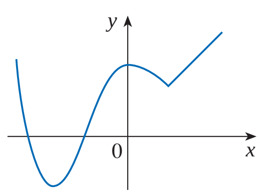


7–11 ▥ Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

7. $f(x) = 5x + 3$

8. $f(x) = 5 - 4x + 3x^2$

9. $f(x) = x^3 - x^2 + 2x$

10. $f(x) = \dfrac{x + 1}{x - 1}$

11. $G(x) = \dfrac{4 - 3x}{2 + x}$

12. A function g is given by the data in the table. Find approximate values for $g'(x)$ when $x = 2, 4, 6, 8, 10, 12$, and 14. then sketch the graph of g'.

x	0	2	4	6	8	10	12	14	16
$g(x)$	1.8	4.7	6.3	6.8	3.9	2.5	2.0	1.8	1.7

13. Let the smoking rate among high-school seniors at time t be $S(t)$. The table (from the Institute of Social Research, University of Michigan) gives the percentage of seniors who reported that they had smoked one or more cigarettes per day during the past 30 days.

t	$S(t)$	t	$S(t)$
1978	27.5	1988	18.1
1980	21.4	1990	19.1
1982	21.0	1992	17.2
1984	18.7	1994	19.4
1986	18.7	1996	22.2

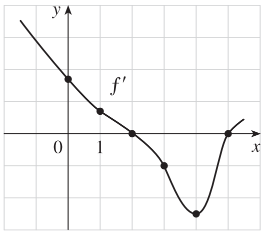
(a) What is the meaning of $S'(t)$? What are its units?
(b) Construct a table of values for $S'(t)$.
(c) Graph S and S'.
(d) How would it be possible to get more accurate values for $S'(t)$?

Answers

E Click here for exercises.

S Click here for solutions.

1. (a) 1.8 (b) 0.8 (c) 0 (d) −1 (e) −2.5 (f) 0



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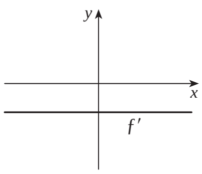

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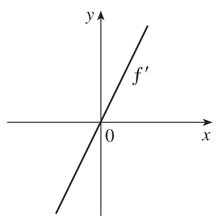

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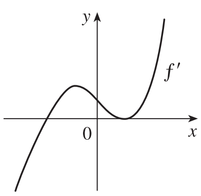

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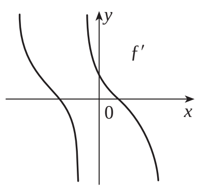

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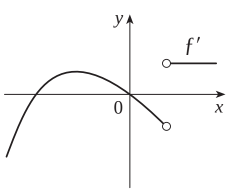


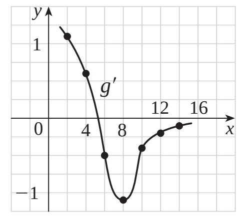
7. $f'(x) = 5, \mathbb{R}, \mathbb{R}$ **8.** $f'(x) = 6x - 4, \mathbb{R}, \mathbb{R}$

9. $f'(x) = 3x^2 - 2x + 2, \mathbb{R}, \mathbb{R}$

10. $f'(x) = \dfrac{-2}{(x-1)^2}, \{x \mid x \neq 1\}, \{x \mid x \neq 1\}$

11. $G'(x) = -10/(2 + x)^2, \{x \mid x \neq -2\}, \{x \mid x \neq -2\}$

12. 1.1, 0.6, −0.5, −1.1, −0.4, −0.2, −0.1

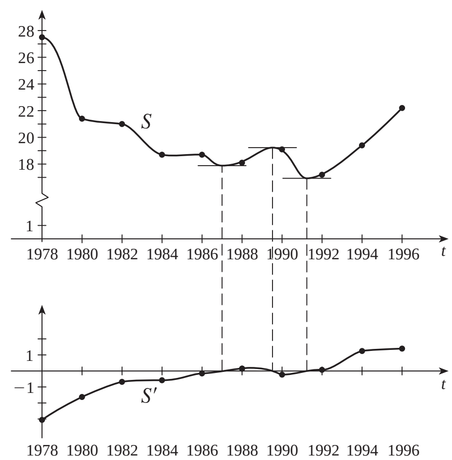


13. (a) The rate at which the smoking rate is changing with respect to time; percent per year

(b)

t	$S'(t)$	t	$S'(t)$
1978	−3.05	1988	0.10
1980	−1.625	1990	−0.225
1982	−0.675	1992	0.075
1984	−0.575	1994	1.25
1986	−0.15	1996	1.40

(c)



(d) By obtaining data for the odd-numbered years

Solutions

E Click here for exercises.

