

2.1 The Tangent and Velocity Problems

A [Click here for answers.](#)

- The point $P(4, 2)$ lies on the curve $y = \sqrt{x}$.
 - If Q is the point (x, \sqrt{x}) , use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x :

(i) 5	(ii) 4.5	(iii) 4.1
(iv) 4.01	(v) 4.001	(vi) 3
(vii) 3.5	(viii) 3.9	(ix) 3.99
(x) 3.999		
 - Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(4, 2)$.
 - Using the slope from part (b), find an equation of the tangent line to the curve at $P(4, 2)$.
- The point $P(0.5, 2)$ lies on the curve $y = 1/x$.
 - If Q is the point $(x, 1/x)$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x :

(i) 2	(ii) 1	(iii) 0.9
(iv) 0.8	(v) 0.7	(vi) 0.6
(vii) 0.55	(viii) 0.51	(ix) 0.45
(x) 0.49		
 - Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(0.5, 2)$.
 - Using the slope from part (b), find an equation of the tangent line to the curve at $P(0.5, 2)$.
 - Sketch the curve, two of the secant lines, and the tangent line.
- The point $P(1, 3)$ lies on the curve $y = 1 + x + x^2$.
 - If Q is the point $(x, 1 + x + x^2)$, find the slope of the secant line PQ for the following values of x :

(i) 2	(ii) 1.5	(iii) 1.1
(iv) 1.01	(v) 1.001	(vi) 0
(vii) 0.5	(viii) 0.9	(ix) 0.99
(x) 0.999		
 - Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(1, 3)$.
 - Using the slope from part (b), find an equation of the tangent line to the curve at $P(1, 3)$.

S [Click here for solutions.](#)

- The point $P(-1, 3)$ lies on the curve $y = 1 - 2x^3$.
 - If Q is the point $(x, 1 - 2x^3)$, find the slope of the secant line PQ for the following values of x :

(i) -2	(ii) -1.5	(iii) -1.1
(iv) -1.01	(v) -1.001	(vi) 0
(vii) -0.5	(viii) -0.9	(ix) -0.99
(x) -0.999		
 - Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(-1, 3)$.
 - Using the slope from part (b), find the equation of the tangent line to the curve at $P(-1, 3)$.
- The displacement (in meters) of a certain particle moving in a straight line is given by $s = t^2 + t$, where t is measured in seconds.
 - Find the average velocity over the following time periods:

(i) $[0, 2]$	(ii) $[0, 1]$
(iii) $[0, 0.5]$	(iv) $[0, 0.1]$
 - Find the instantaneous velocity when $t = 0$.
 - Draw the graph of s as a function of t and draw the secant lines whose slopes are the average velocities found in part (a).
 - Draw the tangent line whose slope is the instantaneous velocity from part (b).
- The experimental data in the table define y as a function of x .

x	0	1	2	3	4	5
y	2.6	2.0	1.1	1.3	2.1	3.5

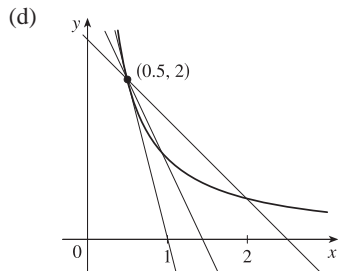
 - If P is the point $(3, 1.3)$, find the slopes of the secant lines PQ when Q is the point on the graph with $x = 0, 1, 2, 4,$ and 5 .
 - Estimate the slope of the tangent line at P by averaging the slopes of two secant lines.
 - Use a graph of the function to estimate the slope of the tangent line at P .

