2.7 Derivatives

A Click here for answers.

S Click here for solutions.

1–7 Find $f'(a)$.

1. $f(x) = 1 + x - 2x^2$

2. $f(x) = x^3 + 3x$

3. $f(x) = \dfrac{x}{2x - 1}$

4. $f(x) = \dfrac{x}{x^2 - 1}$

5. $f(x) = \dfrac{2}{\sqrt{3 - x}}$

6. $f(x) = \sqrt{x - 1}$

7. $f(x) = \sqrt{3x + 1}$

8–13 Each limit represents the derivative of some function f at some number a. State such an f and a in each case.

8. $\displaystyle\lim_{h \to 0} \frac{\sqrt{1 + h} - 1}{h}$

9. $\displaystyle\lim_{h \to 0} \frac{(2 + h)^3 - 8}{h}$

10. $\displaystyle\lim_{x \to 1} \frac{x^9 - 1}{x - 1}$

11. $\displaystyle\lim_{x \to 3\pi} \frac{\cos x + 1}{x - 3\pi}$

12. $\displaystyle\lim_{t \to 0} \frac{\sin\left(\frac{\pi}{2} + t\right) - 1}{t}$

13. $\displaystyle\lim_{x \to 0} \frac{3^x - 1}{x}$

14. A function f is given by the data in the table. Find approximate values for $f'(x)$ when $x = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$, and 0.7.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	5.0	4.1	4.0	4.6	5.5	6.2	6.5	6.1	4.7

15. Let $C(t)$ be the amount of U.S. cash per capita in circulation at time t. The table, supplied by the Treasury Department, gives values of $C(t)$ as of June 30 of the specified year. Interpret and estimate the value of $C'(1980)$.

t	1960	1970	1980	1990
$C(t)$	\$177	\$265	\$571	\$1063

Answers

E Click here for exercises.

S Click here for solutions.

1. $1 - 4a$ **2.** $\dfrac{1}{2\sqrt{a-1}}$ **3.** $-1/(2a-1)^2$

4. $-\dfrac{a^2+1}{(a^2-1)^2}$ **5.** $1/(3-a)^{3/2}$ **6.** $\dfrac{1}{2\sqrt{a-1}}$

7. $\dfrac{3}{2\sqrt{3a+1}}$ **8.** $f(x) = \sqrt{x}$, $a = 1$

9. $f(x) = x^3$, $a = 2$ **10.** $f(x) = x^9$, $a = 1$

11. $f(x) = \cos x$, $a = 3\pi$ **12.** $f(x) = \sin x$, $a = \pi/2$

13. $f(x) = 3^x$, $a = 0$ **14.** $-5, 4, 8, 9, 5, -0.5, -8$

15. The rate at which the cash per capita in circulation is changing in dollars per year; \$39.90/year

Solutions

E Click here for exercises.

A Click here for answers.

1. $$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{1+(a+h)-2(a+h)^2-\left(1+a-2a^2\right)}{h} = \lim_{h\to 0}\frac{h-4ah-2h^2}{h} = \lim_{h\to 0}(1-4a-2h) = 1-4a$$

2. $$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{(a+h)^3+3(a+h)-\left(a^3+3a\right)}{h} = \lim_{h\to 0}\frac{3a^2h+3ah^2+h^3+3h}{h} = \lim_{h\to 0}\left(3a^2+3ah+h^2+3\right) = 3a^2+3\frac{1}{\sqrt{a+h-1}+\sqrt{a-1}} = \frac{1}{\sqrt{a-1}+\sqrt{a-1}} = \frac{1}{2\sqrt{a-1}}$$

3. $$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{\dfrac{a+h}{2(a+h)-1}-\dfrac{a}{2a-1}}{h} = \lim_{h\to 0}\frac{(a+h)(2a-1)-a(2a+2h-1)}{h(2a+2h-1)(2a-1)} = \lim_{h\to 0}\frac{-h}{h(2a+2h-1)(2a-1)} = \lim_{h\to 0}\frac{-1}{(2a+2h-1)(2a-1)} = -\frac{1}{(2a-1)^2}$$

4. $$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{\dfrac{a+h}{(a+h)^2-1}-\dfrac{a}{a^2-1}}{h} = \lim_{h\to 0}\frac{(a+h)\left(a^2-1\right)-a\left(a^2+2ah+h^2-1\right)}{h\left(a^2-1\right)\left(a^2+2ah+h^2-1\right)} = \lim_{h\to 0}\frac{h\left(-a^2-1-ah\right)}{h\left(a^2-1\right)\left(a^2+2ah+h^2-1\right)} = \lim_{h\to 0}\frac{-a^2-1-ah}{\left(a^2-1\right)\left(a^2+2ah+h^2-1\right)} = \frac{-a^2-1}{\left(a^2-1\right)\left(a^2-1\right)} = -\frac{a^2+1}{\left(a^2-1\right)^2}$$

5.

