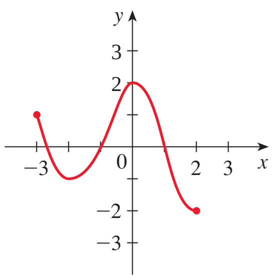
1.1 Four Ways to Represent a Function

A Click here for answers.

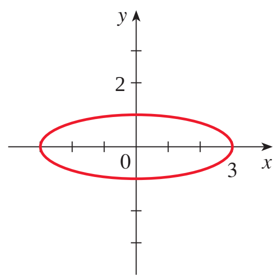
S Click here for solutions.

1–4 Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.

1.



2.



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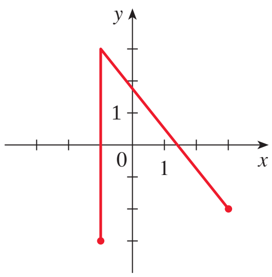

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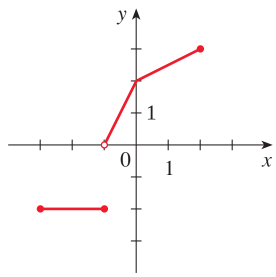


5. Temperature readings T (in °F) were recorded every two hours from midnight to noon in Atlanta, Georgia, on March 18, 1996. The time t was measured in hours from midnight.

t	0	2	4	6	8	10	12
T	58	57	53	50	51	57	61

(a) Use the readings to sketch a rough graph of T as a function of t.
(b) Use the graph to estimate the temperature at 11 A.M.

6. The population P (in thousands) of San Jose, California, from 1984 to 1994 is shown in the table. (Midyear estimates are given.)

t	1984	1986	1988	1990	1992	1994
P	695	716	733	782	800	817

(a) Draw a graph of P as a function of time.
(b) Use the graph to estimate the population in 1991.

7. If $f(x) = 2x^2 + 3x - 4$, find $f(0)$, $f(2)$, $f(\sqrt{2})$, $f(1 + \sqrt{2})$, $f(-x)$, $f(x + 1)$, $2f(x)$, and $f(2x)$.

8. If $g(x) = x^3 + 2x^2 - 3$, find $g(0)$, $g(3)$, $g(-x)$, and $g(1 + h)$.

9–17 Find the domain of the function.

9. $f(x) = \dfrac{x + 2}{x^2 - 1}$

10. $f(x) = \dfrac{x^4}{x^2 + x - 6}$

11. $g(x) = \sqrt[4]{x^2 - 6x}$

12. $h(x) = \sqrt[4]{7 - 3x}$

13. $f(t) = \sqrt[3]{t - 1}$

14. $g(x) = \sqrt{x^2 - 2x - 8}$

15. $\phi(x) = \sqrt{\dfrac{x}{\pi - x}}$

16. $\phi(x) = \sqrt{\dfrac{x^2 - 2x}{x - 1}}$

17. $f(t) = \sqrt{t^2 + 1}$

18–36 Find the domain and sketch the graph of the function.

18. $f(x) = 3 - 2x$

19. $f(x) = x^2 + 2x - 1$

20. $g(x) = \sqrt{-x}$

21. $g(x) = \sqrt{6 - 2x}$

22. $h(x) = \sqrt{x^2 - 4}$

23. $F(x) = \dfrac{1}{x}$

24. $G(x) = |x| + x$

25. $G(x) = |x| - x$

26. $H(x) = |2x|$

27. $f(x) = x/|x|$

28. $H(x) = |2x - 3|$

29. $f(x) = \dfrac{x^2 - 1}{x - 1}$

30. $f(x) = \dfrac{x^2 + 5x + 6}{x + 2}$

31. $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } x \geqslant 2 \end{cases}$

32. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ 1 & \text{if } -1 \leqslant x \leqslant 1 \\ -1 & \text{if } x > 1 \end{cases}$

33. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \leqslant x \leqslant 1 \\ 1 & \text{if } x > 1 \end{cases}$

34. $f(x) = \begin{cases} |x| & \text{if } |x| \leqslant 1 \\ 1 & \text{if } |x| > 1 \end{cases}$

35. $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leqslant 2 \\ 2x - 7 & \text{if } x > 2 \end{cases}$

36. $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ x & \text{if } 0 \leqslant x \leqslant 2 \\ \sqrt{x - 2} & \text{if } x > 2 \end{cases}$

Answers

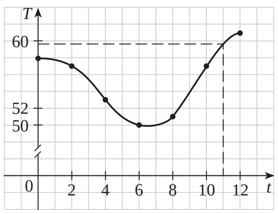
E Click here for exercises.