Answers

E [Click here for exercises.](#)

1. (a) (i) 0.236068 (ii) 0.242641 (iii) 0.248457
 (iv) 0.249844 (v) 0.249984 (vi) 0.267949 (vii) 0.258343
 (viii) 0.251582 (ix) 0.250156 (x) 0.250016
 (b) $\frac{1}{4}$ (c) $y = \frac{1}{4}x + 1$

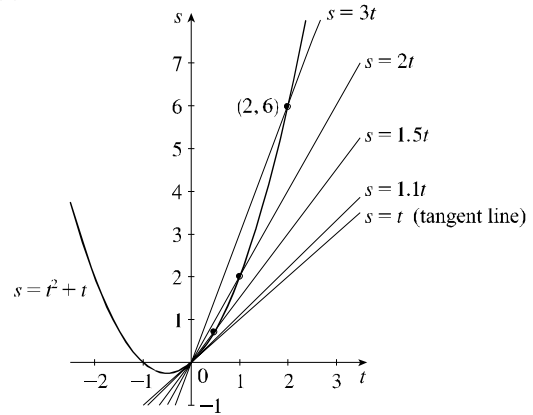
2. (a) (i) -1 (ii) -2 (iii) -2.222222 (iv) -2.5
 (v) -2.857143 (vi) -3.333333 (vii) -3.636364
 (viii) -3.921569 (ix) -4.444444 (x) -4.081633
 (b) -4 (c) $y = -4x + 4$



3. (a) (i) 4 (ii) 3.5 (iii) 3.1 (iv) 3.01 (v) 3.001
 (vi) 2 (vii) 2.5 (viii) 2.9 (ix) 2.99 (x) 2.999
 (b) 3 (c) $y = 3x$

S [Click here for solutions.](#)

4. (a) (i) -14 (ii) -9.5 (iii) -6.62 (iv) -6.0602
 (v) -6.006002 (vi) -2 (vii) -3.5 (viii) -5.42
 (ix) -5.9402 (x) -5.994002
 (b) -6 (c) $y = -6x - 3$
5. (a) (i) 3 m/s (ii) 2 m/s (iii) 1.5 m/s (iv) 1.1 m/s
 (b) 1 m/s
 (c), (d)



6. (a) -0.43, -0.35, 0.2, 0.8, 1.1 (b) 0.5 (c) 0.57

Solutions

[E Click here for exercises.](#)

1. For the curve $y = \sqrt{x}$ and the point $P(4, 2)$:

(a)

	x	Q	m_{PQ}
(i)	5	(5, 2.236068)	0.236068
(ii)	4.5	(4.5, 2.121320)	0.242641
(iii)	4.1	(4.1, 2.024846)	0.248457
(iv)	4.01	(4.01, 2.002498)	0.249844
(v)	4.001	(4.001, 2.000250)	0.249984
(vi)	3	(3, 1.732051)	0.267949
(vii)	3.5	(3.5, 1.870829)	0.258343
(viii)	3.9	(3.9, 1.974842)	0.251582
(ix)	3.99	(3.99, 1.997498)	0.250156
(x)	3.999	(3.999, 1.999750)	0.250016

(b) The slope appears to be $\frac{1}{4}$.

(c) $y - 2 = \frac{1}{4}(x - 4)$ or $y = \frac{1}{4}x + 1$.

2. For the curve $y = 1/x$ and the point $P(0.5, 2)$:

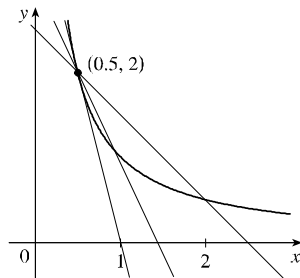
(a)

	x	Q	m_{PQ}
(i)	2	(2, 0.5)	-1
(ii)	1	(1, 1)	-2
(iii)	0.9	(0.9, 1.111111)	-2.222222
(iv)	0.8	(0.8, 1.25)	-2.5
(v)	0.7	(0.7, 1.428571)	-2.857143
(vi)	0.6	(0.6, 1.666667)	-3.333333
(vii)	0.55	(0.55, 1.818182)	-3.636364
(viii)	0.51	(0.51, 1.960784)	-3.921569
(ix)	0.45	(0.45, 2.222222)	-4.444444
(x)	0.49	(0.49, 2.040816)	-4.081633

(b) The slope appears to be -4 .

