2.2 The Limit of a Function

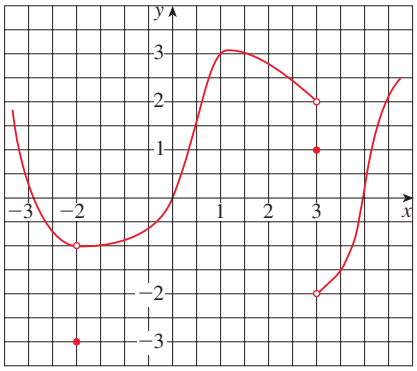
A Click here for answers.

1. For the function f whose graph is given, state the value of the given quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 3^-} f(x)$ (c) $\lim_{x\to 3^+} f(x)$

(d) $\lim_{x\to 3} f(x)$ (e) $f(3)$ (f) $\lim_{x\to -2^-} f(x)$

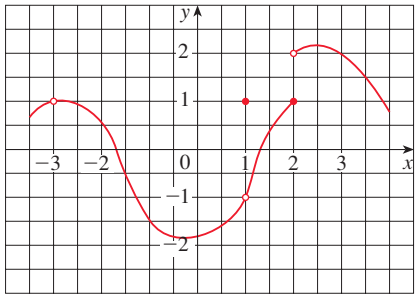
(g) $\lim_{x\to -2^+} f(x)$ (h) $\lim_{x\to -2} f(x)$ (i) $f(-2)$



2. For the function f whose graph is given, state the value of the limit, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 3} f(x)$ (b) $\lim_{x\to 1} f(x)$ (c) $\lim_{x\to -3} f(x)$

(d) $\lim_{x\to 2^-} f(x)$ (e) $\lim_{x\to 2^+} f(x)$ (f) $\lim_{x\to 2} f(x)$

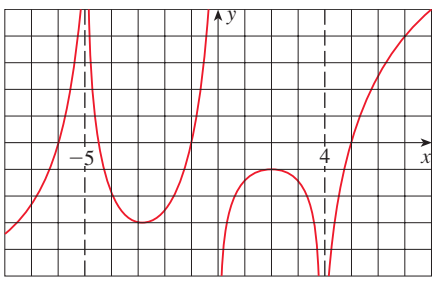


3. For the function g whose graph is shown, state the following.

(a) $\lim_{x\to -6} g(x)$ (b) $\lim_{x\to 0^-} g(x)$

(c) $\lim_{x\to 0^+} g(x)$ (d) $\lim_{x\to 4} g(x)$

(e) The equations of the vertical asymptotes.



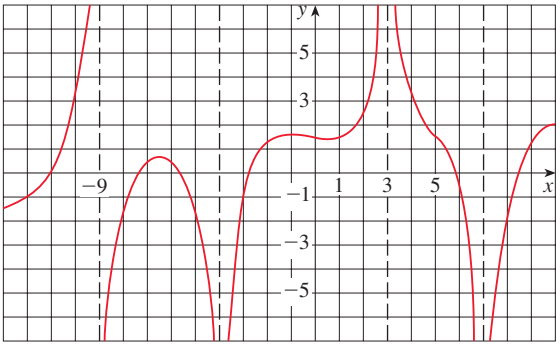
S Click here for solutions.

4. For the function f whose graph is shown, state the following.

(a) $\lim_{x\to 3} f(x)$ (b) $\lim_{x\to 7} f(x)$ (c) $\lim_{x\to -4} f(x)$

(d) $\lim_{x\to -9^-} f(x)$ (e) $\lim_{x\to -9^+} f(x)$

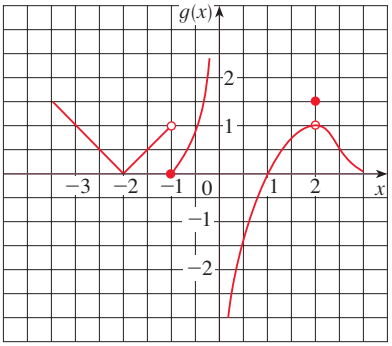
(f) The equations of the vertical asymptotes