S Click here for solutions.

1. Yes, $[-3, 2]$, $[-2, 2]$ **2.** No

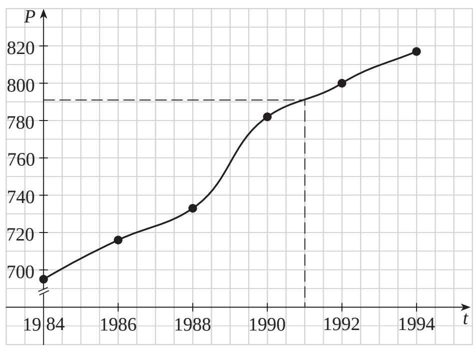
3. No **4.** Yes, $[-3, 2]$, $\{-2\} \cup (0, 3]$

5. (a)



(b) 59°F

6. (a)



(b) 791,000

7. -4, 10, $3\sqrt{2}$, $5 + 7\sqrt{2}$, $2x^2 - 3x - 4$, $2x^2 + 7x + 1$, $4x^2 + 6x - 8$, $8x^2 + 6x - 4$

8. -3, 42, $-x^3 + 2x^2 - 3$, $h^3 + 5h^2 + 7h$

9. $\{x \mid x \neq \pm 1\} = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

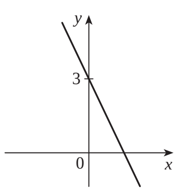
10. $\{x \mid x \neq -3, 2\} = (-\infty, -3] \cup [-3, 2] \cup [2, \infty)$

11. $\{x \mid x \leqslant 0 \text{ or } x \geqslant 6\} = (-\infty, 0] \cup [6, \infty)$

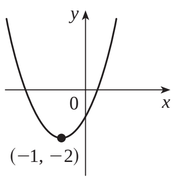
12. $\left(-\infty, \frac{7}{3}\right]$ **13.** $(-\infty, \infty)$ **14.** $(-\infty, -2] \cup [4, \infty)$

