

## ||| 2.6 Tangents, Velocities, and Other Rates of Change

**A** [Click here for answers.](#)

**S** [Click here for solutions.](#)

**1–4** ||| Find an equation of the tangent line to the curve at the given point.

1.  $y = 1 - 2x - 3x^2$ ,  $(-2, -7)$

2.  $y = 1/\sqrt{x}$ ,  $(1, 1)$

3.  $y = 1/x^2$ ,  $(-2, \frac{1}{4})$

4.  $y = x/(1 - x)$ ,  $(0, 0)$

5. (a) Find the slope of the tangent to the curve  $y = 1/\sqrt{5 - 2x}$  at the point where  $x = a$ .

(b) Find equations of the tangent lines at the points  $(2, 1)$  and  $(-2, \frac{1}{3})$ .



(c) Graph the curve and both tangents on a common screen.

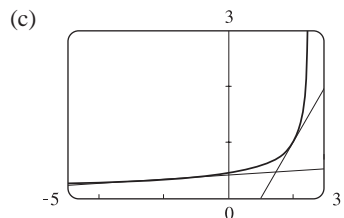
## ||| Answers

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1.  $y = 10x + 13$     2.  $y = -\frac{1}{2}x + \frac{3}{2}$

3.  $y = \frac{1}{4}x + \frac{3}{4}$     4.  $y = x$

5. (a)  $(5 - 2a)^{-3/2}$     (b)  $y = x - 1$ ,  $y = \frac{1}{27}x + \frac{11}{27}$



## Solutions

**E** Click here for exercises.

1. Using (1),

$$\begin{aligned} m &= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \\ &= \lim_{x \rightarrow -2} \frac{(1 - 2x - 3x^2) - (-7)}{x - (-2)} \\ &= \lim_{x \rightarrow -2} \frac{-3x^2 - 2x + 8}{x + 2} \\ &= \lim_{x \rightarrow -2} \frac{(-3x + 4)(x + 2)}{x + 2} \\ &= \lim_{x \rightarrow -2} (-3x + 4) = 10 \end{aligned}$$

Thus, an equation of the tangent is  $y + 7 = 10(x + 2)$  or  $y = 10x + 13$ .

*Alternate Solution:* Using (2),

$$\begin{aligned} m &= \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(-2 + h) - f(-2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[1 - 2(-2 + h) - 3(-2 + h)^2] - (-7)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(-3h^2 + 10h - 7) + 7}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(-3h + 10)}{h} \\ &= \lim_{h \rightarrow 0} (-3h + 10) = 10 \end{aligned}$$

2. Using (1),

$$\begin{aligned} m &= \lim_{x \rightarrow 1} \frac{1/\sqrt{x} - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{-(\sqrt{x} - 1)}{\sqrt{x}(\sqrt{x} - 1)(\sqrt{x} + 1)} \\ &= \lim_{x \rightarrow 1} \frac{-1}{\sqrt{x}(\sqrt{x} + 1)} = -\frac{1}{2} \end{aligned}$$

Thus, an equation of the tangent line is  $y - 1 = -\frac{1}{2}(x - 1)$  or  $y = -\frac{1}{2}x + \frac{3}{2}$ .

3. Using (1),

$$\begin{aligned} m &= \lim_{x \rightarrow -2} \frac{1/x^2 - \frac{1}{4}}{x - (-2)} = \lim_{x \rightarrow -2} \frac{4 - x^2}{4x^2(x + 2)} \\ &= \lim_{x \rightarrow -2} \frac{(2 - x)(2 + x)}{4x^2(x + 2)} = \lim_{x \rightarrow -2} \frac{2 - x}{4x^2} = \frac{1}{4} \end{aligned}$$

Thus, an equation of the tangent line is  $y - \frac{1}{4} = \frac{1}{4}(x + 2)$   
 $\Rightarrow y = \frac{1}{4}x + \frac{3}{4}$ .

**A** Click here for answers.

4. Using (1),

$$\begin{aligned} m &= \lim_{x \rightarrow 0} \frac{x/(1-x) - 0}{x - 0} = \lim_{x \rightarrow 0} \frac{x}{x(1-x)} \\ &= \lim_{x \rightarrow 0} \frac{1}{1-x} = 1 \end{aligned}$$

Thus, an equation of the tangent line is  $y - 0 = 1(x - 0)$   
 $\Rightarrow y = x$ .

5. (a) Using (1),

$$\begin{aligned} m &= \lim_{x \rightarrow a} \frac{\frac{1}{\sqrt{5-2x}} - \frac{1}{\sqrt{5-2a}}}{x - a} \\ &= \lim_{x \rightarrow a} \frac{\sqrt{5-2a} - \sqrt{5-2x}}{(x-a)\sqrt{5-2x}\sqrt{5-2a}} \\ &= \lim_{x \rightarrow a} \frac{2(x-a)}{(x-a)\sqrt{5-2x}(5-2a)(\sqrt{5-2a} + \sqrt{5-2x})} \\ &= \lim_{x \rightarrow a} \frac{2}{(x-a)\sqrt{5-2x}(5-2a)(\sqrt{5-2a} + \sqrt{5-2x})} \\ &= \frac{2}{2(5-2a)^{3/2}} = (5-2a)^{-3/2} \end{aligned}$$

(b) At  $(2, 1)$ :  $m = [5 - 2(2)]^{-3/2} = 1 \Leftrightarrow$   
 $y - 1 = 1(x - 2) \Leftrightarrow y = x - 1$ .

At  $(-2, \frac{1}{3})$ :  $m = [5 - 2(-2)]^{-3/2} = \frac{1}{27} \Leftrightarrow$   
 $y - \frac{1}{3} = \frac{1}{27}[x - (-2)] \Leftrightarrow y = \frac{1}{27}x + \frac{11}{27}$ .

(c)