5. State the value of the limit, if it exists, from the given graph.

(a) $\lim_{x\to 1} g(x)$ (b) $\lim_{x\to 0} g(x)$ (c) $\lim_{x\to 2} g(x)$

(d) $\lim_{x\to -2} g(x)$ (e) $\lim_{x\to -1^-} g(x)$ (f) $\lim_{x\to -1} g(x)$



6–11 Evaluate the function at the given numbers (correct to six decimal places). Use the results to guess the value of the limit, or explain why it does not exist.

6. $g(x) = \dfrac{x-1}{x^3-1}$;

$x = 0.2, 0.4, 0.6, 0.8, 0.9, 0.99, 1.8, 1.6, 1.4, 1.2, 1.1, 1.01$;

$\lim_{x\to 1} \dfrac{x-1}{x^3-1}$

7. $g(x) = \dfrac{1-x^2}{x^2+3x-10}$;

$x = 3, 2.1, 2.01, 2.001, 2.0001, 2.00001$;

$\lim_{x\to 2^+} \dfrac{1-x^2}{x^2+3x-10}$

8. $F(x) = \dfrac{(1/\sqrt{x}) - \frac{1}{5}}{x - 25}$;

$x = 26, 25.5, 25.1, 25.05, 25.01, 24, 24.5, 24.9, 24.95, 24.99$;

$\lim\limits_{x \to 25} \dfrac{(1/\sqrt{x}) - \frac{1}{5}}{x - 25}$

9. $F(t) = \dfrac{\sqrt[3]{t} - 1}{\sqrt{t} - 1}$;

$t = 1.5, 1.2, 1.1, 1.01, 1.001$;

$\lim\limits_{t \to 1} \dfrac{\sqrt[3]{t} - 1}{\sqrt{t} - 1}$

10. $f(x) = \dfrac{1 - \cos x}{x^2}$;

$x = 1, 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, 0.01$;

$\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$

11. $g(x) = \sqrt{x} \ln x$;

$x = 1, 0.5, 0.1, 0.05, 0.01, 0.005, 0.001$;

$\lim\limits_{x \to 0^+} \sqrt{x} \ln x$

12–13 Determine the infinite limit.

12. $\lim\limits_{x \to 3} \dfrac{1}{(x - 3)^8}$

13. $\lim\limits_{x \to 1^+} \dfrac{x + 1}{x \sin \pi x}$

14. (a) By graphing the function $f(x) = (\tan 4x)/x$ and zooming in toward the point where the graph crosses the y-axis, estimate the value of $\lim_{x \to 0} f(x)$.

(b) Check your answer in part (a) by evaluating $f(x)$ for values of x that approach 0.

15. (a) Estimate the value of

$$\lim_{x \to 0} \frac{6^x - 2^x}{x}$$

by graphing the function $y = (6^x - 2^x)/x$. State your answer correct to two decimal places.

(b) Check your answer in part (a) by evaluating $f(x)$ for values of x that approach 0.

Answers

E Click here for exercises.

S Click here for solutions.

1. (a) 3 (b) 2 (c) -2 (d) Does not exist (e) 1 (f) -1 (g) -1 (h) -1 (i) -3

2. (a) 2 (b) -1 (c) 1 (d) 1 (e) 2 (f) Does not exist

3. (a) 0 (b) ∞ (c) $-\infty$ (d) $-\infty$ (e) $x = -5$, $x = 0$, $x = 4$

4. (a) ∞ (b) $-\infty$ (c) $-\infty$ (d) ∞ (e) $-\infty$ (e) $x = -9$, $x = -4$, $x = 3$, $x = 7$