15. $[0, \pi)$ **16.** $[0, 1) \cup [2, \infty)$ **17.** $(-\infty, \infty)$

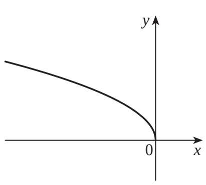
18. $(-\infty, \infty)$



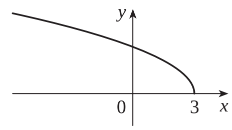
19. $(-\infty, \infty)$



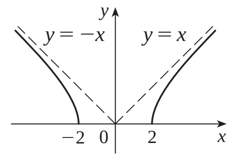
20. $(-\infty, 0]$



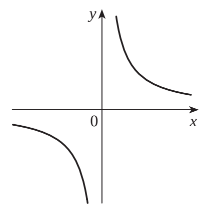
21. $(-\infty, 3]$



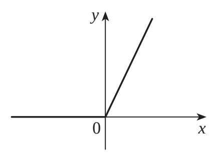
22. $(-\infty, -2] \cup [2, \infty)$



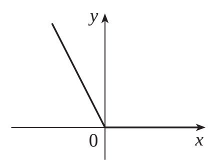
23. $\{x \mid x \neq 0\}$



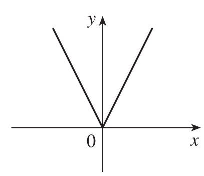
24. $(-\infty, \infty)$



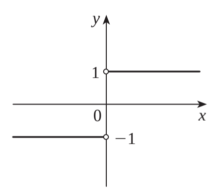
25. $(-\infty, \infty)$



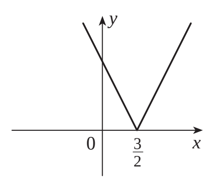
26. $(-\infty, \infty)$



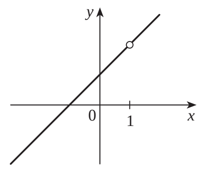
27. $(-\infty, 0) \cup (0, \infty)$



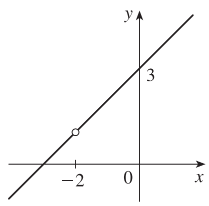
28. $(-\infty, \infty)$



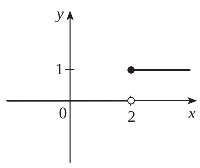
29. $(-\infty, 1) \cup (1, \infty)$



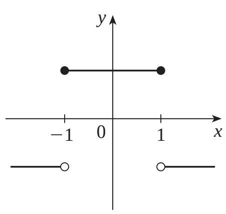
30. $(-\infty, -2) \cup (-2, \infty)$



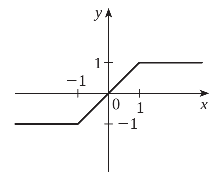
31. $(-\infty, \infty)$



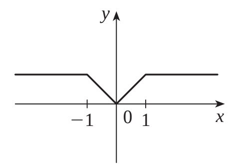
32. $(-\infty, \infty)$



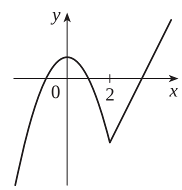
33. $(-\infty, \infty)$



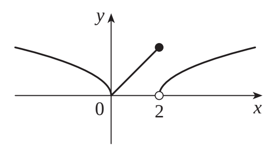
34. $(-\infty, \infty)$



35. $(-\infty, \infty)$



36. $(-\infty, \infty)$



Solutions

E Click here for exercises.

A Click here for answers.

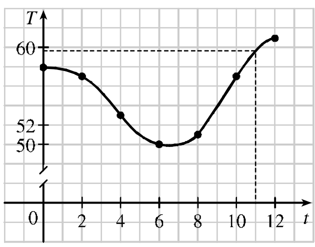
1. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-3, 2]$ and the range is $[-2, 2]$.

2. No, the curve is not the graph of a function because a vertical line intersects the curve more than once and hence, the curve fails the Vertical Line Test.

3. No, the curve is not the graph of a function since for $x = -1$ there are infinitely many points on the curve.

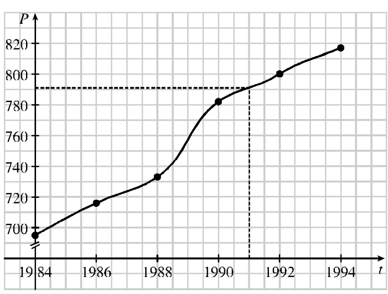
4. Yes, the curve is the graph of a function with domain $[-3, 2]$ and range $\{-2\} \cup (0, 3]$.

5. (a)



(b) $T(11) \approx 59\,^\circ\mathrm{F}$

6. (a)



(b) $P(1991) \approx 791{,}000$ people

7. $f(x) = 2x^2 + 3x - 4$, so $f(0) = 2(0)^2 + 3(0) - 4 = -4$,

$f(2) = 2(2)^2 + 3(2) - 4 = 10$,

$f(\sqrt{2}) = 2(\sqrt{2})^2 + 3(\sqrt{2}) - 4 = 3\sqrt{2}$,

$$\begin{aligned} f(1+\sqrt{2}) &= 2(1+\sqrt{2})^2 + 3(1+\sqrt{2}) - 4 \\ &= 2\left(1 + 2 + 2\sqrt{2}\right) + 3 + 3\sqrt{2} - 4 \\ &= 5 + 7\sqrt{2} \end{aligned}$$

$f(-x) = 2(-x)^2 + 3(-x) - 4 = 2x^2 - 3x - 4$,

$$\begin{aligned} f(x+1) &= 2(x+1)^2 + 3(x+1) - 4 \\ &= 2\left(x^2 + 2x + 1\right) + 3x + 3 - 4 \\ &= 2x^2 + 7x + 1 \end{aligned}$$

$2f(x) = 2\left(2x^2 + 3x - 4\right) = 4x^2 + 6x - 8$, and

$$\begin{aligned} f(2x) &= 2(2x)^2 + 3(2x) - 4 \\ &= 2\left(4x^2\right) + 6x - 4 \\ &= 8x^2 + 6x - 4 \end{aligned}$$

8. $g(x) = x^3 + 2x^2 - 3$, so $g(0) = 0^3 + 2(0)^2 - 3 = -3$,

$g(3) = 3^3 + 2(3)^2 - 3 = 42$,

$g(-x) = (-x)^3 + 2(-x)^2 - 3 = -x^3 + 2x^2 - 3$, and

$g(1+h) = (1+h)^3 + 2(1+h)^2 - 3 = h^3 + 5h^2 + 7h$.

9. $f(x) = \dfrac{x+2}{x^2-1}$ is defined for all x except when $x^2 - 1 = 0$ $\Leftrightarrow$ $x = 1$ or $x = -1$, so the domain is $\{x \mid x \neq \pm 1\}$.