(c) $y - 2 = -4(x - 0.5)$ or $y = -4x + 4$

(d)



[A Click here for answers.](#)

3. For the curve $f(x) = 1 + x + x^2$ and the point $P(1, 3)$:

(a)

	x	Q	m_{PQ}
(i)	2	(2, 7)	4
(ii)	1.5	(1.5, 4.75)	3.5
(iii)	1.1	(1.1, 3.31)	3.1
(iv)	1.01	(1.01, 3.0301)	3.01
(v)	1.001	(1.001, 3.003001)	3.001
(vi)	0	(0, 1)	2
(vii)	0.5	(0.5, 1.75)	2.5
(viii)	0.9	(0.9, 2.71)	2.9
(ix)	0.99	(0.99, 2.9701)	2.99
(x)	0.999	(0.999, 2.997001)	2.999

(b) The slope appears to be 3.

(c) $y - 3 = 3(x - 1)$ or $y = 3x$

4. For the curve $y = 1 - 2x^3$ and the point $P(-1, 3)$:

(a)

	x	Q	m_{PQ}
(i)	-2	(-2, 17)	-14
(ii)	-1.5	(-1.5, 7.75)	-9.5
(iii)	-1.1	(-1.1, 3.662)	-6.62
(iv)	-1.01	(-1.01, 3.060602)	-6.0602
(v)	-1.001	(-1.001, 3.006006)	-6.006002
(vi)	0	(0, 1)	-2
(vii)	-0.5	(-0.5, 1.25)	-3.5
(viii)	-0.9	(-0.9, 2.458)	-5.42
(ix)	-0.99	(-0.99, 2.940598)	-5.9402
(x)	-0.999	(-0.999, 2.994006)	-5.994002

(b) The slope appears to be -6 .

(c) $y - 3 = -6(x + 1)$ or $6x + y + 3 = 0$

5. (a) The average velocity between times 0 and h is

$$\frac{s(h) - s(0)}{h} = \frac{h^2 + h - 0}{h} = h + 1.$$

(i) $[0, 2]$: $2 + 1 = 3$ m/s

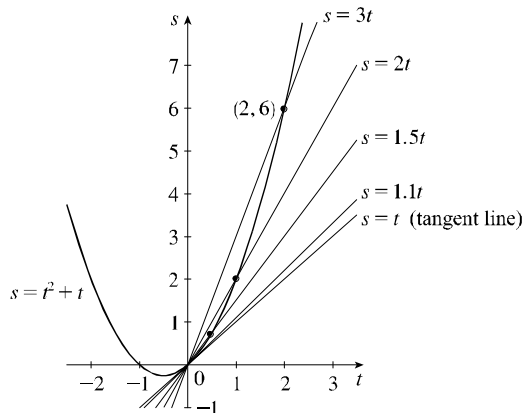
(ii) $[0, 1]$: $1 + 1 = 2$ m/s

(iii) $[0, 0.5]$: $0.5 + 1 = 1.5$ m/s

(iv) $[0, 0.1]$: $0.1 + 1 = 1.1$ m/s

(b) As h approaches 0, the velocity approaches 1 m/s.

(c), (d)



6. (a) Slopes of the secant lines:

x	m_{PQ}
0	$\frac{2.6 - 1.3}{0 - 3} \approx -0.43$
1	$\frac{2.0 - 1.3}{1 - 3} = -0.35$
2	$\frac{1.1 - 1.3}{2 - 3} = 0.2$
4	$\frac{2.1 - 1.3}{4 - 3} = 0.8$
5	$\frac{3.5 - 1.3}{5 - 3} = 1.1$

(b) We average the slopes of the two closest secant lines from part (a): $\frac{1}{2} (0.2 + 0.8) = 0.5$.

(c) Using the points $(0.6, 0)$ and $(5, 2.5)$ from the graph, the slope of the tangent line at P is about $\frac{2.5 - 0}{5 - 0.6} \approx 0.57$.