5. (a) 0 (b) Does not exist (c) 1 (d) 0 (e) 1 (f) Does not exist

6. 0.806452, 0.641026, 0.510204, 0.409836, 0.369004, 0.336689, 0.165563, 0.193798, 0.229358, 0.274725, 0.302115, 0.330022; $\frac{1}{3}$

7. -1, -4.8028, -43.368, -429.08, -4286.2, -42858; $-\infty$

8. -0.003884, -0.003941, -0.003988, -0.003994, -0.003999, -0.004124, -0.004061, -0.004012, -0.004006, -0.004001; -0.004

9. 0.643905, 0.656488, 0.661358, 0.666114, 0.666611; $\frac{2}{3}$

10. 0.459698, 0.489670, 0.493369, 0.496261, 0.498336, 0.499583, 0.499896, 0.499996; 0.5

11. 0, -0.490129, -0.728141, -0.669866, -0.460517, -0.374648, -0.218442; 0

12. ∞ **13.** $-\infty$ **14.** 4 **15.** (a) 1.10s

Solutions

E Click here for exercises.

A Click here for answers.

1. (a) $\lim_{x \to 1} f(x) = 3$

(b) $\lim_{x \to 3^-} f(x) = 2$

(c) $\lim_{x \to 3^+} f(x) = -2$

(d) $\lim_{x \to 3} f(x)$ doesn't exist because the limits in part (b) and part (c) are not equal.

(e) $f(3) = 1$

(f) $\lim_{x \to -2^-} f(x) = -1$

(g) $\lim_{x \to -2^+} f(x) = -1$

(h) $\lim_{x \to -2} f(x) = -1$

(i) $f(-2) = -3$

2. (a) $\lim_{x \to 3} f(x) = 2$

(b) $\lim_{x \to 1} f(x) = -1$

(c) $\lim_{x \to -3} f(x) = 1$

(d) $\lim_{x \to 2^-} f(x) = 1$

(e) $\lim_{x \to 2^+} f(x) = 2$

(f) $\lim_{x \to 2} f(x)$ doesn't exist because the limits in part (d) and part (e) are not equal.

3. (a) $\lim_{x \to -6} g(x) = 0$

(b) $\lim_{x \to 0^-} g(x) = \infty$

(c) $\lim_{x \to 0^+} g(x) = -\infty$

(d) $\lim_{x \to 4} g(x) = -\infty$

(e) The equations of the vertical asymptotes: $x = -5$, $x = 0$, $x = 4$

4. (a) $\lim_{x \to 3} f(x) = \infty$

(b) $\lim_{x \to 7} f(x) = -\infty$

(c) $\lim_{x \to -4} f(x) = -\infty$

(d) $\lim_{x \to -9^-} f(x) = \infty$

(e) $\lim_{x \to -9^+} f(x) = -\infty$

(f) The equations of the vertical asymptotes: $x = -9$, $x = -4$, $x = 3$, $x = 7$

5. (a) $\lim_{x \to 1} g(x) = 0$

(b) $\lim_{x \to 0} g(x)$ does not exist

(c) $\lim_{x \to 2} g(x) = 1$

(d) $\lim_{x \to -2} g(x) = 0$

(e) $\lim_{x \to -1^-} g(x) = 1$

(f) $\lim_{x \to -1} g(x)$ does not exist

6. For $g(x) = \dfrac{x-1}{x^3-1}$:

x	$g(x)$
0.2	0.806452
0.4	0.641026
0.6	0.510204
0.8	0.409836
0.9	0.369004
0.99	0.336689

x	$g(x)$
1.8	0.165563
1.6	0.193798
1.4	0.229358
1.2	0.274725
1.1	0.302115
1.01	0.330022

It appears that $\lim_{x \to 1} \dfrac{x-1}{x^3-1} = 0.\overline{3} = \frac{1}{3}$.

7. For $g(x) = \dfrac{1-x^2}{x^2+3x-10}$:

x	$g(x)$
3	-1
2.1	-4.8028
2.01	-43.368
2.001	-429.08
2.0001	-4286.2
2.00001	-42858

It appears that $\lim_{x \to 2^+} \dfrac{1-x^2}{x^2+3x-10} = -\infty$.