10. $f(x) = x^4 / \left(x^2 + x - 6\right)$ is defined for all x except when $0 = x^2 + x - 6 = (x+3)(x-2)$ $\Leftrightarrow$ $x = -3$ or 2, so the domain is $\{x \mid x \neq -3, 2\}$.

11. $g(x) = \sqrt[4]{x^2 - 6x}$ is defined when $0 \le x^2 - 6x = x(x-6)$ $\Leftrightarrow$ $x \ge 6$ or $x \le 0$, so the domain is $(-\infty, 0] \cup [6, \infty)$.

12. $h(x) = \sqrt[4]{7 - 3x}$ is defined when $7 - 3x \ge 0$ or $x \le \frac{7}{3}$, so the domain is $\left(-\infty, \frac{7}{3}\right]$.

13. $f(t) = \sqrt[3]{t-1}$ is defined for every t, since every real number has a cube root. The domain is the set of all real numbers, $\mathbb{R}$.

14. $g(x) = \sqrt{x^2 - 2x - 8}$ is defined when $0 \le x^2 - 2x - 8 = (x-4)(x+2)$ $\Leftrightarrow$ $x \ge 4$ or $x \le -2$, so the domain is $(-\infty, -2] \cup [4, \infty)$.

15. $\phi(x) = \sqrt{\dfrac{x}{\pi - x}}$ is defined when $\dfrac{x}{\pi - x} \ge 0$. So either $x \le 0$ and $\pi - x < 0$ ($\Leftrightarrow$ $x > \pi$), which is impossible, or $x \ge 0$ and $\pi - x > 0$ ($\Leftrightarrow$ $x < \pi$), and so the domain is $[0, \pi)$.

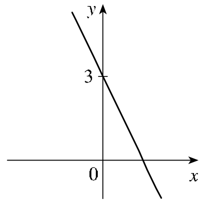
16. $\phi(x) = \sqrt{\dfrac{x^2 - 2x}{x - 1}}$ is defined when $0 \le \dfrac{x^2 - 2x}{x-1} = \dfrac{x(x-2)}{x-1}$. Constructing a table:

Interval	x	$x-1$	$x-2$	$x(x-2)/(x-1)$
$x < 0$	$-$	$-$	$-$	$-$
$0 < x < 1$	$+$	$-$	$-$	$+$
$1 < x < 2$	$+$	$+$	$-$	$-$
$x > 2$	$+$	$+$	$+$	$+$

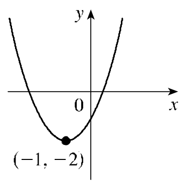
So the domain is $[0, 1) \cup [2, \infty)$.

17. $f(t) = \sqrt{t^2 + 1}$ is defined for every t, since $t^2 + 1$ is always positive. The domain is the set of all real numbers.

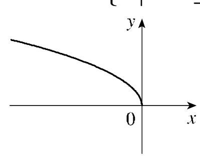
18. $f(x) = 3 - 2x$. Domain is $\mathbb{R}$.



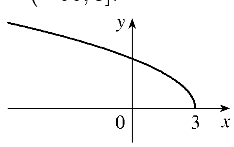
19. $f(x) = x^2 + 2x - 1 = (x^2 + 2x + 1) - 2 = (x+1)^2 - 2$, so the graph is a parabola with vertex at $(-1, -2)$. The domain is $\mathbb{R}$.



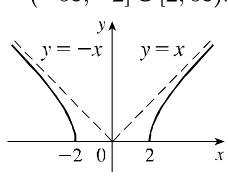
20. $g(x) = \sqrt{-x}$. The domain is $\{x \mid -x \geq 0\} = (-\infty, 0]$.



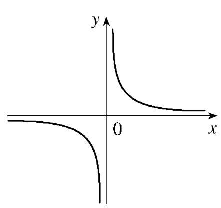
21. $g(x) = \sqrt{6 - 2x}$. The domain is $\{x \mid 6 - 2x \geq 0\} = (-\infty, 3]$.



22. $h(x) = \sqrt{x^2 - 4}$. Now $y = \sqrt{x^2 - 4} \;\Rightarrow\; y^2 = x^2 - 4 \Leftrightarrow \; x^2 - y^2 = 4$, so the graph is the top half of a hyperbola. The domain is $\{x \mid x^2 - 4 \geq 0\} = (-\infty, -2] \cup [2, \infty)$.