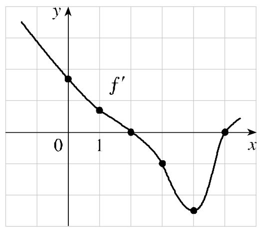
A Click here for answers.

1. From the graph of f, it appears that

(a) $f'(0) \approx 1.8$ (b) $f'(1) \approx 0.8$

(c) $f'(2) \approx 0$ (d) $f'(3) \approx -1$

(e) $f'(4) \approx -2.5$ (f) $f'(5) \approx 0$



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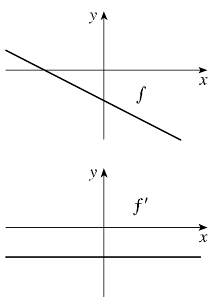

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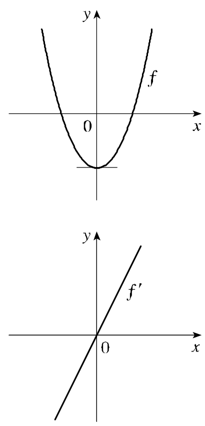

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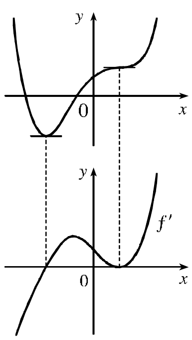

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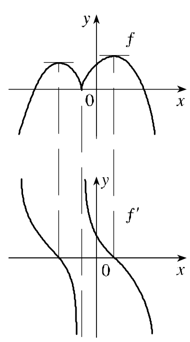

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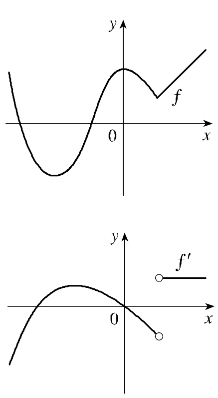


7. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{[5(x+h)+3] - (5x+3)}{h}$$

$$= \lim_{h\to 0} \frac{5h}{h} = \lim_{h\to 0} 5 = 5$$

Domain of f = domain of $f' = \mathbb{R}$.

8. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{\left[5-4(x+h)+3(x+h)^2\right] - \left[5-4x+3x^2\right]}{h}$$

$$= \lim_{h\to 0} \frac{\left[5-4x-4h+3x^2+6xh+3h^2\right] - \left[5-4x+3x^2\right]}{h}$$

$$= \lim_{h\to 0} \frac{-4h+6xh+3h^2}{h}$$

$$= \lim_{h\to 0} (-4+6x+3h) = -4+6x$$

Domain of f = domain of $f' = \mathbb{R}$.

9. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{\left[(x+h)^3-(x+h)^2+2(x+h)\right] - \left(x^3-x^2+2x\right)}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3-2xh-h^2+2h}{h}$$

$$= \lim_{h\to 0} \left(3x^2+3xh+h^2-2x-h+2\right)$$

$$= 3x^2 - 2x + 2$$

Domain of f = domain of $f' = \mathbb{R}$.

10. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{\dfrac{x+h+1}{x+h-1} - \dfrac{x+1}{x-1}}{h}$$

$$= \lim_{h\to 0} \frac{(x+h+1)(x-1) - (x+1)(x+h-1)}{h(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{-2h}{h(x+h-1)(x-1)}$$

$$= \lim_{h\to 0} \frac{-2}{(x+h-1)(x-1)} = \frac{-2}{(x-1)^2}$$

Domain of f = domain of $f' = \{x \mid x \neq 1\}$.