8. For $F(x) = \dfrac{(1/\sqrt{x}) - \frac{1}{5}}{x-25}$:

x	$F(x)$
26	-0.003884
25.5	-0.003941
25.1	-0.003988
25.05	-0.003994
25.01	-0.003999

x	$F(x)$
24	-0.004124
24.5	-0.004061
24.9	-0.004012
24.95	-0.004006
24.99	-0.004001

It appears that $\lim_{x \to 25} F(x) = -0.004$.

9. For $F(t) = \dfrac{\sqrt[3]{t}-1}{\sqrt{t}-1}$:

t	$F(t)$
1.5	0.643905
1.2	0.656488
1.1	0.661358
1.01	0.666114
1.001	0.666611

It appears that $\lim\limits_{t\to 1} \dfrac{\sqrt[3]{t}-1}{\sqrt{t}-1} = 0.\overline{6} = \frac{2}{3}$.

10. For $f(x) = \dfrac{1-\cos x}{x^2}$:

x	$f(x)$
1	0.459698
0.5	0.489670
0.4	0.493369
0.3	0.496261
0.2	0.498336
0.1	0.499583
0.05	0.499896
0.01	0.499996

It appears that $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} = 0.5$.

11. For $g(x) = \sqrt{x}\ln x$:

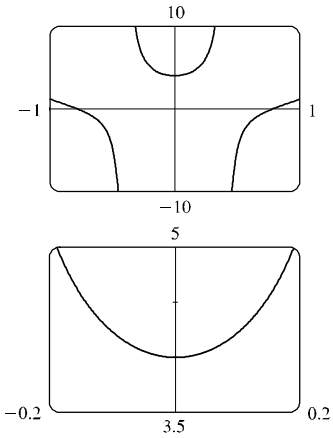
x	$g(x)$
1	0
0.5	−0.490129
0.1	−0.728141
0.05	−0.669866
0.01	−0.460517
0.005	−0.374648
0.001	−0.218442

As x gets smaller, $g(x)$ is increasing through negative values and slowly approaches 0. It appears that $\lim\limits_{x\to 0^+} \sqrt{x}\ln x = 0$.

12. $\lim\limits_{x\to 3} \dfrac{1}{(x-3)^8} = \infty$ since $(x-3) \to 0$ as $x \to 3$ and $\dfrac{1}{(x-3)^8} > 0$.

13. $\lim\limits_{x\to 1^+} \dfrac{x+1}{x\sin \pi x} = -\infty$ since $\dfrac{x+1}{x} \to 2$ as $x \to 1^+$ and $\sin \pi x \to 0$ through negative values as $x \to 1^+$.

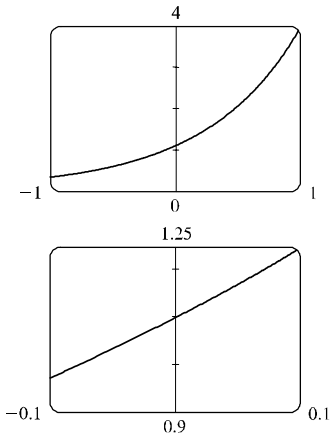
14. (a) From the following graphs, it seems that $\lim\limits_{x\to 0} \dfrac{\tan(4x)}{x} = 4$.



(b)

x	$f(x)$
±0.1	4.227932
±0.01	4.002135
±0.001	4.000021
±0.0001	4.000000

15. (a) From the following graphs, it seems that $\lim\limits_{x\to 0} \dfrac{6^x - 2^x}{x} \approx 1.10$.



(b)

x	$f(x)$
−0.01	1.085052
−0.001	1.097248
−0.0001	1.098476
0.0001	1.098749
0.001	1.099978
0.01	1.112353