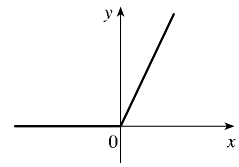
23. $F(x) = \dfrac{1}{x}$. The domain is $\{x \mid x \neq 0\}$.



24. $G(x) = |x| + x$. Since $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ we have

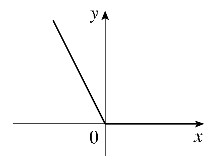
$$G(x) = \left\{ \begin{array}{ll} x + x & \text{if } x \geq 0 \\ -x + x & \text{if } x < 0 \end{array} \right\} = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Domain is $\mathbb{R}$. Note that the negative x-axis is part of the graph of G.



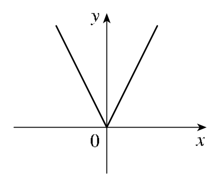
25. $G(x) = |x| - x = \begin{cases} 0 & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$

Domain is $\mathbb{R}$.



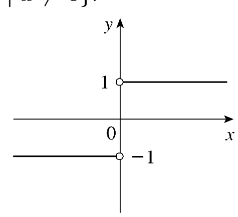
26. $H(x) = |2x| = \left\{ \begin{array}{ll} 2x & \text{if } 2x \geq 0 \\ -2x & \text{if } 2x < 0 \end{array} \right\} = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$

Domain is $\mathbb{R}$.



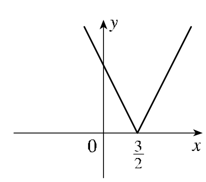
27. $f(x) = \dfrac{x}{|x|} = \left\{ \begin{array}{ll} x/x & \text{if } x > 0 \\ x/(-x) & \text{if } x < 0 \end{array} \right\} = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$

Note that we did not use $x \geq 0$, because $x \neq 0$. Hence, the domain of f is $\{x \mid x \neq 0\}$.

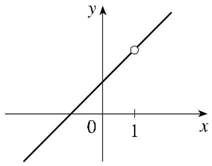


28. $H(x) = |2x - 3| = \begin{cases} 2x - 3 & \text{if } x \geq \frac{3}{2} \\ 3 - 2x & \text{if } x < \frac{3}{2} \end{cases}$

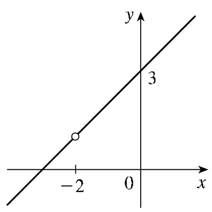
Domain is $\mathbb{R}$.



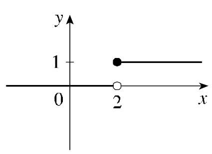
29. $f(x) = \dfrac{x^2 - 1}{x - 1} = \dfrac{(x+1)(x-1)}{x-1}$, so for $x \neq 1$, $f(x) = x + 1$. Domain is $\{x \mid x \neq 1\}$.



30. $f(x) = \dfrac{x^2 + 5x + 6}{x + 2} = \dfrac{(x+3)(x+2)}{x+2}$, so for $x \neq -2$, $f(x) = x + 3$. Domain is $\{x \mid x \neq -2\}$. The hole in the graph can be found using the simplified function, $h(x) = x + 3$. $h(-2) = 1$ indicates that the hole has coordinates $(-2, 1)$.

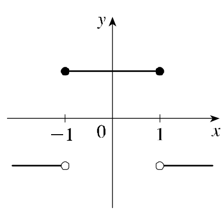


31. $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$ Domain is $\mathbb{R}$.

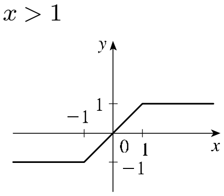


32. $f(x) = \begin{cases} 1 & \text{if } -1 \leq x \leq 1 \\ -1 & \text{if } x > 1 \text{ or } x < -1 \end{cases}$

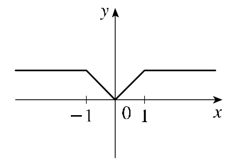
Domain is $\mathbb{R}$.



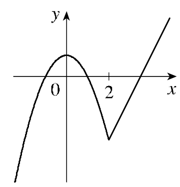
33. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$ Domain is $\mathbb{R}$.



34. $f(x) = \begin{cases} |x| & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \text{ or } x < -1 \end{cases}$ Domain is $\mathbb{R}$.



35. $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 2 \\ 2x - 7 & \text{if } x > 2 \end{cases}$ Domain is $\mathbb{R}$.



36. $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 2 \\ \sqrt{x - 2} & \text{if } x > 2 \end{cases}$ Domain is $\mathbb{R}$.