11. $G'(x) = \lim_{h\to 0} \frac{G(x+h) - G(x)}{h}$

$$= \lim_{h\to 0} \frac{\frac{4-3(x+h)}{2+(x+h)} - \frac{4-3x}{2+x}}{h}$$

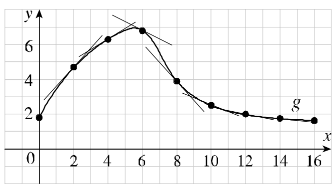
$$= \lim_{h\to 0} \frac{(4-3x-3h)(2+x) - (4-3x)(2+x+h)}{h(2+x+h)(2+x)}$$

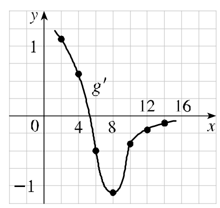
$$= \lim_{h\to 0} \frac{-10h}{h(2+x+h)(2+x)}$$

$$= \lim_{h\to 0} \frac{-10}{(2+x+h)(2+x)} = -\frac{10}{(2+x)^2}$$

Domain of G = domain of $G' = \{x \mid x \neq -2\}$.

12. We plot the points given by the data in the table, then sketch the rough shape of the curve. To estimate the derivative $f'(x)$, we draw the tangent line to the curve at x. It appears that $g'(2) \approx 1.1$, $g'(4) \approx 0.6$, $g'(6) \approx -0.5$, $g'(8) \approx -1.1$, $g'(10) \approx -0.4$, $g'(12) \approx -0.2$, and $g'(14) \approx -0.1$.





13. (a) $S'(t)$ is the rate at which the smoking rate is changing with respect to time. Its units are percent per year.

(b) To find $S'(t)$, we use

$$\lim_{h\to 0} \frac{S(t+h) - S(t)}{h} \approx \frac{S(t+h) - S(t)}{h}$$

for small values of h.

For 1978:

$$S'(1978) \approx \frac{S(1980) - S(1978)}{1980 - 1978} = \frac{21.4 - 27.5}{2} = -3.05$$

For 1980: We estimate $S'(1980)$ both using $h = -2$ and using $h = 2$, and then average the two results to obtain a final estimate.

$h = -2 \Rightarrow$

$$S'(1980) \approx \frac{S(1978) - S(1980)}{1978 - 1980} = \frac{27.5 - 21.4}{-2} = -3.05$$

$h = 2 \Rightarrow$

$$S'(1980) \approx \frac{S(1982) - S(1980)}{1982 - 1980} = \frac{21.0 - 21.4}{2} = -0.20$$

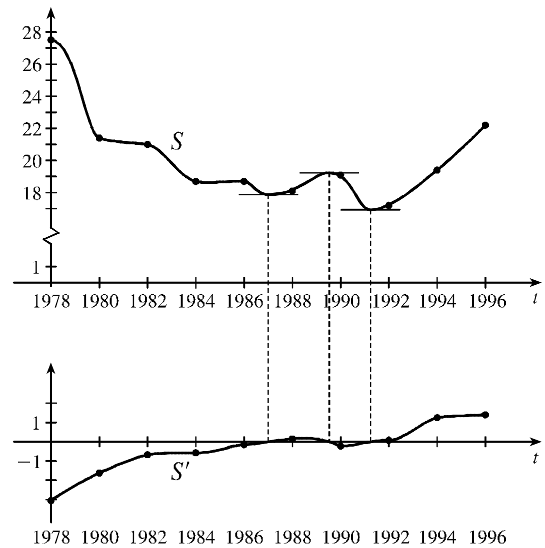
So we estimate that

$S'(1980) \approx \frac{1}{2}(-3.05 - 0.20) = -1.625$.

t	1978	1980	1982	1984	1986
$S'(t)$	−3.05	−1.625	−0.675	−0.575	−0.15

t	1988	1990	1992	1994	1996
$S'(t)$	0.10	−0.225	0.075	1.25	1.40

(c)



(d) We could get more accurate values for $S'(t)$ by obtaining data for the odd-numbered years.