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{\dfrac{2}{\sqrt{3-(a+h)}}-\dfrac{2}{\sqrt{3-a}}}{h} = \lim_{h\to 0}\frac{2\left(\sqrt{3-a}-\sqrt{3-a-h}\right)}{h\sqrt{3-a-h}\sqrt{3-a}} = \lim_{h\to 0}\frac{2\left(\sqrt{3-a}-\sqrt{3-a-h}\right)}{h\sqrt{3-a-h}\sqrt{3-a}}\cdot\frac{\sqrt{3-a}+\sqrt{3-a-h}}{\sqrt{3-a}+\sqrt{3-a-h}} = \lim_{h\to 0}\frac{2\left[3-a-(3-a-h)\right]}{h\sqrt{3-a-h}\sqrt{3-a}\left(\sqrt{3-a}+\sqrt{3-a-h}\right)} = \lim_{h\to 0}\frac{2}{\sqrt{3-a-h}\sqrt{3-a}\left(\sqrt{3-a}+\sqrt{3-a-h}\right)} = \frac{2}{\sqrt{3-a}\sqrt{3-a}\left(2\sqrt{3-a}\right)} = \frac{1}{(3-a)^{3/2}}$$

6.

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim\frac{\sqrt{a+h-1}-\sqrt{a-1}}{h} = \lim_{h\to 0}\frac{\sqrt{a+h-1}-\sqrt{a-1}}{h}\cdot\frac{\sqrt{a+h-1}+\sqrt{a-1}}{\sqrt{a+h-1}+\sqrt{a-1}} = \lim_{h\to 0}\frac{(a+h-1)-(a-1)}{h\left(\sqrt{a+h-1}+\sqrt{a-1}\right)} = \lim_{h\to 0}\frac{1}{\sqrt{a+h-1}+\sqrt{a-1}} = \frac{1}{\sqrt{a-1}+\sqrt{a-1}} = \frac{1}{2\sqrt{a-1}}$$

7.

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{\sqrt{3(a+h)+1}-\sqrt{3a+1}}{h} = \lim_{h\to 0}\frac{\left(\sqrt{3a+3h+1}-\sqrt{3a+1}\right)\left(\sqrt{3a+3h+1}+\sqrt{3a+1}\right)}{h\left(\sqrt{3a+3h+1}+\sqrt{3a+1}\right)} = \lim_{h\to 0}\frac{(3a+3h+1)-(3a+1)}{h\left(\sqrt{3a+3h+1}+\sqrt{3a+1}\right)} = \lim_{h\to 0}\frac{3}{\sqrt{3a+3h+1}+\sqrt{3a+1}} = \frac{3}{2\sqrt{3a+1}}$$

8. By Equation 1, $\lim\limits_{h\to 0} \dfrac{\sqrt{1+h}-1}{h} = f'(1)$, where $f(x) = \sqrt{x}$.
Or: $f'(0)$, where $f(x) = \sqrt{1+x}$

9. By Equation 1, $\lim\limits_{h\to 0} \dfrac{(2+h)^3-8}{h} = f'(2)$, where $f(x) = x^3$.

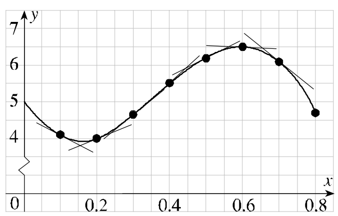
10. By Equation 3, $\lim\limits_{x\to 1} \dfrac{x^9-1}{x-1} = f'(1)$, where $f(x) = x^9$.

11. By Equation 3, $\lim\limits_{x\to 3\pi} \dfrac{\cos x+1}{x-3\pi} = f'(3\pi)$, where $f(x) = \cos x$.

12. By Equation 1, $\lim\limits_{t\to 0} \dfrac{\sin\left(\frac{\pi}{2}+t\right)-1}{t} = f'\left(\frac{\pi}{2}\right)$, where $f(x) = \sin x$.

13. By Equation 3, $\lim\limits_{x\to 0} \dfrac{3^x-1}{x} = f'(0)$, where $f(x) = 3^x$.

14. We plot the points given by the data in the table, then sketch the rough shape of the curve. To estimate the derivative $f'(x)$, we draw the tangent line to the curve at x. It appears that $f'(0.1) \approx -5$, $f'(0.2) \approx 4$, $f'(0.3) \approx 8$, $f'(0.4) \approx 9$, $f'(0.5) \approx 5$, $f'(0.6) \approx -0.5$, and $f'(0.7) \approx -8$.



15. $C'(1980)$ is the rate of change of U.S. cash per capita in circulation with respect to time. To estimate the value of $C'(1980)$, we will average the difference quotients obtained using the years 1970 and 1990.

Let $A = \dfrac{C(1970)-C(1980)}{1970-1980} = \dfrac{265-571}{-10} = 30.6$ and $B = \dfrac{C(1990)-C(1980)}{1990-1980} = \dfrac{1063-571}{10} = 49.2$. Then

$$C'(1980) = \lim_{t\to 1980} \frac{C(t)-C(1980)}{t-1980} \approx \frac{A+B}{2} = 39.9 \text{ dollars per year}$$